MURARI

FM is branch of science which deals with property of fluids (liquids and gases), behaviour of fluids at Rest, under motion with and without forces causing motion.

App of FM

1. Hydraulic Analysis of turbine, pumps, Hydraulic equipment Flow Analysis in pipes and Hydraulic loses in pipes for Optimisation of the pipe size etc.

unit I

1. Properties of Fluids
2. Fluid statics
3. Fluid kinematics
4. Fluid dynamics
5. Laminar Flow of Incompressible fluids through pipes and in between plates
6. Turbulent flow thru pipes
7. Boundary Layer theory concepts
8. Turbo machinery (Hydraulic turbines)

PROPERTIES OF FLUIDS

→ Property of fluids

→ Newton's law of viscosity

→ Compressibility and Bulk modulus

... intensity of fluid and vapour pressure

→ classification of fluid Based on fluid power Law

[Rhe logical eqn of Fluids]

Properties

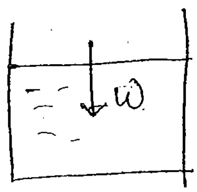
there are 4 properties

1. Mass of the motter
2. weight of the fluid
3. Area
4. Volume.
5. Specific mass [mass density]
6. weight density [unit weight or sp weight]
7. Specific Gravity or Relative density
8. Specific volume
9. Dynomic viscosity
10. kinematic viscosity
11. compressibility of liquids [Bulk Modulus]
12. Surface Tension [surface Energy]
13. Pressure Intensity
14. Vapour pressure

Weight

A force due to gravitational pull.

$$W = mg$$



Newton's 2nd Law

$F = ma$

$1N = 1kg \times 1\frac{m}{sec^2}$

$N = kg\,m/sec^2$

$W = mg$

$kg.F = m \times 9.81$

$kg\left[\frac{m}{sec^2}\right]$

$$\boxed{kgf = 9.81\,N}$$

$= 10N$

3. Area

Area is the space occupied by the Matter in 2D

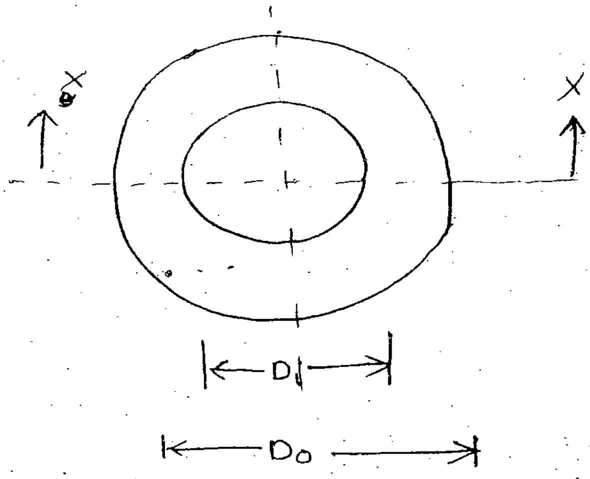
- Normal Area
- c/s Area
- plane Area
- Resisting Area
- Surface Area
- shear Area
- crushing Area
- tearing Area
- projected Area
- Flow Area

~~[illegible]~~

Normal Area

without touching the body

2 dimensions visible



$A_N = \frac{\pi}{4}\left[D_o^2 - D_i^2\right]$

$A_{c/s} = \frac{\pi}{4} \times d^2 \times 2$

Flow Area

Depending on the fluid toching the sides

Volume

It is the space occupied by the matter in 3D

units

m^3, mm^3, cm^3

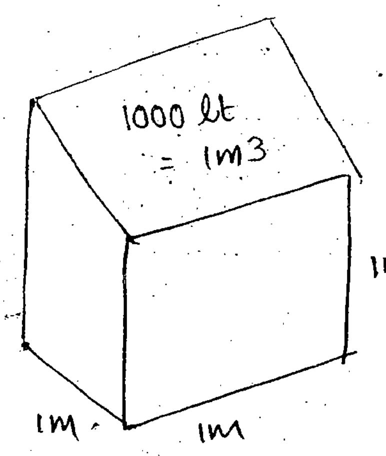
$V = (A \times l)$

$V = l \times b \times h$

→ litres (lt)

For water

$1 m^3 = 1000$ lit



$1000\ cm^3 = 1\ lit$

$cm^3 = \frac{1}{1000}\ lit$

$= 10^{-3}\ lit$

$=$ milli lit

$$\boxed{cm^3 = ml}$$

Specific Mass or Mass density

- Property of matter

- $\frac{\text{matter occupied}}{1\ \text{volume}}$

$$\rho = \frac{m}{v} \left[\frac{kg}{m^3}\right]$$

Ex:

$\rho_{air} = 1.2\ kg/m^3$

$\rho_{water} = 1000\ kg/m^3$

$\rho_{Hg} = 13600\ kg/m^3$

$\rho_{gold} : 19200\ kg/m^3$

$\rho_{Pt} : 22500\ kg/m^2$

↘ Highest

$\rho_{oil} < \rho$

1) A steel plate 10 mm thick, 1m width and length 2m. cost of 1 tonne steel is Rs 30,000. Estimate cost of plate

$$\text{Volume} = \cancel{10}\ 0.01 \times 1 \times 2$$

$$= \underline{0.02\,m^3}$$

$$= \cancel{20}\ \cancel{20m}$$

$$\rho = \frac{m}{v}$$

$$7850 \left(\frac{kg}{m^3}\right) = \frac{m\ (kg)}{\cancel{20}\ 0.02}$$

$$m = 7850 \times \cancel{20}\ 0.02$$

$$\underline{157\,kg}$$

1 Ton = 1000 kg

~~1 kg =~~

1000 kg = 30,000

1 kg = 30 Rs

Cost = 157×30

= 4710 Rs

2) A water ~~tank~~ sump of inner dimensions 6 ft, 10 ft, 5 ft, Estimate how many lit of

4

~~Volume~~: ~~10~~ A family consists 4 people and Consumption per Head 150 lit/day. How many day the sump can serve ?

Volume

$$\rho_w = \frac{m_w}{\text{Volume}}$$

Volume =

$$1000 = \frac{m}{5ft \times 6ft \times 10ft}$$

1 ft = 12 In

Inch = 25.4 mm

= 12x25.4 mm

$= \frac{12 \times 25.4}{1000}$

= 0.3048

$$m = \frac{1000 \times (5\times6\times10)\,ft^3}{0\cancel{.3048 \times 1000}}$$

$$= \underset{kg/\cancel{m^3}}{1000} \times \frac{3000}{35} (\cancel{m^3})$$

~~m = 304.8 kg~~

8570 kg = 8750 lt

$$\boxed{35\,ft^3 = 1\,m^3}$$

kg →

$$\frac{8750}{600} = \underline{15\ days}$$

<u>weight density</u> or sp weight -

It is the weight of the matter occupied per unit volume.

$$\frac{\text{weight}}{\text{Volume}} = \frac{w}{V} = \frac{N}{m^3}$$

$$\boxed{w = \gamma = \rho g \ (N/m^3)}$$

$$\gamma_{water} = \rho_{water} \times g$$

$$= 1000 \times 9.81$$

$$= 9810 \ \frac{N}{m^3}$$

$$= \cancel{98} \ 9.81 \ \frac{kN}{m^3}$$

Relative density or Sp gravity $[S]$

It is the Ratio of mass density of any matter
" " " std fluid - → water

$$\boxed{S = R.D = \frac{\rho_x}{\rho_{water}}}$$

$$S_{steel} = \frac{\rho_{steel}}{\rho_{water}} = \frac{7850}{1000} = 7.85$$

ex: $S_{mercury} = \frac{\rho_{mercury}}{\rho_{water}} = \frac{13600}{1000}$

$$= 13.6$$

$$S_{air} = \frac{\rho_{air}}{\rho} = \frac{1.2}{1000} = 0.001$$

Specific volume [Vs]

Volume of matter for unit mass.

→ very useful for gases

$$\vartheta_s = \frac{\text{Volume}}{\text{Mass}} \quad \frac{m^3}{kg}$$

$$= \frac{V}{m} = \frac{1}{\rho}$$

$$V_{s\,water} = \frac{1}{1000} = 0.001 \frac{m^3}{kg}$$

$$V_{s\,air} = \frac{1}{1.2} = 0.8733 \frac{m^3}{kg}$$

Q) 1 lit of an oil weighing 7N Det

1. mass of oil in kg
2. vol of oil in m3
3. Mass density of oil in kg/m3, g/cc, g/m
4. sp weight of oil in $\frac{kN}{m^3}$
5. sp gravity of oil
6. sp vol of oil. m3/kg, $\frac{lit}{kg}$, $\frac{ml}{g}$

$w = 7N$

$w = mg$

$7 = m \times 9.81$

$m = \frac{7}{} = 0.713 \text{ kg}$

$$\rho = \frac{m}{volume}$$

$Vol = \frac{0.713}{1 lit}$ ~~Vol =~~ ~~0.713~~ ~~1 lit~~

Vol in m3

$= 10^{-3}$ m3

$1 m^3 = 1000$ lit

Vol = ~~0.713 lit~~

= ~~0.713×10^{-3} m3~~

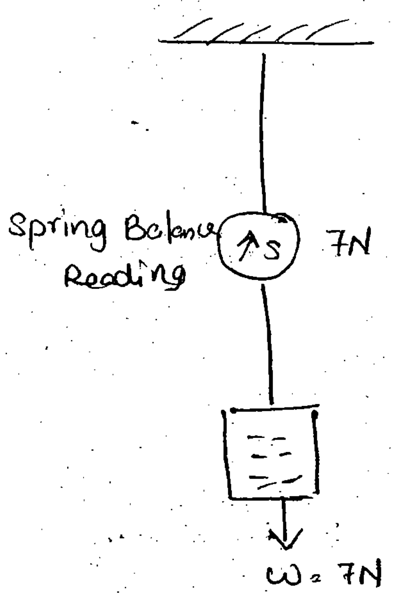
3. Mass density

$$\rho = \frac{m}{v}$$

$$= \frac{0.713}{10^{-3}}$$

$$= 0.713 \times 10^3 \frac{kg}{m3}$$

PHYSICAL REPRESENTATION



in g/cc : $0.713 \times 10^6 \frac{g}{m3}$

$$= \frac{0.713 \times 10^6}{10^6}$$

$$= 0.714 \text{ g/cc}$$

in g/ml = $\frac{0.713 \times 10^3 \times 10^3}{10^6 \text{ ml}}$

4) <u>weight density</u> (γ)

$$\gamma = \frac{W}{\text{Volume}} = \rho g$$

$$= 714 \times 9.81$$

$$= 7000 \text{ N/m}^3$$

5) <u>Specific Gravity</u> (S)

$$S_{oil} = \frac{\rho_{oil}}{\rho_{water}} = \frac{714}{1000} \frac{(\cancel{kg/m^3})}{(\cancel{kg/m^3})}$$

$$= 0.714$$

6) <u>Sp volume</u> (V_s)

$$V_s = \frac{V_{oil}}{m_{oil}} = \frac{1}{\rho_{oil}}$$

$$= \frac{1}{714} = 1.400 \times 10^{-3} \frac{m^3}{kg}$$

$$= 1.4 \frac{lt}{kg}$$

<u>Viscosity</u>

It is a property of fluids [liquids and gases only] which offers resistance to the flow of fluids.

forces and adhesive forces]

Cohesive – attraction b/w same molecules

adhesive – attracting with other molecules

ie water with oil etc.

mercury – cohesive forces

It never wets any substance.

– calibevation fluid.

–

$$\boxed{\frac{\text{Energy}}{\text{unit weight}} \quad \frac{Nm}{N} = m}$$

$\frac{100\ J}{1000\ N}$ $\frac{Nm}{}$

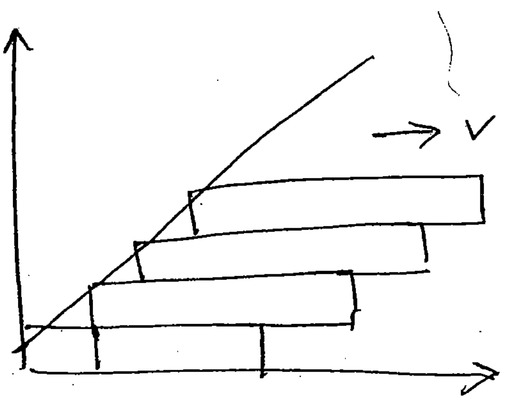
$= 100m$ ✗

– Imp for pump selection.

lift against gravity

NEWTON'S LAW OF VISCOSITY.

$\tau = \left(\frac{F}{A}\right)$



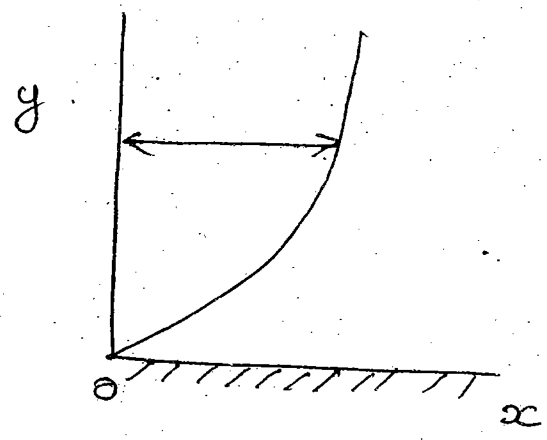
$\tau \propto \left(\frac{du}{dy}\right)$ → vel graidient

$$\boxed{\tau = \mu \frac{du}{dy}}$$

μ = dynamic viscosity

$$\boxed{\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}}$$

→ vel Gradient or Rate of shear strain.



~~SHEAR STRESS~~

[SHEAR FORCE IS MEASURED BY FRUSTUM TUBE]

→ Shearstress on fluid layer $\propto$ vel gradient [Rate of deformation or Rate of shear strain]

<u>unit</u> $= \dfrac{\frac{N}{m^2}}{\frac{\cancel{m}/sec}{\cancel{m}}} = \dfrac{Ns}{m^2}$

$$\mu = \frac{\text{Force}}{\text{Area}} \cdot \text{Time}$$

Dynamic viscosity - study under forces.

<u>KINEMATIC VISCOSITY</u>

$$\vartheta = \frac{\mu}{\rho}$$

Ratio of dynamic viscosity to density

No force involved

$$\nu = \frac{Ns}{\cancel{m^2} \times kg} \times m^{\cancel{3}}$$

$$N = kg\frac{m}{s^2}$$

$$m \cdot \frac{Ns}{kg} = \frac{\cancel{kg}\frac{m}{s^{\cancel{2}}} \times \cancel{s} \cdot m}{\cancel{kg}}$$

$$\boxed{\text{Unit} = \frac{m^2}{sec}}$$

Ex:

Water

$$\mu_{water} = \frac{1}{1000}\ \frac{N\ sec}{m^2}$$

$$= 1 \times 10^{-3}\ \frac{Ns}{m^2}$$

$$\nu = \frac{\frac{1}{1000}}{1000}\ m^2/sec$$

$$= 1 \times 10^{-6}\ \frac{Ns}{m^2}$$

AIR

$$\mu_{AIR} = 1.8 \times 10^{-5}\ \frac{N}{m^2}\ sec$$

$$18 \times 10^{-6}\ \frac{N}{m^2}\ sec$$

[18 yrs 10 lakhs]

$$\nu_{AIR} = \frac{\mu}{\rho} = \frac{18 \times 10^{-6}}{1.2}$$

$$= 1.5 \times 10^{-5}\ \frac{m^2}{sec}$$

$$= 15 \times 10^{-6}\ \frac{m^2}{sec}$$

(1) $\mu_{water} > \mu_{AIR}$

$\nu_{WATER} < \nu_{AIR}$

NEWTONS LAW OF VISCOSITY

made into 5 models.

$$\tau = \mu \frac{du}{dy}$$

$$F = \mu A \frac{du}{dy}$$

A – surface Area

Model 1

An incompressible fluid of sp gravity (S) 0.8 and kinematic viscosity $7.4 \times 10^{-7} \frac{m^2}{s}$ is subjected to tensional force on 1unit Area. the fluid is moving b/w a fixed plates of gap 1mm with velocity $30 \frac{m}{min}$. Estimate the τ caused on the plate?

$$\tau = \mu \frac{du}{dy}$$

$S = 0.8$

$\nu = \frac{\mu}{\rho}$

$0.8 = \frac{\rho_{oil}}{\rho_{water}}$

$\mu = 7.4 \times 10^{-7} \times 800$

$\mu = \nu \times \rho$

$dA = 1m^2$.

$\rho = 0.8 \times 1000$

$dy = 1mm = 10^{-3} m$

$= 800 \frac{kg}{m^3}$

$V = \frac{30}{60} = 0.5 m/s$.

$$Z = \frac{7.4\times10^{-7}\times0.5\times800}{10^{-3}}$$

$$Z = \cancel{3.7\times10^{-4}\,N/m^2}\; 0.296\,N/m^2$$

A Newtonian filled b/w a shaft and Sleeve Bearing. A force 800N is applied on shaft. The shaft gains 1.5 cm/s velocity. If force is increased to 2.4 kN then new speed will be

a) 1.5 cm/s

b) 13.5 cm/s

c) 0.5 cm/s

d) 4.5 cm/s

Case1

$F = 800N$

$V = 1.5\,\frac{cm}{s} = \cancel{1.5\times10^{-2}\,\frac{m}{s}}\; 1.5\,\frac{cm}{s}$

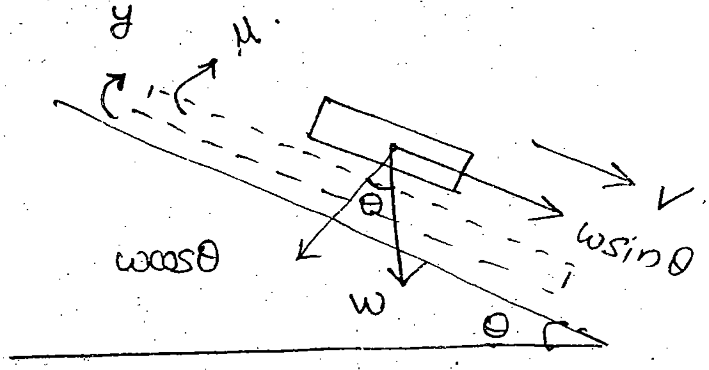
$\tau = \mu \frac{du}{dy}$

$\frac{F}{A} \propto \frac{du}{dy}$

$\frac{F_1}{F_2} = \frac{V_1}{V_2}$

$\frac{800}{2400} = \frac{1.5}{V_2}$ $\quad V_2 = \frac{2400\times1.5}{800}$

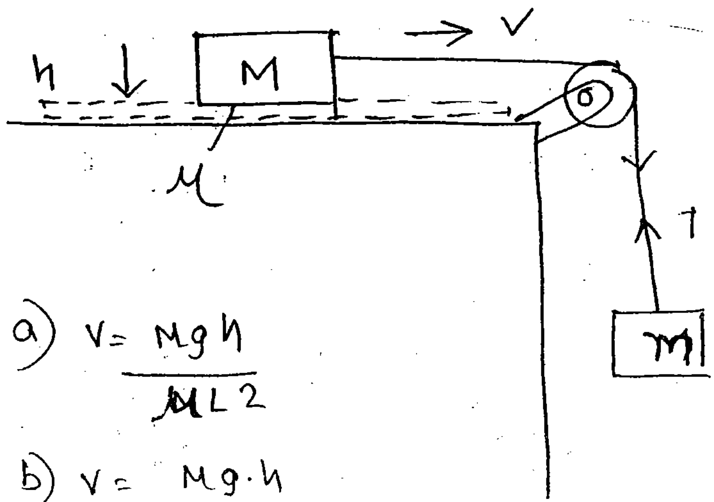
Inclined plane Model



$$F = \mu A \cdot \frac{V}{y}$$

$$\boxed{w\sin\theta = \mu \cdot A \cdot \frac{V}{y}}$$

Q.) A cubic Block of each side 'L' and mass 'm' is pulled over an oil film across a Table shown in fig. A small mass 'm' is connected to a string which passes over a smooth pulley. what is the steady velocity of the Block (M) against the Newtonian fluid resisting Force.



$\Sigma F = ma$

$= m \times 0$

$= 0$

a) $V = \frac{Mgh}{\mu L^2}$

b) $V = \frac{Mg \cdot h}{\mu}$

d) $V = mgh$

$$F = \mu A \times \frac{V}{y}$$

$$T = \mu \times L^2 \times \frac{V}{h}$$

Tension

Dalemberts principle

$$\Sigma F = ma$$

$$mg - T = 0$$

$$\underline{\underline{T = mg}}$$

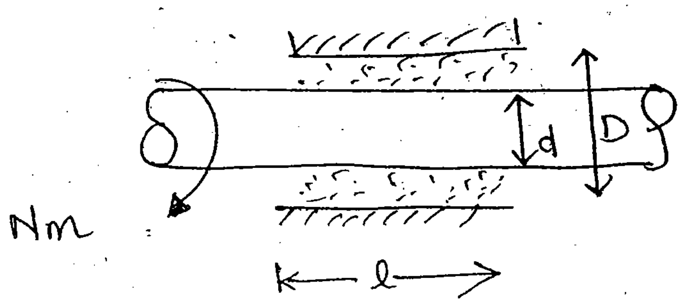
$$mg = \mu \times L^2 \times \frac{V}{h}$$

$$\boxed{V = \frac{mgh}{\mu L^2}}$$

Model-III

Power lost in viscosity

Subjected to a Torque



T = Torque

N = Speed in rpm

$$\text{Power Lost} = \frac{2\pi N T}{60}$$

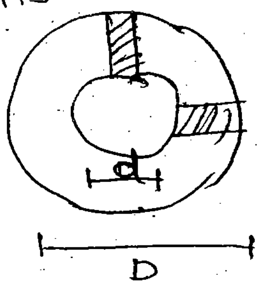
$$T = F \times \frac{d}{}$$

surface area

$$T = \mu A \frac{v}{y} \cdot \frac{d}{2} \quad (Nm)$$

$$T = \frac{\mu \times \pi d \times l \times \left(\frac{\pi d N}{60}\right)}{\left(\frac{D-d}{2}\right)} \cdot \frac{d}{2}$$

$$= \frac{2\pi N}{60}\left[\mu \cdot \pi d l \cdot \frac{\frac{\pi d N}{60}}{\left(\frac{D-d}{2}\right)}\right] \cdot \frac{d}{2} \quad \text{watts}$$



Radial gap is only read .

A loaded Bearing has shaft dia 50 mm, Bush length 20 mm & Bush Bore 50.05 mm Subjected to 1200 rpm with a Newtonian Fluid of viscosity 0.03 Pa sec. what is the power lost due to viscosity in watts ?

a) 37 watt
b) 74
c) 118
d) 237

$$P = \frac{2\pi N T}{60}$$

$D = 50.05$

$d = 50$

$\frac{D-d}{2} = 0.025$

$= 2 \times \pi \times 1200 \cdot \times$

$\mu = 0.03 \frac{N}{m^2} sec.$

$$F = \mu A \frac{v}{y}$$

$$V = \frac{\pi d N}{60} = \frac{\pi \times 50 \times 120}{60}$$

$V = 3141.59\,r$

$$T = F \times \frac{d}{2} = \mu A \frac{v}{y} \cdot \frac{d}{2}$$

$\pi \times r$

$2 \times \pi \times 1200 \times 0.03 \times 3141.59 \times 50$

$$\frac{\quad}{sec}$$

J/sec² · rad

= watts · rad = $\underline{\text{watts}}$

Points:-

Fluids at Rest: Fluid statics

Fluids in motion: Fluid kinematics

Here pressure forces not considered

Fluid dynamics:

study of Fluids in motion when pressure forces are also considered.

Density:-

For liquids ρ = const

$$\rho = \frac{m}{V}$$

$\rho_{WATER} = 1000 \, kg/m^3$
$= 1 \, gm/cm^3$

$$\frac{10^3 \times 10^3}{10^6}$$

Specific weight or weight density

$$w = \gamma = \frac{\text{Weight of fluid}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid} \times g}{\text{Vol of fluid}} = \rho g$$

Specific volume:- (Vs)

$$V_s = \frac{Vol}{Mass} = \frac{1}{\rho}$$

unit $\frac{m^3}{kg}$ [Applicable to Gases]

Specific Gravity

$$S = \frac{\rho_{liq}}{\rho_{std\ liquid}}$$

std liquid → water

$$S = \frac{\rho_{liq}}{1000}$$

$$S = \frac{\text{weight density of liquid}}{\text{weigh density of water}}$$

For mercury $S = 13.6$

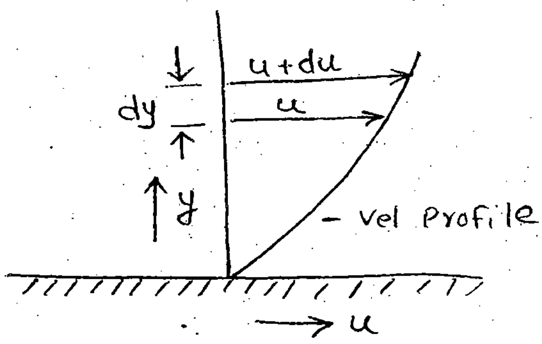
Viscosity

dy = dist b/w layers

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

Rate of shear strain or vel Gradient



Unit

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{N/m^2}{\frac{\cancel{m}}{s} \times \frac{1}{\cancel{m}}} = \frac{Ns}{m^2} \text{ - SI unit}$$

MKS unit. $\mu = \frac{\text{kgf} \cdot \text{sec}}{m^2}$

CGS unit $\mu = \frac{\text{dyne} \cdot \text{sec}}{cm^2} = \text{Poise}$

$$\boxed{1 \text{ POISE} = \frac{1}{10} \frac{Ns}{m^2}}$$

$\frac{1}{CP}$ = $1 CP = \frac{1}{100}$ POISE

KINEMATIC VISCOSITY

$$\nu = \frac{\text{viscosity}}{\rho} = \frac{\mu}{\rho} = \frac{Ns \times m^3}{m^2 \times kg}$$

$$= \frac{\cancel{kg}}{\cancel{m^2}} \frac{m}{s^2} \times \cancel{s} \times \frac{m^3}{\cancel{kg}}$$

$$= \frac{m^2}{s}$$

CGS unit - STOKE - cm^2/s

$$\boxed{1\,\text{STOKE} = 10^{-4}\,\frac{m^2}{s}}$$

$$1\,CS = \frac{1}{100}\,\text{STOKE}$$

FLUIDS

NEWTONIAN

OBEY NEWTONS LAW OF VISCOSITY

NON NEWTONIAN

DOESN'T OBEY NEWTON'S LAW OF VISCOSITY

VARIATION OF VISCOSITY WITH TEMPERATURE

VISCOSITY ↓ WITH ↑ in TEMP

$$\mu = \mu_0\left[\frac{1}{1+\alpha t+\beta t^2}\right]$$

μ_0 = viscosity of liq. at 0°c in poise

μ = " " " at t°c in poise

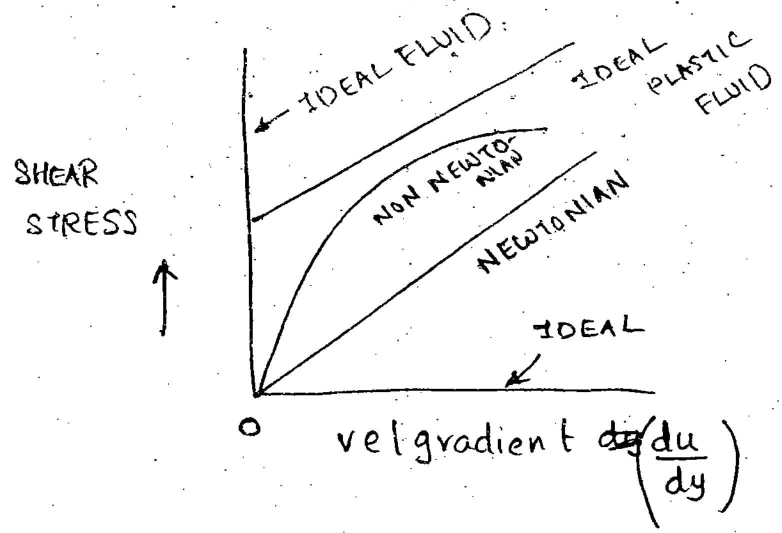
α, β = constants

$$\mu_{air} = 18\times10^{-6}\,\frac{Ns}{m^2}$$

$$\mu_{water} = \frac{1}{1000}\,\frac{Ns}{m^2}$$

Fluid types

1. Ideal Fluid : Incompressible, No viscosity, only Imaginary
2. Real " - Posess viscosity, All fluids in Practice are real.
3. Newtonian " - obeys Newton law of viscosity
4. Non Newtonian - doesn't obey Newton's Law of viscosity



PROBLEMS

HN TEXT 1.3

$u = \frac{2}{3}y - y^2$

u = velocity in m/s

$\tau = ?$

At $y = 0$, $y = 0.15$ m

$\mu = 8.63$ Poise $= \frac{8.63}{10}\frac{Ns}{m^2} = 0.863\frac{Ns}{m^2}$

we have $\tau = \mu\left[\frac{du}{dy}\right]$

$\frac{du}{dy} = \frac{2}{3} - 2y$

At $y = 0$ $\quad \frac{du}{dy} = \frac{2}{3}$

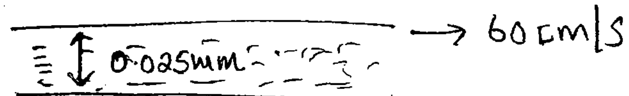
$\therefore \tau = 0.863 \times \frac{2}{3} = 0.5756\ N/m^2$

At $y = 0.15$ $\quad \frac{du}{dy} = \frac{2}{3} - 2 \times 0.15$

$= 0.367$

$\therefore \tau = 0.863 \times 0.367 = 0.3167\ N/m^2$

1.4



$\tau = \frac{2N}{m^2} \qquad \mu = ? \qquad V = 60$ cm/s $= 0.6$ m/s

dy = 0.025 mm

$$\tau = \mu \left(\frac{du}{dy}\right)$$

$$\frac{F}{A} = \mu \left(\frac{du}{dy}\right), \quad 2\frac{N}{m^2} = \mu \left[\frac{0.6}{0.025\times10^{-3}}\right]$$

$$\therefore \mu = 8.33\times10^{-5}\ \frac{Ns}{m^2}$$

(1.5) $A = 1.5\times10^{6}\ mm^2 = 1.5\times\cancel{10^3}\ m^2$

$du = 0.4\ m/s$

$dy = 0.15\ mm = 0.15\times10^{-3}\ m$

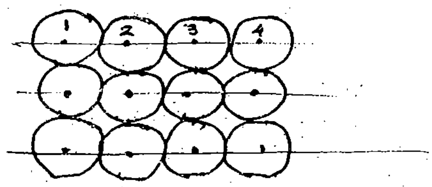
J - Energy

$$W = F\times S$$

$$N\times m$$

$$J$$

Model IV



Fluids made of sphere molecule centre doesn't chang

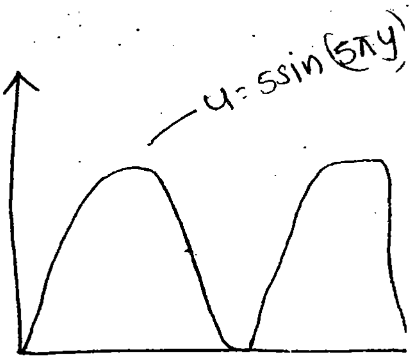
Q.) The velocity distribution in a laminar flow at given section is given by $u = \cancel{\sin y}\ 5\sin(5\pi y)$ where u is in m/s and y is in m. The viscosity of the fluid is 0.5 Pa s. Then the shear stress at $y = 5$ cm ~~as~~ in N/m² is

(a) 39.27 N/m² (b) 27.76 N/m² (c) 38.9 N/m² (d) None

Ans:

$$\tau = \mu \cdot \left[\frac{du}{dy}\right] \qquad \mu = 0.5\,\frac{Ns}{m^2}$$

$$u = 5\sin(5\pi y)$$



$$\frac{du}{dy} = 5 \times \cos(5\pi y) \times 5\pi$$

$$= 25\cos(5\pi y) \times \pi$$

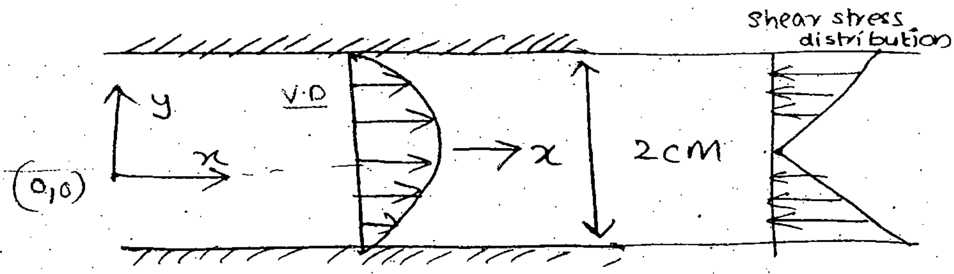
$$\tau = 0.5 \times 25\cos(5\pi y) \times \pi$$

in Radians

$$\tau\Big]_{y=0.05} = 0.5 \times 25 \times \cos(5 \times \pi \times 0.05) \times \pi$$

$$= \cancel{\ \ } \quad 27.768\ N/m^2$$

4) The velocity profile for Newtonian fluid in Laminar flow b/w 2 ||el plates shown in fig is given by $u = 0.01\left[1 - 10000y^2\right]$ where u is in m/s and y is in m.



$$\frac{du}{dy} = 0.01\left[-20{,}000y\right]$$

The viscosity of fluid is assumed to be water whose value is 1×10^{-3} Pas.

Then shear stress on the plates will be.

origin specified

$$\tau = \mu\left[\frac{du}{dy}\right]$$

$y = 1cm$

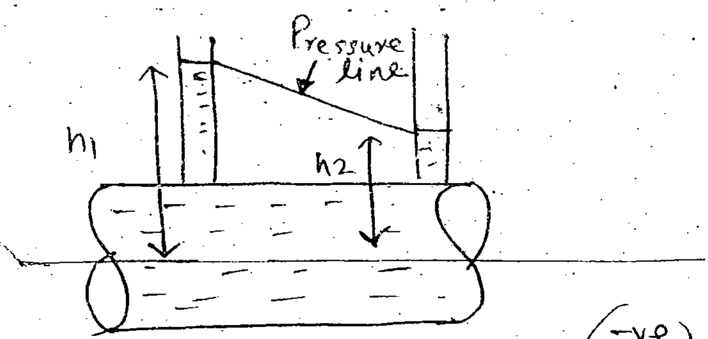
$$\tau\Big]_{y=1} = 10^{-3} \times 0.01\left[-20{,}000 \times 1\times10^{-2}\right]$$

$$\underline{\tau = 0.002 \, N/m^2}$$

NOTE!

In Laminar flow velocity distribution is parabolic

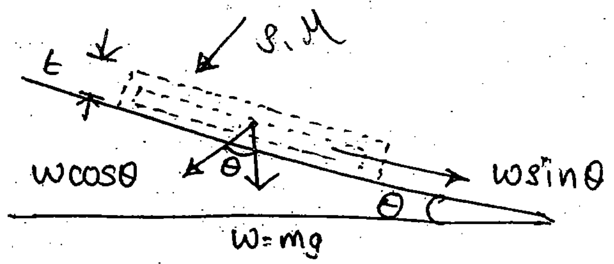
shear stress distribution is linear.



$$Pa = \frac{N}{m^2}$$

(−ve) $\frac{dp}{dx}$

Q.) A liquid of density ρ, dynamic viscosity μ moves steadily down the inclined plane in a thin sheet of const thickness 't'. Neglect friction, and the shear stress at the bottom of liquid surface due to flow parallel to the inclined plane will be



a) $\tau = \rho g t \sin\theta$ b) $\tau = \rho g t \cos\theta$ c) $\tau = \rho g \tan\theta$

d) $\tau = \mu \sqrt{\frac{g}{t}}$

$$\tau = \mu \times \left[\frac{du}{dy}\right]$$

~~μ neglected~~

$$\tau = \frac{F}{\text{Area}}$$ — surface area

$$\tau = \frac{w\sin\theta}{A} = \frac{mg\sin\theta}{A}$$

$$\rho = \frac{m}{v}$$

$$= \frac{\rho v g \sin\theta}{A}$$

$$\frac{\rho \times (\cancel{A_s} \times t) g\sin\theta}{\cancel{A_s}}$$

$$\rho t g \sin\theta$$

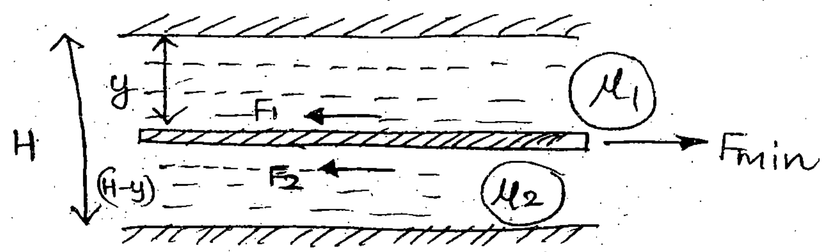
$$\frac{\rho \times (A_s \times t) \times g \sin\theta}{A}$$

$= \rho g t \sin\theta$

ⓐ

V. Multi fluid Flow

(1)



Q.) A Thin plate is placed b/w 2 flat surfaces which are 'H' metres apart such that the viscosity of liquids on the Top and Bottom of plates are μ_1 and μ_2 respectively. what is the position of the thin plate Such that the viscous resistance to have the uniform motion velocity 'V' of the plate is minimum.

a) $\dfrac{y}{H-y} = \sqrt{\dfrac{\mu_1}{\mu_2}}$ b) $\dfrac{H-y}{y} = \sqrt{\dfrac{\mu_1}{\mu_2}}$

c) $\dfrac{y}{H} = \sqrt{\dfrac{\mu_1}{\mu_2}}$ d) $\dfrac{y}{H} = \sqrt{\dfrac{\mu_2}{\mu_1}}$

$F = F_1 + F_2$

$= \mu_1 A_1 \dfrac{V}{y} + \mu_2 A_2 \cdot \dfrac{V}{(H-y)}$

to minimise F

$\dfrac{dF}{dy} = 0$

$\mu_1 A V \left[\dfrac{-1}{y^2}\right] + \mu_2 A V \left[\dfrac{1 \times 1}{(H-y)^2}\right]$

At 20°C $\quad \nu = \frac{\mu}{\rho} = 1.6\times10^{-5}$

AIR – GAS

At 70°C = $\quad \mu \uparrow =$

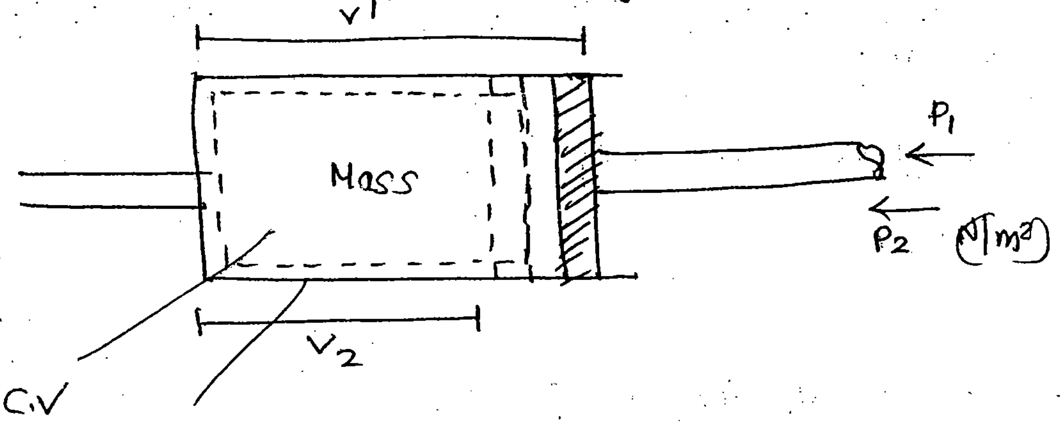
so when temp ↑ viscosity ↑

so $\underline{\nu > 1.6\times10^{-5}}$

So (a)

→ **Compressibility and Bulk Modulus of elasticity.**

In Case of Hydraulic systems the working fluid is incompressible (ρ = const) fluid [water, oils, Mercury etc] subjected to the external pressures causing the change in volumetric strain [decreased] or increased density strain due to the fluid behaviour compressibility as shown in fig



...es pressure due to ↓ in ...cosity of oil

C.S $\qquad dv = V_1 - V_2$

$\qquad = -[V_2 - V_1]$

$\rho = \frac{m}{V}$

$\boxed{\beta = \frac{1}{K}}$

K = Bulk Modulus of elasticity of liquid

$= \frac{\Delta P}{\ldots}$

$$\mu_1 A V \times \frac{-1}{y^2} + \mu_2 A V \times \frac{1}{(H-y)^2} = 0$$

$$-\frac{\mu_1}{y^2} + \frac{\mu_2}{(H-y)^2} = 0 \qquad \underline{AV \neq 0}$$

$$\frac{\mu_1}{y^2} = \frac{\mu_2}{(H-y)^2}$$

$$\frac{\mu_1}{\mu_2} = \frac{y^2}{(H-y)^2}$$

$$\boxed{\sqrt{\frac{\mu_1}{\mu_2}} = \frac{y}{H-y}}$$

VARIATION OF VISCOSITY WITH RESPECT TO TEMPERATURE

NOTE: $\mu = \mu_0 \left[\frac{1}{1+\alpha t + \beta t^2} \right]$

1. For liquids viscosity decreases with the increase of temp

2. For gases viscosity increases with the increases of temp

1. the viscosity of air at 20° is given to be $1.6 \times 10^{-5}\ m^2/s$. Its ν at 70° will vary approximately — 99 GATE

a) $2.2 \times 10^{-5}\ m^2/sec$

b) $1.6 \times 10^{-5}\ m^2/sec$

c) ~~1.6x~~ $1.2 \times 10^{-5}\ m^2/sec$

d) 2.0 ...

At 20°C $\nu = \frac{\mu}{\rho} = 1.6 \times 10^{-5}$

AIR – GAS

At 70°C = $\mu \uparrow$ =

so when temp ↑ viscosity ↑

so $\nu > 1.6 \times 10^{-5}$

So (a)

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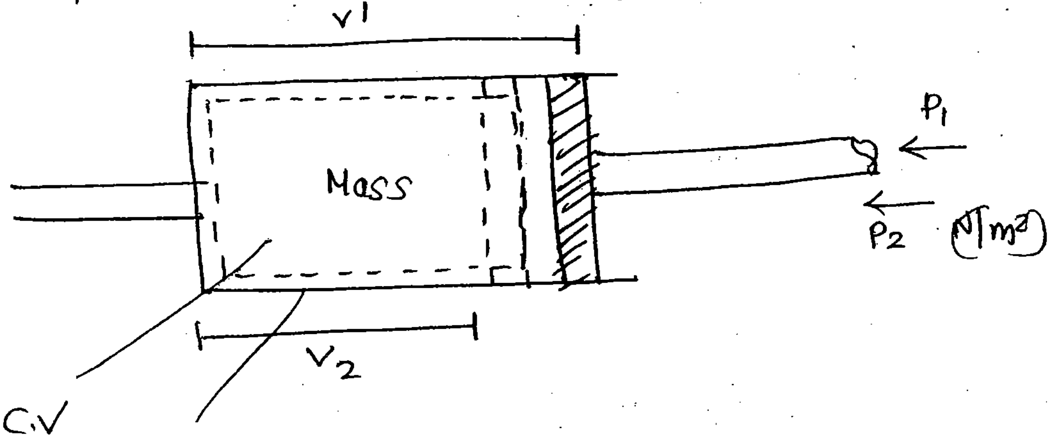


Loss pressure due to ↓ in viscosity of oil

$dv = V_1 - V_2$

$- [V_2 - V_1]$

$\rho = \frac{m}{v}$

$$\boxed{\beta = \frac{1}{K}}$$

K = Bulk Modulus of elasticity of liquid

$= \Delta P$

where $dv = v_2 - v_1$ UNIT

$d\rho = \rho_2 - \rho_1$ $\frac{N/m^2}{\frac{m^3}{m^3}} =$

Unit $= \frac{N}{m^2}$

DRDO

The increase in pressure required to decrease unit volume of mercury fluid by 0.1% is $\frac{dv}{}$

(a) 285 GPa (b) 285 MPa (c) 2.85 MPa (d) 28.5

Bulk modulus of mercury is 28.5 ~~GPa~~ MPa

Ans:

$K = 28.5 \frac{N}{mm^2}$

$K = \frac{\Delta P}{-\cancel{0.999}\frac{dV}{V}}$ b

$V = \cancel{\phi}$

~~Final volume~~

$\cancel{V - \frac{0.1 V}{100}}$

$=$

$\cancel{\Delta P = \frac{0.999}{28.5}}$

b $28.5 MPa = \frac{\Delta P}{-[0.1\%]}$

$10^6 \times 28.5 \left(\frac{N}{m^2}\right) = \frac{\Delta P}{-\left[\frac{0.1}{1000}\right]}$

$\Delta P = 28.5 \times 10^6 \times \frac{0.1}{100}$

$= 28.5 \times 10^3 \times \cancel{0.1}$

$= 28.5 \times 10^3 \ N/m^2$

$= 28.5 kPa$

Q) When the pressure on a given mass of liquid is increased from 30 bar to 35 bar, the density of the liquid increases from 500 kg/m3 to 501 kg/m3. What is the value of Bulk modulus of the given tested liquid for the given range of pressure?

(a) 700 MPa (b) 600 MPa (c) 500 MPa

(d) 250 MPa.

Ans:

$P\uparrow$ 30 → 35 Bar $\quad dP = 5$ bar

$m = const$ $\quad d\rho = 1$ kg/m3

$\rho\uparrow = 500 \rightarrow 501$

$$K = \frac{P_2 - P_1}{\frac{\rho_2 - \rho_1}{\rho_1}} = \frac{5 \text{ bar}}{\frac{501 - 500}{500}}$$

$$= \frac{5 \times 10^5}{\frac{1}{500}}$$

$$= 250 \text{ MPa}$$

Q. Which one of the following is represented by Bulk modulus of a fluid (symbols have the usual meaning) 97- P.1

a) $\frac{d\rho}{\rho \cdot dp}$ b) $\frac{P \cdot d\rho}{dp}$ $\quad K = \left(\frac{dP}{d\rho/\rho}\right)$

c) $\frac{dp}{\rho \cdot d\rho}$ d) ✓ $\frac{\rho \cdot dp}{d\rho}$

water to that of Mercury.

Bulk Modulus of Mercury = 30 MPa

Bulk " " water = 2 GPa $k_{MERCURY} = 30\,MPa$

$\boxed{K_{WATER} = 2\,GPa}$

$\cancel{\frac{2\,MPa}{30\,MPa}}$

$$\frac{2\,GPa}{30\,MPa} = \frac{2\times10^{9}\,Pa}{30\times10^{6}\,Pa} = \frac{200}{3} = \underline{\underline{66.67}}$$

$$\boxed{k_w = (60-70)\,k_m}$$

SURFACE ENERGY OF FLUIDS OR SURFACE TENSION

All the fluids in Nature made from tiny molecules and the surface of the fluids exibits a tensile force ~~or~~ in energy form which makes the fluid to hold other matter easily. this is due to the forces of molecules. [cohesive forces and adhesive forces].

Cohesive forces are those forces caused b/w own molecules attraction.

ex: mercury - liquid.

Adhesive forces are those forces ~~cause~~ which are attraction b/w 2 diff matter molecules.

[water and other similar liquids]

Surface Tension of Liquids > that of Gases

It is represented by σ

$$\boxed{\sigma = \frac{\text{Force (F)}}{\text{unit length surface of liquid}}}$$

Unit

$$\frac{N}{m}$$

EIL

Hygrometer is the device Used to measure Surface Tension

Specific Gravity

$$\sigma = \frac{\text{Energy possessed (Joules)}}{\text{unit surface Area } (m^2)}$$

$$\frac{Nm}{m^2} = \frac{\cancel{Nm}}{} \frac{N}{m}$$

units of surface Tension

$$\frac{N}{m} = \frac{kg\,m}{s^2 \times m} \quad \frac{kg}{sec^2}$$

$$= M^1 L^0 T^{-2}$$

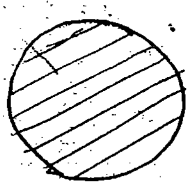
$\sigma_{Water-Air}$ = $0.073 \frac{N}{m}$

$\sigma_{Mercury}$ = 0.44 [6 times Higher]

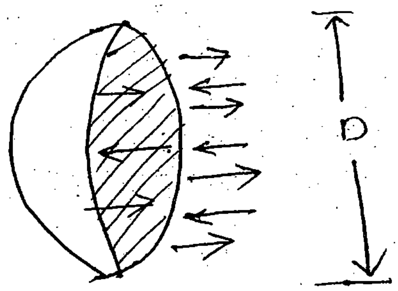
$\boxed{\sigma_{Mercury} > \sigma_{Water}}$ $\boxed{\sigma_M = 6\sigma_W}$

1. Droplet formation
2. Bubble formation
3. Jet travelling
4. Capillarity phenomena.

Sol :- D = dia of liquid droplet (metres)
σ = Surface tension of liquid (N/m)
P = Diff in pressure of liquid exerted (N/m²)



Droplet of liquid



Pressure exerted by fluid due to surface Tension.

(a) Droplet of liquid

$$\Delta P = \frac{4\sigma}{D} \left(\frac{N/m}{m}\right) = \frac{N}{m^2}$$

$$\Delta P = \frac{\text{Force}}{\text{Area (Normal)}} = \frac{F}{\frac{\pi}{4} \times D^2}$$

$$\boxed{F = \Delta P \times \frac{\pi}{4} \times D^2} \quad \text{——— (1)}$$

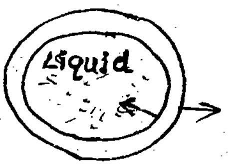
σ = surface Tension of liquid formed in drop

$$\sigma = \frac{F}{\text{unit length}} = \frac{F}{\pi D}$$

$$\boxed{F = \sigma \cdot \pi D} \quad \text{—— (2)}$$

(1) = (2)

[inside & outside]
2 surfaces



Liquid Bubble

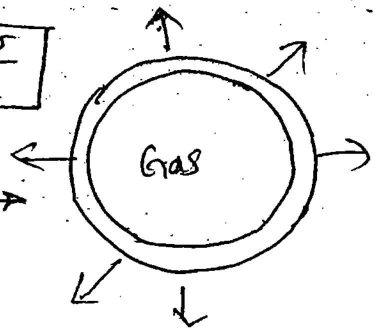
$$\Delta P \times \frac{\pi}{4} \times D^2 = \cancel{2}\sigma \cdot \cancel{\pi} \cdot D$$

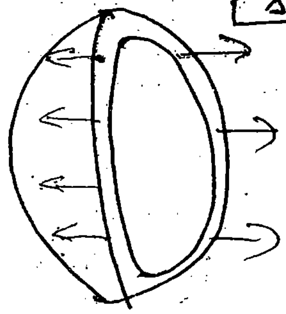
$$\therefore \boxed{\Delta P = \frac{4\sigma}{D}}$$

Bubble

$\boxed{\Delta P = \frac{4\sigma}{D}}$

AIR BUBBLE





AIR BUBLE

$\Delta P = F / \frac{\pi}{4} \times D^2$

$\sigma = \frac{F}{\pi D}$

$\sigma \times \cancel{\pi} \cancel{D} = \Delta P \times \frac{\cancel{\pi}}{4} \times D^{\cancel{2}}$

$\Delta P = 4\sigma/D$

LIQUID BUBBLE

$F = \Delta P / \pi/4\, D^2$

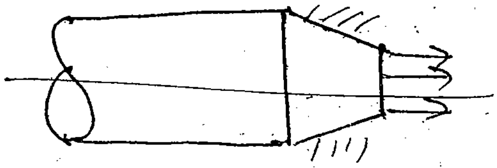
$\sigma = \frac{F}{2\pi D}$

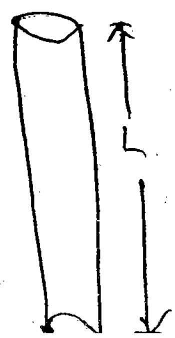
$\Delta P = \frac{8\sigma}{D}$

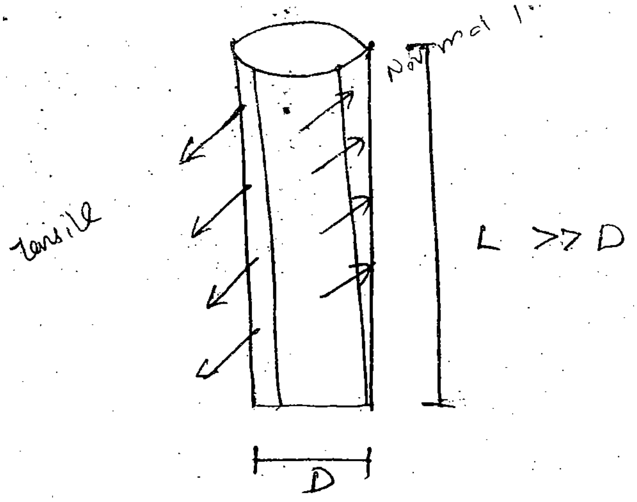
For Liquid Bubble $\boxed{\Delta P = \frac{8\sigma}{D}}$

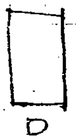
$$\boxed{\Delta P_{Bubble} = 2 \times \Delta P_{droplet}}$$

Jet of Liquid









Pressure difference $(\Delta P) = \dfrac{F}{\boxed{\text{Area}_{\text{Normal}}}}$

surface tension $= \sigma = \dfrac{F}{\text{unit length}}$

$F = \Delta P \cdot A_N = \sigma \cdot \text{UNIT LENGTH}$

$\Delta P \times [D \times L] = \sigma \cdot [D + L + D + L]$

$\Delta P \times (D \times L) = \sigma (2D + 2L)$

$L >> D$

$2L >>> 2D$

$\Delta P (D \times L) = \sigma (2L)$

$$\boxed{\Delta P = \frac{2\sigma}{D}}$$

$$\boxed{\Delta P_{Jet} = \frac{\Delta P_{DROPLET}}{2}}$$

Q.) The surface Tension of water with air interface is 0.073 N/m. the Gauge pressure inside a rain droplet of diameter 1mm will be

ⓐ 0.146 N/m^2 ⓑ 73 N/m^2 ⓒ 146 N/m^2

ⓓ 292 N/m^2

Ans:—

$\sigma = 0.073$

$D = 1mm$

$$P = \frac{4\sigma}{D}$$

$$= \frac{4 \times 0.073}{10^{-3}}$$

$$= \underline{292\ N/m^2}$$

Q.) If 'P' is the Gauge pressure within a spherical droplet then the gauge pressure within a Bubble of same fluid and of same size will be

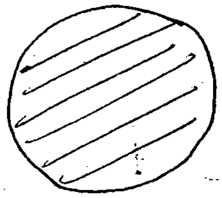
ⓐ P/4

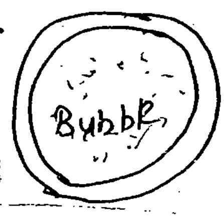
ⓑ P/2

ⓒ P

ⓓ 2P

2 surfaces so $\frac{8\sigma}{D}$





$P_1 = \frac{4\sigma}{D}$

$P_2 = \frac{8\sigma}{D}$

$= 2 \times 4\sigma$

what is the S.T. of soap film ?

$D = 50\,mm = 50 \times 10^{-3}\,m$

$P = 25\,N/m^2$

$P = 25\,N/m^2$

$\sigma = ?$

Bubble

$P = 2 \times \frac{4\sigma}{D}$

$\sigma = \frac{P \times D}{8} = \frac{25 \times 50 \times 10^{-3}}{8}$

$= \underline{0.156\,N/m^2}$

(a) A water jet issued from a nozzle of diameter 2mm and length 25mm. the surface Tension of water with air interface = $0.075\frac{N}{m}$. what is the Pressure inside the jet above the atmosphere in N/m^2 ?

J - Nm

$\sigma = 0.075\,N/m$

$l = 25\,mm = 25 \times 10^{-3}\,m$

$D = 2\,mm = 2 \times 10^{-3}\,m$

$\frac{J}{s} = watt$

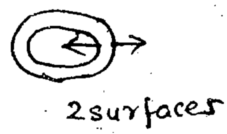
$\frac{Nm}{s}$

$\Delta P = \frac{2\sigma}{D} - \frac{2 \times 0.075}{2 \times 10^{-3}}$

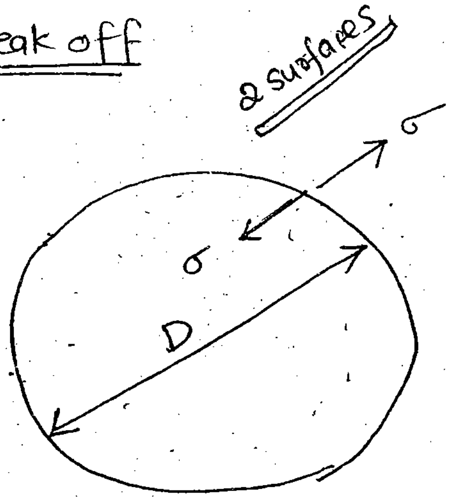
$= \underline{75\,N/m^2}$

1. Bubble Break off
2. Large size Bubble convert into no of Bubbles

RING



→ **Bubble Break off**



LIQUID BUBBLE

$D_o = D_i = D$

So thin film

Work Reqd to Break a liquid Bubble

$$\sigma = \frac{F}{\text{unit length}} : \frac{\text{Energy Possesed}}{\text{unit Area}}$$

$$\sigma = \frac{\text{Energy}}{\text{Area}}$$

$$\text{Energy} = \sigma \times \text{Area}$$

$$\frac{J}{m^2} \times m^2 = \underline{J}$$

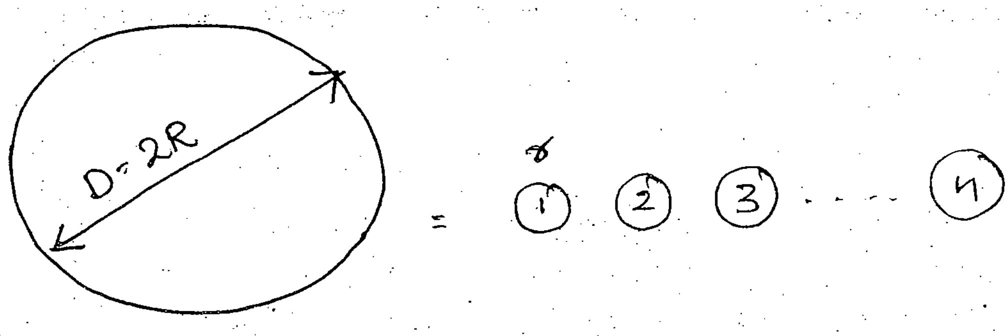
$$E = \sigma\left[4\pi R^2 + 4\pi R^2\right]$$

Joules

→ $$\boxed{E = \sigma\left[8\pi R^2\right]}$$

$$\boxed{\begin{aligned} S.A &= 4\pi R^2 \ (m^2) \\ V &= \frac{4}{3}\pi R^3 \ (m^3) \end{aligned}}$$

Break off into small Bubbles



LAW OF CONSERVATION OF MASS

mass Before collapse = mass After collapse

Total mass remains conserved always

$$\cancel{\rho} V = n \cdot \cancel{\rho} v$$

Mass Big Bubble = n × Mass small Bu

$$V = nv$$

$$\frac{4}{3}\pi R^3 = n \cdot \frac{4}{3}\pi r^3$$

$$R^3 = n \cdot r^3$$

$$\boxed{r = \frac{R}{\sqrt[3]{n}}}$$

Work reqd to Break 'n' Bubbles

$$\sigma = \frac{\text{Energy}}{\Delta A} \quad \text{change in Area}$$

$$\Delta A = n \cdot a - A$$

Area of small Bubbles; n: no. of Bubbles; A: Big Bubb. Area

$$\text{Energy} = \sigma \times (\Delta A)$$

$$= \sigma \left[n a - A \right] = \sigma \left[n 4\pi r^2 - 4\pi R^2 \right]$$

1. What is the energy Reqd to Break the Soap liquid Bubble of dia 100 mm. S.T of soap soln is 0.05 $\frac{N}{m}$ is

a) 0.00157 Joule (N-m)
b) 0.00314 Joule
c) 0.00628 Joule
d) None

$\frac{J}{m^2} = \frac{Nm}{m^2}$

$$\sigma = \frac{\text{Energy}}{\text{Area}}$$

$$\text{Energy} = \sigma \times \left[4\pi R^2 + 4\pi R^2\right]$$

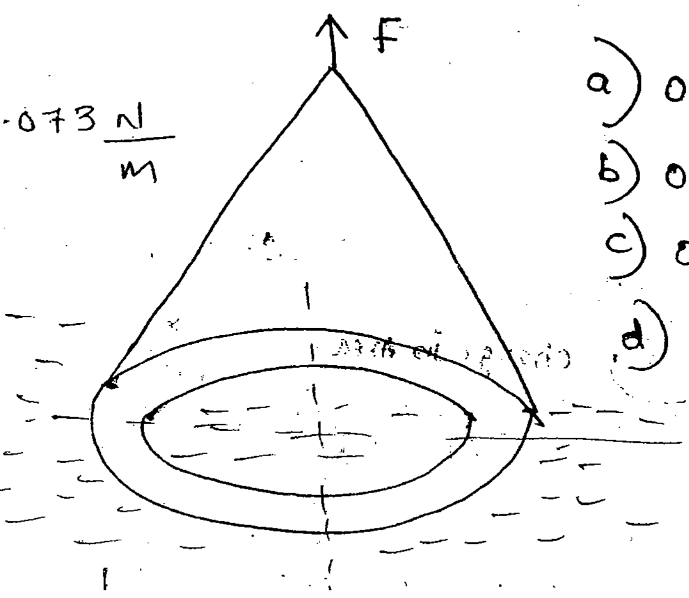
$$= 8\pi R^2 \times \sigma$$

$$= 8 \times \pi \times \left[\frac{100 \times 10^{-3}}{2}\right]^2 \times 0.05$$

$$= \underline{3.14 \times 10^{-3} \text{ Joules}}$$

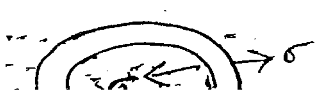
2. What is the force necessary to lift a Platinum wire from water surface shown in fig. Neglect self weight of Ring and Bouyancy force.

$\sigma = 0.073 \frac{N}{m}$



a) 0.00685 N
b) 0.0137 N
c) 0.0274 N
d) 0.0175 N

Like a Ring



$$\sigma = \frac{\text{Force applied}}{\text{unit length}} \quad \frac{N}{m}$$

$$0.073 = \frac{\text{Force applied}}{2 \cdot \pi D} \qquad D = 0.04\,m$$

$$0.073 = \frac{F}{2\pi \times 0.04}$$

2 - inside and outside surfc

$$F = 2\pi \times 0.04 \times 0.073$$

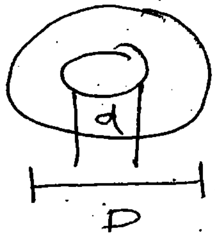
$$= \underline{\underline{0.0175 \text{ (N)}}}$$

(4) PLANE SHEET [LIKE METAL WASHER]

$= \pi D$

unit length $= \pi r$

surface



(3) A spherical liquid Bubble of Radius 'r' is blown into 64 small bubbles of equal size and volume - what is the work reqd to convert large size into small no's in terms of S.T.

a) $12\pi R^2 \sigma$

b) $16\pi R^2 \cdot \sigma$

c) $4\pi R^2 \sigma$

d) None.

$$\text{Work} = \sigma\left[n \cdot a - A\right]$$

$$\sigma\left[n \frac{4\pi r^2}{\cancel{3}} - \frac{4}{\cancel{3}}\pi R^2\right]$$

$r = \quad R \qquad r = R$

$$\sigma\left[64\times\frac{4\pi\times R^2}{16}-4\pi\times R^2\right]$$

$$= 12\pi R^2.\sigma$$

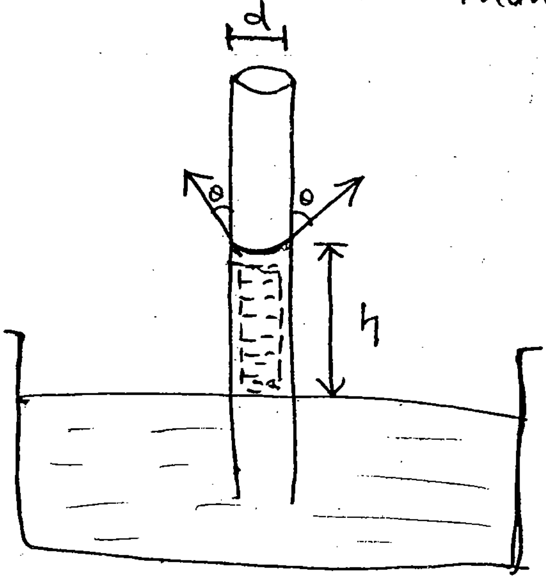
CAPILARITY PHENOMENA

[SAP- plants receiving minerals from Beneath]

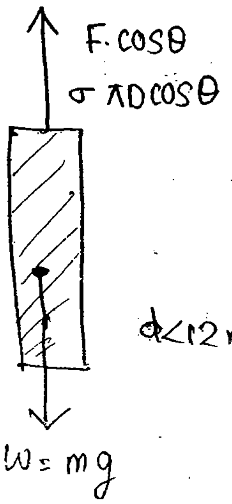
CAPILARITY is the phenomena of liquid due to which the liquid rises their level in small narrow conduit [Tube, channel] expect mercury where the capillarity effect is the depression or fall of mercury coloumn. This phenomenon is observed in narrow tubes

$$\boxed{\text{ie} \quad D < 12\,mm}$$

Expression for Capillary Rise of liquid other than Mercury



$$\sigma = \frac{F}{l} = \frac{F}{\pi D}$$



d<12mm

$$\sigma = \frac{\text{Force}}{\text{unit length}} = \frac{F}{\pi D}$$

[$10-30^\circ$]

$F \cdot \cos\theta = mg$

$\sigma \times \pi d \cos\theta = \rho \cdot v \cdot g$

$= \rho \cdot \underset{c/s}{A} \times h \cdot g$

$\rho \times \frac{\pi d^2}{4} \cdot h \cdot g$

$\sigma \times \cancel{\pi} \cos\theta = \rho \times \frac{\cancel{\pi} d}{4} \cdot h \cdot g$

$h = \frac{4\sigma\cos\theta}{\rho g d} = \boxed{\frac{4\sigma\cos\theta}{\gamma d}}$

$\boxed{h = \frac{4\sigma\cos\theta}{\gamma d}}$

$\frac{\frac{N}{m} \times}{\frac{N}{m^3} \times m}$

[if High dia tube is used
Rise will be very small]

unit $= m$

$w = \rho g$

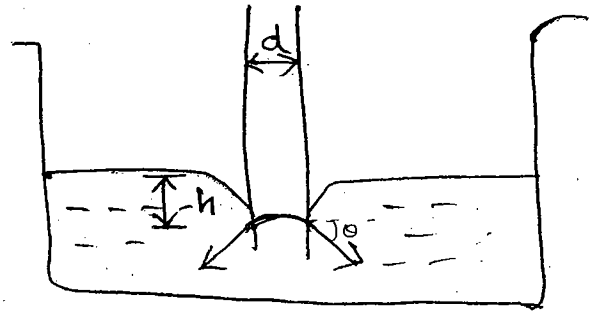
$\frac{kg}{m^3} \times \frac{m}{s}$

$\boxed{\text{if } d\uparrow \text{ then } h\downarrow \text{ capillary fall.}}$

Capillary action of Mercury.

θ = angle of contact

capillary fall



$\theta > 90$

(135°)

1) A capillarity Rise in 3mm dia of Tube immersed in a liquid is 1.5 cm. If another tube of dia 4mm is immersed in same liquid then capillary Rise would be in mm.

$d = 3mm$

$h = 1.5 \times 10 \, mm$

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

$$h \propto \frac{1}{d}$$

$$h_1 d_1 = h_2 d_2$$

$$\frac{h_1}{h_2} = \frac{d_2}{d_1}$$

$$\frac{h_1}{h_2} = \frac{d_2}{d_1}$$

$$\frac{15}{h_2} \quad \frac{4}{3}$$

$$h_2 = \frac{15 \times 3}{4} = \frac{45}{4} = \underline{\underline{11.25\,mm}}$$

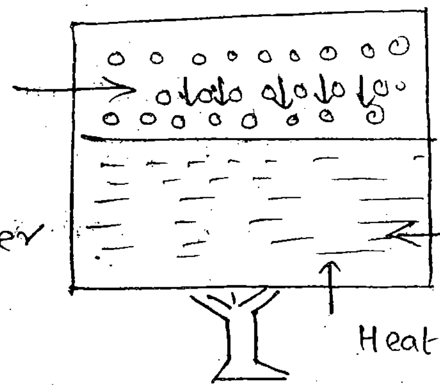
3)

VAPOUR PRESSURE

It is a property of fluid due to which the fluid surface exerts additional Pressure intensties (Partial Pressures) due to which bubbles in liquid are formed. the Bubbles rises from the liquid and collapse on the metal surfaces due to which the metal surface eroded. this the sharp Bends of the pipes

Every liquid and gas has its own vapour pressure.

ex: Water = 25kPa [2.5 m of water column]



1 atm
= 101 kN/m^2
= 10.33m of water
= 76mm of Hg

20°C → 100°C

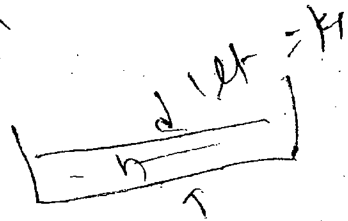
Heat energy

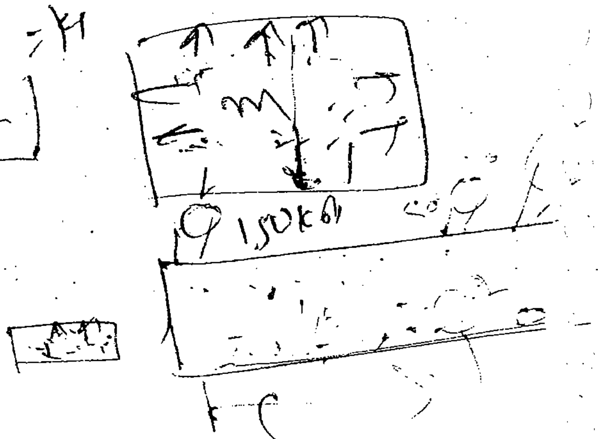
$P = \rho g h$

h

Boyles Law

$P \propto T$ ↑

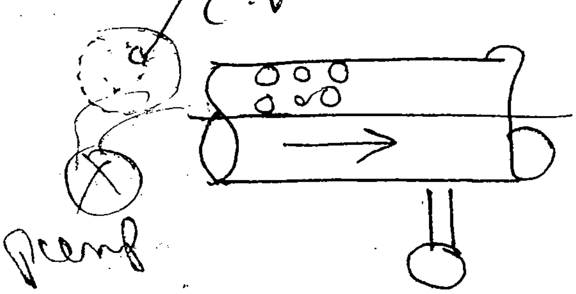




In FM

Temp remains constant

C.V



At low pressure, liquid molecules can't exist, so they jump into higher state, ie vapour.

$P \propto T$ ↓ ↓

During state change Temp will not change

In fluid mechanics the vapour pressure is concerned with the liquids where the region pressure is Below atmos pressure or reaches the fluid self liquid pressure which leads to Bubble formation

Thermodynamic properties

Gases – Compressible fluids

$PV = mRT$

$\boxed{P v_s = RT}$ $\rightarrow v_s =$ specific volume $= \frac{V}{m} = \frac{1}{\rho}$

T- absolute temp in °K

R = Gas const

ρ = density

$\frac{P}{\rho} = RT$

[When temp and Pressure changes 'ρ' varies]

So

R

$\frac{P}{\rho} = RT$

in SI $\boxed{R = 287\ J/kg\,K}$

R-unit

UNIT

$R = \frac{P}{\rho T} = \frac{N}{m^2 \times \frac{kg}{m^3} \times K}$

$\frac{N\,m^3}{\cancel{m^2}\,kg\,K}$

$\frac{Nm}{kg\,K} = \underline{J/kg\,K}$

Processes

→ Isothermal

$\frac{P}{\rho} = $ const

→ Adiabatic

$\frac{P}{\rho^K} = $ const

$K = \frac{c_p}{c_v} = 1.4$ for air

$$\boxed{= M \times R}$$ M = Molecular Mass

$= 8314 \, J/kg\text{-mole}$ in SI unit

$M = \dfrac{\text{mass of Gas molecules}}{\text{Mass of a Hydrogen atom}}$

Compressibility

$$\boxed{\beta = \frac{1}{K}}$$

$$\boxed{K = \frac{\text{Increase in Pressure}}{\text{volumetric strain}} = \frac{dP}{-\frac{dV}{V}} \text{ or } \frac{dP}{\frac{d\rho}{\rho}}}$$

Relationship b/w K and Pressure of Gas

Isothermal process

$$\frac{P}{\rho} = \text{const}$$

$$P v_s = \text{const.}$$

Diff $\quad P dV_s + V_s dP = 0$

$$P dV_s = -V_s dP$$

$$P = -V_s \frac{dP}{dV_s} = \frac{dP}{-\frac{dV_s}{V_s}}$$

$$\boxed{K = P}$$

Adiabatic

$$\frac{P}{\rho^K} = \text{const}$$

$$\boxed{K = PK}$$

$$\cancel{P dv^k + v^k dP = 0}$$

$$\cancel{P dv^k = -v^k dP}$$

$$\cancel{P = \frac{-v^k dP}{dv^k}}$$

$$P v_s^k = \text{const}$$

diff

$dv_s \; P \cdot K \, v_s^{k-1} = -v_s \, dp$

$PK = -\dfrac{v_s \, dp}{v_s^{k-1}}$ $\quad v_s^k$

$PK = -\dfrac{dp}{v_s^k}$

$P \cdot K \, dv_s^{k-1} \cdot dv_s = -v_s^k \, dp$

$PK = \dfrac{-v_s^k \, dp}{dv_s^{k-1} \cdot dv_s} = \dfrac{dp}{-v_s^k}$

PROBLEMS

TEXT 1.5

$A = 1.5 \times 10^6 \, mm^2$

$V = 0.4 \, m/s$

$y = 0.15 \, mm$

$F = ?$

Power $P = ?$

$\mu = 1 \text{ POISE} = \dfrac{1}{10} \dfrac{NS}{m^2}$

$\dfrac{F}{A} = \mu \dfrac{V}{y}$

$F = \dfrac{1}{10} \times 1.5 \times \dfrac{0.4}{0.15 \times 10^{-3}}$

SHEAR FORCE $= \underline{\underline{400N}}$

(ii) Power required to maintain this speed

$= F \times \text{velocity} = 160$

$= 400 \times 0.4$

$\dfrac{N \times m}{s} \cdot \dfrac{J}{s} = \underline{\underline{W}}$ $\quad \underline{\underline{160W}}$

16

$\mu = 1 \text{ POISE} = \dfrac{1}{10} \dfrac{NS}{m^2}$

$d = 10 \, cm = 10 \times 10^{-2} \, m$

$= 0.1 \, m$

$dy = 1.5 \, mm = 1.5 \times 10^{-3} \, m$

$N = 150 \, rpm$

$V = \pi D N = \pi \times 0.1 \times 150 = 0.785 \, m/s$

$$\tau = \mu \times \frac{du}{dy} = \frac{1}{10} \times \frac{0.785}{1.5\times10^{-3}}$$

$$= \underline{52.33\ N/m^2}$$

1.7 $\mu = ?$

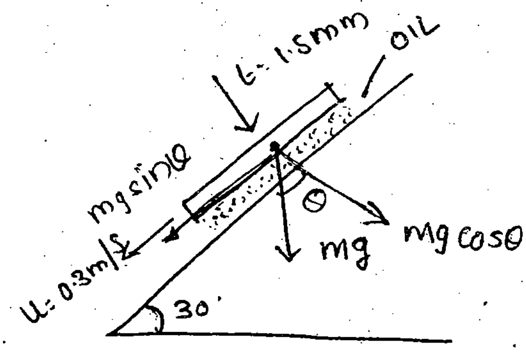
$A = 0.8 \times 0.8$

$\theta = 30^\circ$

$W = 300\ N$

$V = 0.3\ m/s$

$y = 1.5\times10^{-3}\ m$



$$\tau = \mu \times \frac{du}{dy}$$

$$\mu = \tau \times \frac{dy}{du}$$

$$\tau = \frac{F}{A} = \frac{mg\sin\theta}{0.64}$$

$$= \frac{300\sin 30}{0.64} = \frac{150}{0.64}$$

$$=$$

$$\mu = 1.17 \times \cancel{10}\ \frac{Ns}{m^2}$$

$$= \cancel{1.17\ poise}\ \underline{11.7\ POISE}$$

$$1\ Poise = \frac{1}{10}\frac{Ns}{m^2}$$

1.8 $y = 1.25\times10^{-2}\ m = 0.0125\ m$

oil $\mu = 14\ poise = \frac{14}{10}\frac{NS}{m^2} = \underline{1.4}\ N\ s/m^2$

$\tau = ?$

$V = 2.5\ m/s$

$$\tau = \mu \cdot \frac{du}{dy} = \underline{280\ N/m^2}$$

19 Area $= 60\times60 = 3600\ cm^2$

oil.

$y = t = 12.5\ mm$

$du = 2.5\ m/s$ $\mu = ?$

$F = 98.1\ N.$ $\nu = ?$

$$\frac{F}{A} = \mu \cdot \frac{du}{dy}$$

$$\mu = \frac{F}{A} \times \frac{dy}{du}$$

$$\mu = \frac{98.1}{3600\times10^{-4}} \times \frac{12.5\times10}{2.5}$$

$$= 1.36\ NS = 13.6\ P$$

~~s~~ $\frac{\rho_{liq}}{\rho_w} = \frac{\rho_{oil}}{1000}$

$\rho_{oil} = 1000 \times 0.95$

$= \underline{950\ kg/m^3}$

$\vartheta = \frac{\mu}{\rho} = \underline{\hspace{3cm}}\ m^2/s$

~~(ii)~~

(1.10) $\rho =$ ~~90~~ $981\ kg/m^3$

$\frac{m}{s\,m}$

$\vartheta = ?$

$\tau = 0.2452$

$\frac{du}{dy} = 0.2$

$\tau = \mu \frac{du}{dy}$

$\vartheta = \frac{\mu}{\rho} =$

$\underline{12.5 cm^2/s} = \underline{12.5\ stoke}$

1.11 $S = ?$

$\mu =$

$\mu = 0.05$ Poise

$= 0.005$ ~~$\frac{Nm^2}{s}$~~ $\frac{Ns}{m^2}$

$\vartheta = 0.035$ stoke

$= 0.034 \times 10^{-4} \frac{m^2}{s}$

$\rho_{fluid} = ?$ $\vartheta = \frac{\mu}{\rho_{fluid}}$

$S = \frac{\rho_{fluid}}{\rho_{water}}$

$S = \underline{\hspace{2cm}}$

1.12 $\vartheta = 6$ stokes $= 6 \times 10^{-4} \frac{m^2}{s}$

$S = 1.9$

$\vartheta = \frac{\mu}{\ }$

$\mu = 8.6\ \text{poise} = \frac{8.6}{10}\ Ns/m^2$

u in m/s

y in m

$\tau]_{y=0.15} = ?$

$\tau = \mu \times \frac{du}{dy}$

$\tau \times \frac{dy}{du}$

$\cancel{\frac{N}{m^2}} \times \cancel{\frac{m}{s}}$

$\frac{N}{m^2} \times \frac{m}{1}$

$\frac{N}{m^2} \times \frac{\cancel{m} \times s}{\cancel{m}}$

$\frac{Ns}{m^2}$

$\frac{du}{dy} = \frac{3}{4} - 2y$

$\tau = \mu \times \frac{du}{dy}$

$\tau = \frac{8.6}{10} \times \frac{3}{4} - 2y$

$\tau = 0.86 \times \frac{3}{4} - 2 \times 0.15$

$= 0.3825\ \frac{N}{m^2}$

1.14 $\mu = 6\ \text{poise}$

$= \frac{6}{10}\ \frac{Ns}{m^2}$

$d = 0.4\ m$

$N = 190\ r.p.m$

$P = ?$

$l = 90\ mm = 90 \times 10^{-3}\ m$

$P = \frac{2\pi NT}{60}$

$T = F \times r_{\perp}$

$= F \times \frac{d}{2} =$

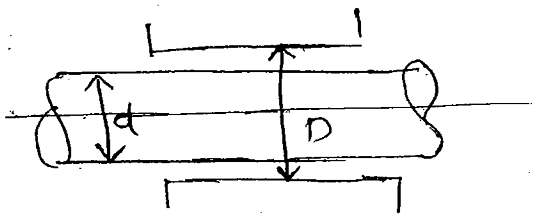
$\frac{F}{A} = \mu \frac{du}{dy}$

$\frac{F}{\frac{\pi}{4} \times D^2} = 0.6 \times$

$V = \frac{\pi DN}{60}$

$= \frac{\pi \times 0.4 \times 190}{60}$

$= 3.98\ m/s$



$\frac{F}{A} = \mu \times$

$\frac{2 \times \pi \times N \times F \times \frac{D}{2}}{60}$

$$y = \frac{D-d}{2} \qquad \frac{F}{A} = \mu \frac{du}{dy}$$

$$A = \pi d L$$

$$P = \frac{2 \times \pi \times N \times}{60} \quad \mu \times \pi \times d L \times \frac{\pi D N}{60} \times \frac{2}{D-d}$$

$\boxed{1.14}$

$\mu : 0.6 \frac{Ns}{m^2}$

$d : 0.4 m$

$N = 120$ rpm

$P = ?$

$L = 90 \times 10^{-3} m$

$t = 1.5 \times 10^{-3}$ mm

$$P = \frac{2 \pi N T}{60}$$

$$T = F \times \frac{D}{2}$$

$$v = \frac{\pi D N}{60} = \frac{\pi \times 0.4 \times 190}{60}$$

$$= \cancel{2.51 m/s}$$

$$\underline{3.98 \ m/s}$$

$$F = \mu \times A \times \frac{du}{dy}$$

$$= 0.6 \times \frac{\pi}{4} \times (0.4)^{2} \times \frac{\cancel{2.51}\ 3.98}{1.5 \times 10^{-3}} \times 90 \times 10^{-3}$$

Area $= \pi d L$

1.15 $\quad V = 120 \frac{cm}{sec} = 120 \times 10^{-2}$ m/s

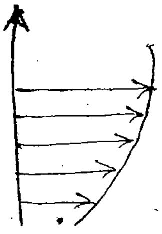
$\frac{du}{dy}$

NOTE :-

> EQUATION OF VELOCITY PROFILE (PARABOLIC) is GIVEN BY -
>
> $u = ay^2 + by + C$
>
> a, b, c are constants

These values determined by Boundary conditions

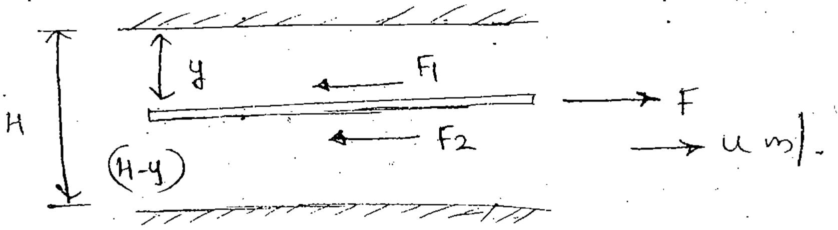
ie at : $y = 0$; $u = 0$



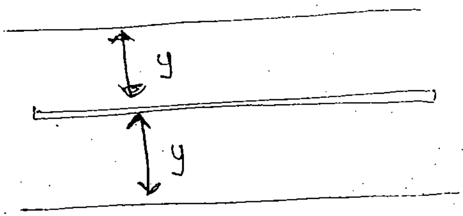
q $\quad \frac{du}{dy} = 0$

NOTE:-

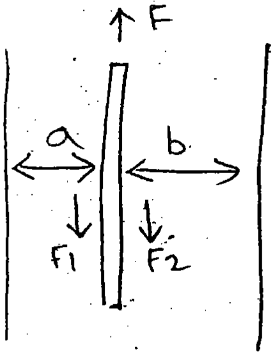
If plate is very thin then thickness can be neglected.



~~For~~



Vertical plates



forces
upward thrust
weight acting downwards.

Force required to pull the plate upwards

Here the upthrust is also to be considered since weight of plate is acting in opposite direction

$$\boxed{\begin{aligned} \text{upward thrust} &= \text{weight of fluid displaced} \\ &= \text{weight density of fluid} \times \text{volume of plate} \\ &= \rho \times g \times \text{volume} \end{aligned}}$$

this upward thrust is due to the weight

the Net force acting in downward direction due to weight and upward thrust

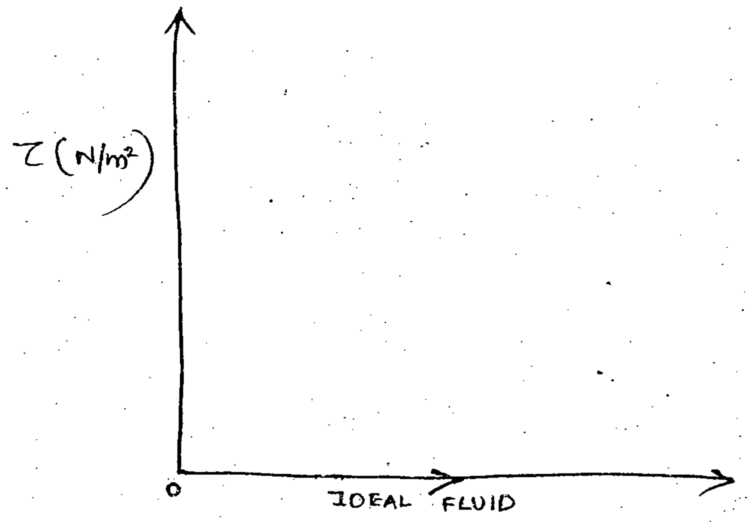
= weight − upward thrust.

$$\boxed{\text{Total Force Reqd to lift plate} = \text{Shear Force} + \text{Net force} = (F = F_1 + F_2) + W - \text{upward thrust}}$$

due to viscosity

$$\text{Total Force} = (F_1 + F_2) + [mg - \text{upward thrust}]$$

Classification of Fluid Based on "Fluid Power Law" or Rheological Eq of Fluid



velocity gradient or
Rate of deformation
$\left(\frac{du}{dy}\right) = sec^{-1}$

$$\tau = \tau_{yp} + \left(\frac{du}{dy}\right)^n \cdot \mu$$

$$\boxed{\tau = B + A\left(\frac{du}{dy}\right)^n}$$

$$\tau = B + A\left(\frac{du}{dy}\right)^n$$

where n = flow behaviour index

Classification:-

- Ideal fluid
- Ideal solid
- Real Fluids → [Paste]
  - Bingham's fluids
  - Thixiotropic Fluids (scent, lipstic)
  - Ideal plastic fluid
  - Newtonian fluids
  - Non Newtonian Fluids

theoritical fluid. Never exists in Nature

$$\tau = B + A\left(\frac{du}{dy}\right)^n$$

$B = 0$

$A = \mu = 0$

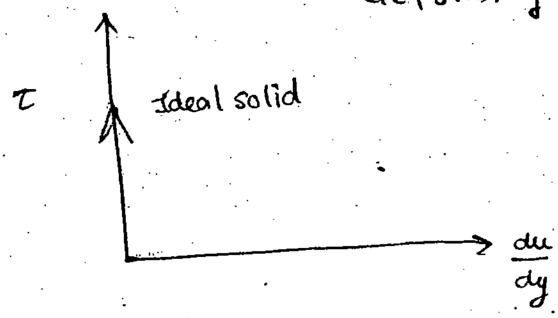
$\Rightarrow \tau = 0$

2.) Ideal solid :-

$$\tau \rightarrow \infty \ [\text{Large}]$$

$$\frac{du}{dy} = 0 \Rightarrow$$

$B \rightarrow \infty$ $\qquad \tau_{yp} = \infty$ [very Large forces needs will be applied for deforming]



3.) Newtonian Fluids

→ Fluids which follows Newtons law of viscosity

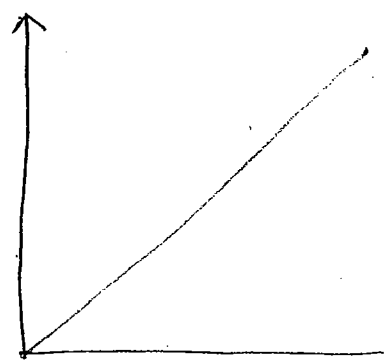
$$\tau = B + A\left[\frac{du}{dy}\right]^n \qquad \text{ie} \quad \tau \propto \frac{du}{dy}$$

$B = \tau_y = 0$

$A = \mu$

$n = 1$

$$\boxed{\tau = \mu \frac{du}{dy}}$$



slope $\mu = \dfrac{\frac{du}{dy}}{\tau} = \text{const}$

1) Air

Non Newtonian Fluids :-

Fluids which does not obeys Newton's Law

Dilatent Fluid

$B = \tau_{yp} = 0$

$A = \mu$

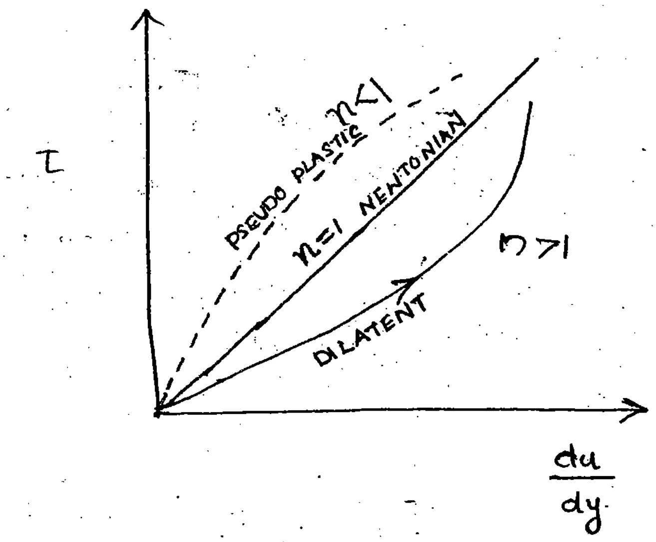
$n > 1$

DILATENT FLUID.

$n > 1$

$B = \tau_{yp} = 0$

$A = \mu$



Ex : sugar water soln

salt water soln.

Ex imp : Polymer solution [Rubber solutions].

Butter.

Rice starch.

Two diff Fluids mixed to form a new fluid.

PSEUDO FLUIDS :-

$n < 1$

$B = 0$

$A = \mu$

ex: Blood

Milk

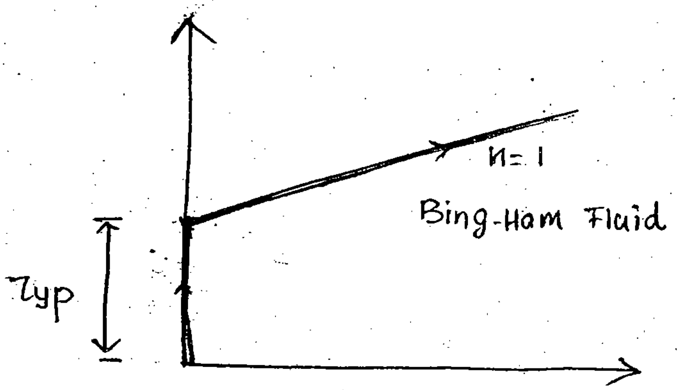
Suspension Fluids

$$\tau = B + A\left[\frac{du}{dy}\right]^n$$

$B = \tau_{yp}$ = yeild shear stress.

$A = \mu$

$n = 1$



→ Tooth paste

Creams in tube

Sewage sludge

Drilling mud

GREASES

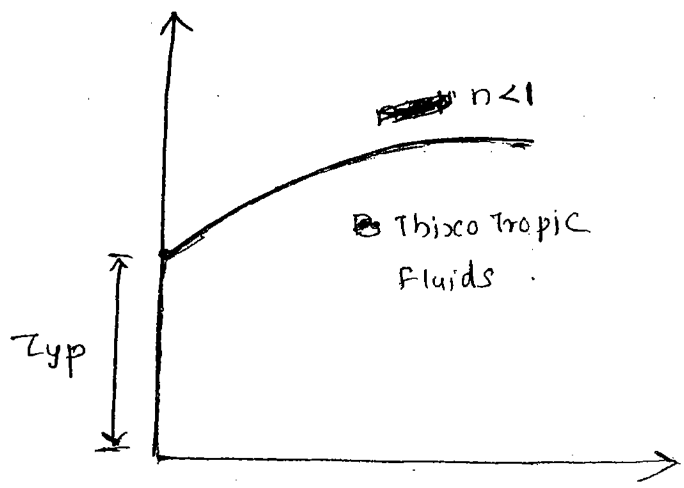
Thixo tropic Fluids:-

$$\tau = B + A\left(\frac{du}{dy}\right)^n$$

$B = \tau_{yp}$

$A = \mu$

$n < 1$



- Fluids which are in evaporative Nature

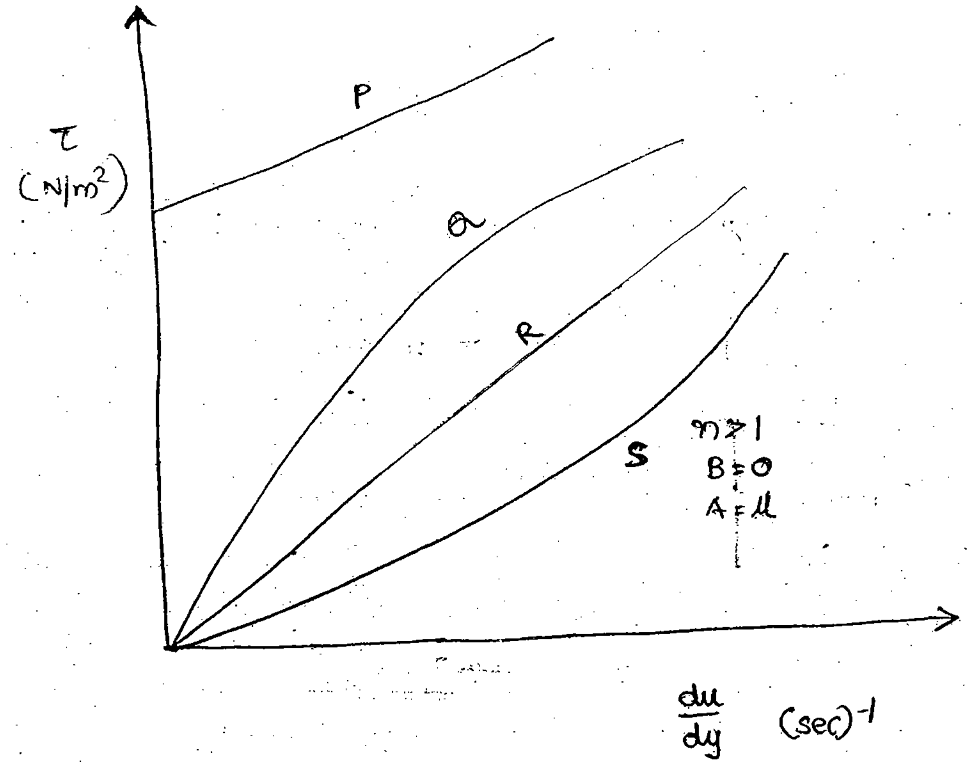
Lipstics solutions

Print

Scent

Sprays deodorants

The Rheological diagram, the Relation b/w shear stress and shear strain Rate for diff Types of Fluids shown below.



For S:

$$\tau = B + A \left(\frac{du}{dy}\right)^n$$

$B = 0$

$A = \mu$

$n > 1$

For R:

Newtonian Fluid

$B = 0$

$A = \mu$

$n = 1$

$$\boxed{\tau = \mu \frac{du}{dy}}$$

Pseudoplastic Fluid :

$B = 0$

$A = \mu$

$n < 1$

ex: Blood, Milk, Paint,

For P

$B = \tau_{xy}$

$A = \mu$

$n = 1$

Bingham Fluids

Tooth paste.
Sewage sludge
Drilling mud

[Dry Fluids | Force has to be applied]

$$\tau = B + A\left(\frac{du}{dy}\right)^n$$

Q) The Most suitable Relation for the flow of tooth paste being squeezed out of the Tube is given by the curve

Ans: ie Bingham Fluids

[So curve P]

Q) Match List I with List II and select correct code given Below.

List I (Fluid Power eqn)	List II (Fluid)
(A) $\tau = \mu\left[\frac{du}{dy}\right]^n$; $n=1$	(1) Bingham Fluids
(B) $\tau = \mu\left[\frac{du}{dy}\right]^n$; $n<1$	(2) Dilatent Fluids
(C) $\tau = \mu\left[\frac{du}{dy}\right]^n$; $n>1$	(3) Newtonian Fluids
(D) $\tau = \tau_{yp} + \mu\left(\frac{du}{dy}\right)^n$; $n=1$	(4) Pseudo-Fluids

List I	List II
	Variation of shear stress
A) Ideal Fluid	1.) shear stress varies linearly with rate of strain
B) Newtonian Fluid	2.) Shear stress doesn't vary Linearly with rate of strain
C) Non-Newtonian	3.) Fluid behaves like a solid until a min yeild stress beyond which it exibits a linear relationship b/w shear stress and rate of strain
D.) Bing Ham Fluid	4.) shear stress is zero

Ans:

B - 1

C - 2

D - 3

A - 4

x————————x

Q.) In an expt conducted to determine the Rhelogical behaviour of a material, it is observed that the Relation b/w shear stress τ and rate of shear strain $\frac{du}{dy}$ is given as

$$\tau = \tau_0 + C\left[\frac{du}{dy}\right]^{0.5}$$

The material is ·

a) Newtonian fluid

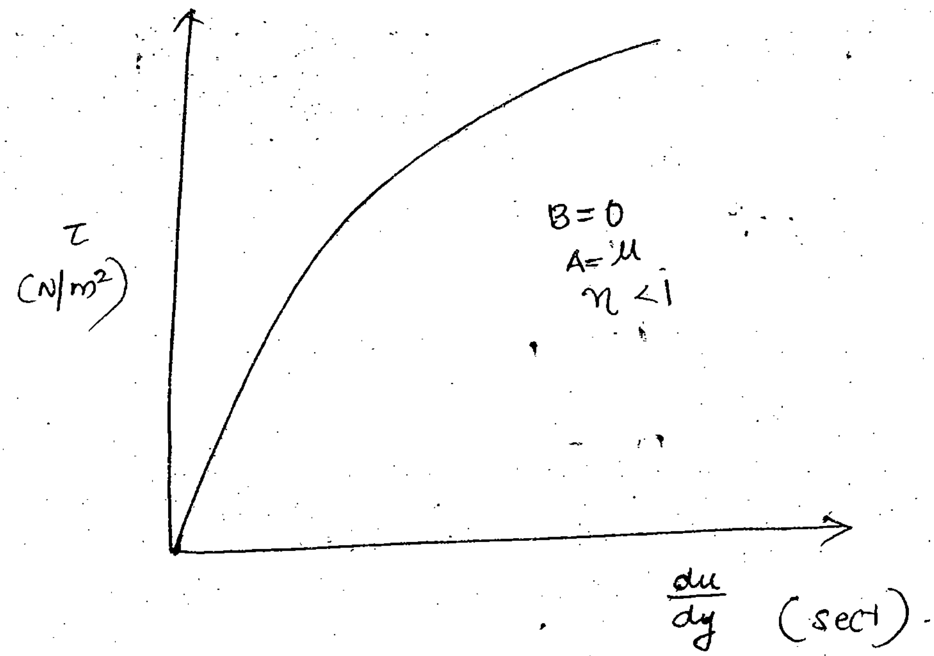
b) Bingham plastic fluid

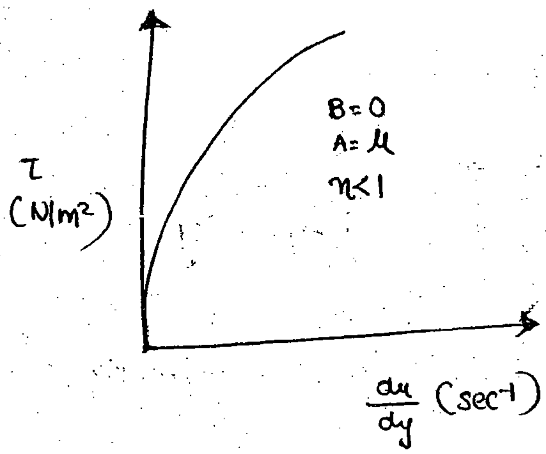
c) An ideal plastic fluid

D.) thixo Tropic fluid

Ans: D $\quad [n<1] \quad n<1$

$\eta < 1$





(*) POISE

$$\boxed{1 \text{ POISE} = \frac{1}{10} \frac{Ns}{m^2}}$$

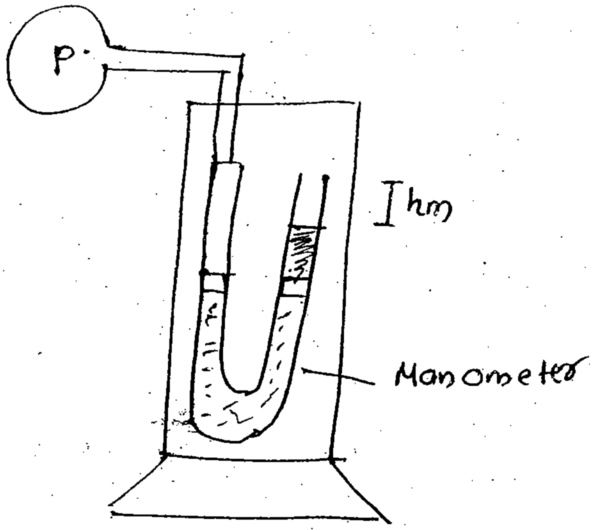
$$\boxed{1 \text{ STOKE} = 10^{-4} \frac{m^2}{s}}$$

List I	List II
Q.) A Sp Gravity	1) $M^0 L^2 T^{-1}$
B Coeff of viscosity	2) $M^0 L^0 T^0$
C Kinematic viscosity	3) $M L^{-1} T^{-1}$
D Stress	4) $M L^{-1} T^{-2}$

$N = kgm$ $\quad \frac{F}{A} = \frac{N}{m^2}$

CAPILLARITY EFFECT

$h_{actual} = h_m \pm h_{capillarity}$



If mercury is manometer fluid

~~hact = hm~~

$$\boxed{h_{act} = h_m + h_{capillarity}}$$

If other than mercury is manometer fluid

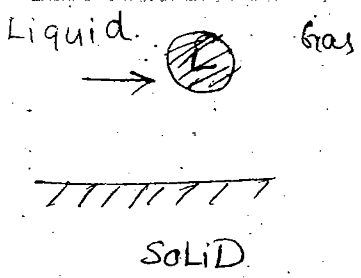
$$\boxed{h_{act} = h_m - h_{capillarity}}$$

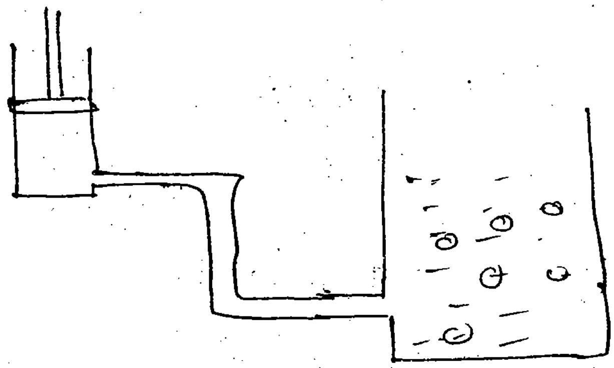
Oil drop falling on solid surface and liquid surface

σ

S L G

σ_{GG} = neglected





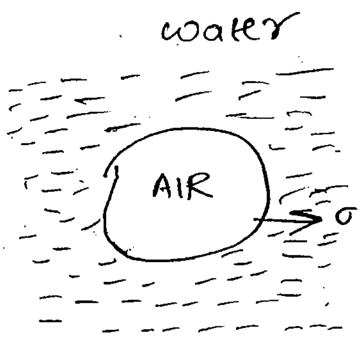
$$P_{AIR\ BUBLE} = K\frac{\sigma}{D}$$

$$\boxed{K = 4}$$

$P_{AIR\ BUBBLE}$

$P_{LIQ\ BUBLE}$

$$\boxed{\begin{aligned} P_{droplet} &= \frac{4\sigma}{D} \\ P_{Bubble} &= \frac{8\sigma}{D} \\ P_{jet} &= \frac{2\sigma}{D} \end{aligned}}$$

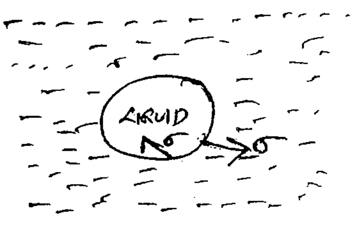


only one surface

$\boxed{\sigma_{GG} \text{ - ignore}}$

A.K JAIN

{ SOM & BISWAS }



2 surfaces

S.S.M

UNIT-II

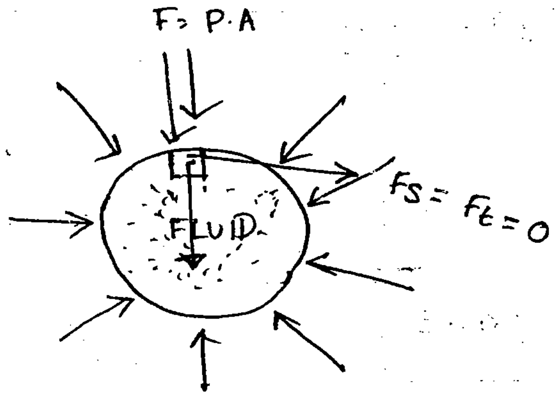
FLUID STATICS

1) Bouyancy force
2) Hydrostatic force
3) Pressure measurement

Fluid statics deals with fluid Behaviour and its effects when it is at Rest.

A fluid is said to be Rest if no shear stresses acting on fluid particle ie no viscosity.

only 2 forces involved
1. Body Force (gravity force) [self wt of fluid matter]
2. Normal Force on the Surface due to Pressure intensity of the fluid.



$P = \frac{F}{A}$ [N/m^2]

Pressure → scalar qty

Compressive stress.

$M = \rho V$

Bouyancy principle

Bouyancy is a phenomena which is observed when a Solid body interact with any liquid. There are 3 possibilities

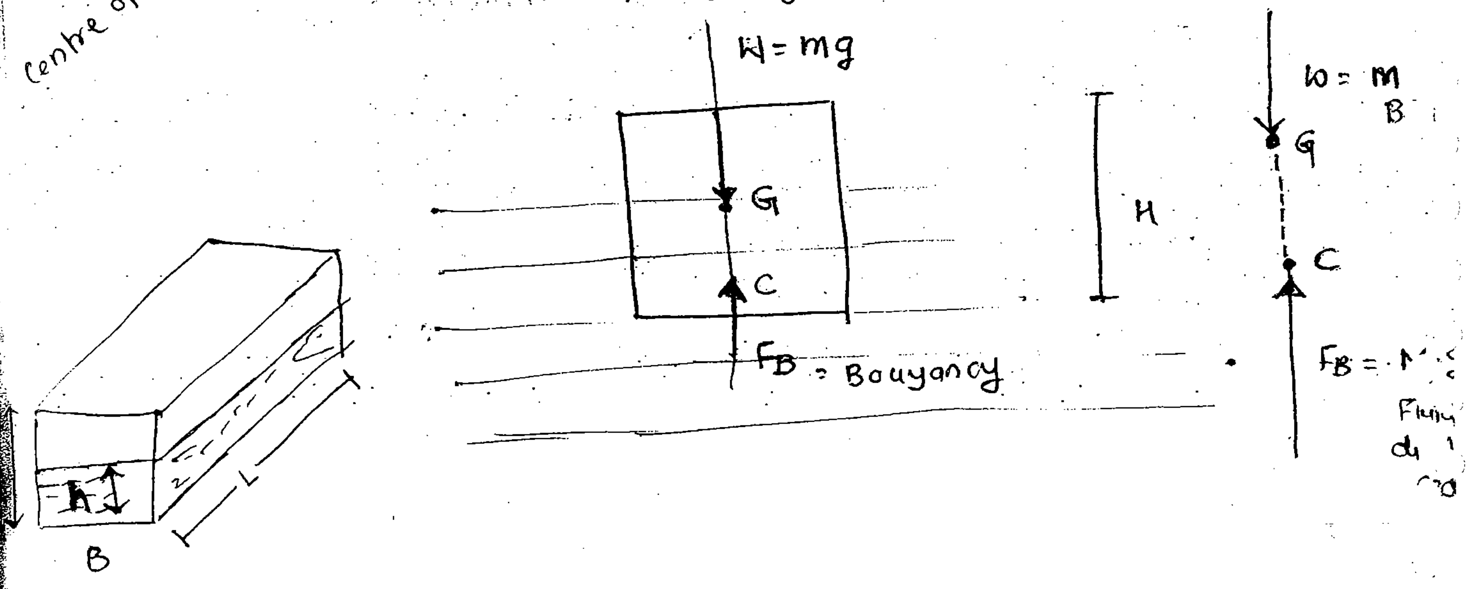
1) the body can float (partially submerged)
2) the Body can sink or (submerged fully)
3) Body top surface coincide with liquid surface

EIC QUES

centre of Bouyancy

G = centre of Gravity

C = Centre of Bouyancy



$$W = M_{body} \cdot g$$

$$= \rho_{Body} \cdot V \cdot g = \rho_{Body} \cdot A \cdot H \cdot g \text{ (Newton)}$$

$$F_B = \underset{\text{displaced}}{M_{Fluid}} \cdot g = \rho_{Fluid} \underset{\text{displaced}}{\forall} \cdot g$$

$$\rho_{Fluid} \cdot A \cdot h \; g \text{ (Newton)}$$

Under Equilibrium

$$F_B = W$$

$$\rho_{fluid} \cdot \cancel{A} \cdot h \cdot \cancel{g} = \rho_{Body} \times \cancel{A} \cdot H \cdot \cancel{g}$$

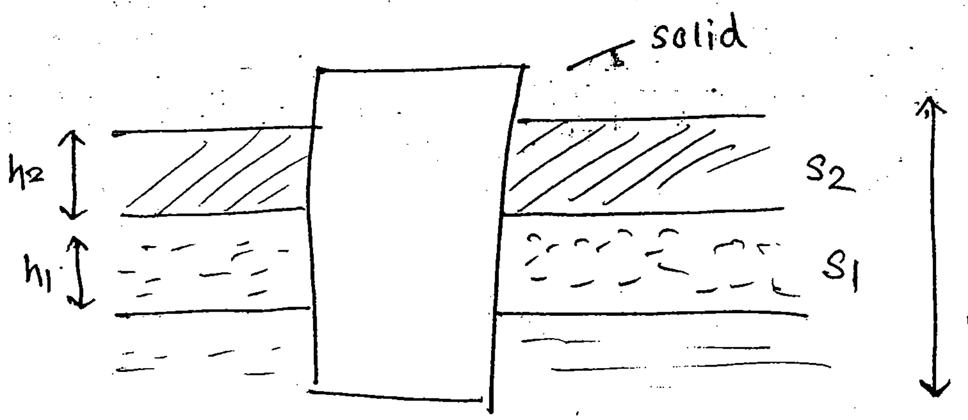
$$\frac{\rho_{Fluid}}{\rho_{water}} \cdot h = \frac{\rho_{Body}}{\rho_{water}} \times H$$

$$S_{Fluid} \cdot h = S_{BODY} \cdot H$$

$$\boxed{\frac{S_{FLUID}}{S_{BODY}} = \frac{H}{h}}$$

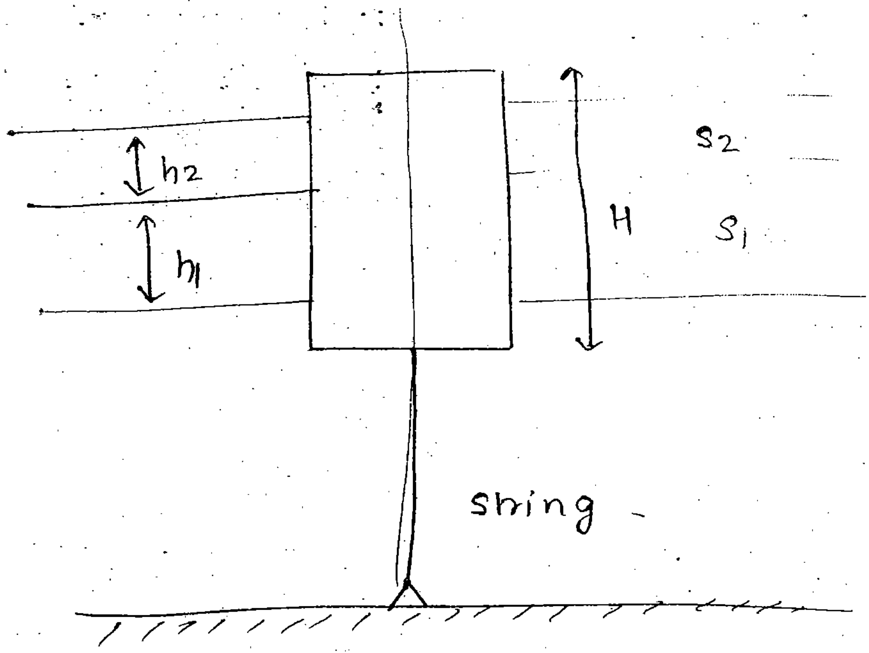
h = depth of immersion

Model 2



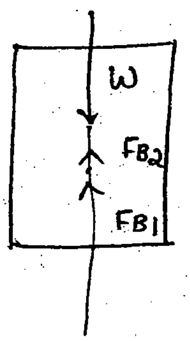
$$\boxed{S_1 \cdot h_1 + S_2 \cdot h_2 = S_{SOLID} \cdot H}$$

$$\boxed{S_{Matter} = \frac{\rho_{MATTER}}{\rho_{WATER}}}$$

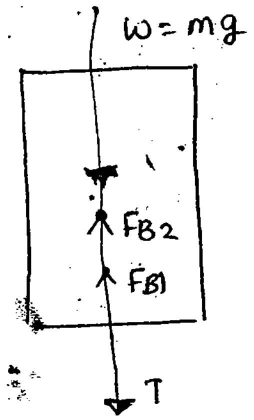


Tension take w.r.t Body

Free Body diagram



Let T = Tension in string



$$\Sigma F_y = 0$$

$$T + mg = F_{B1} + F_{B2}$$

$\underline{T = ?}$

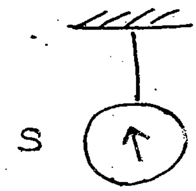
$$T + M_{Body} \cdot g = \rho_{fluid1} \cdot \forall_1 \cdot g + \rho_{Fluid2} \cdot g \cdot \forall_2$$

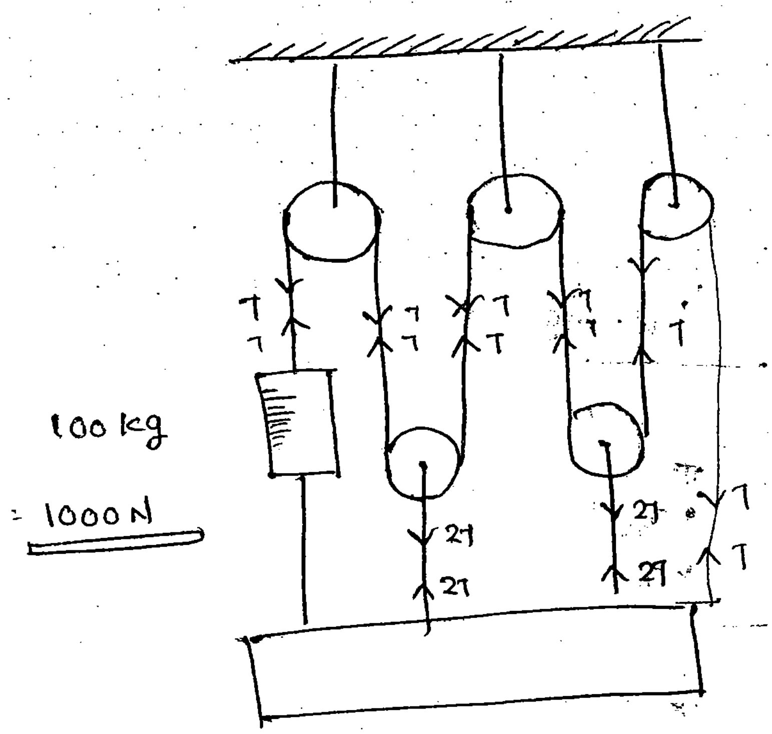
$\forall_1 = A \cdot h_1$ wetted volume of fluid 1

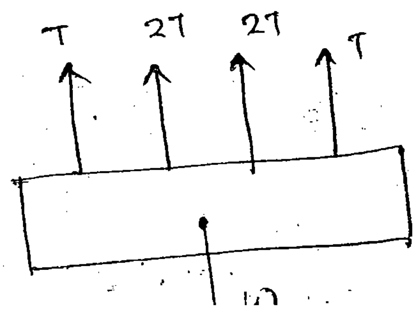
$\forall_2 = A \cdot h_2$,, wetted volume ob fluid 2

Model 4:-

Spring Balance







$6 \times 1000 = W$

$\underline{W = 6000}$

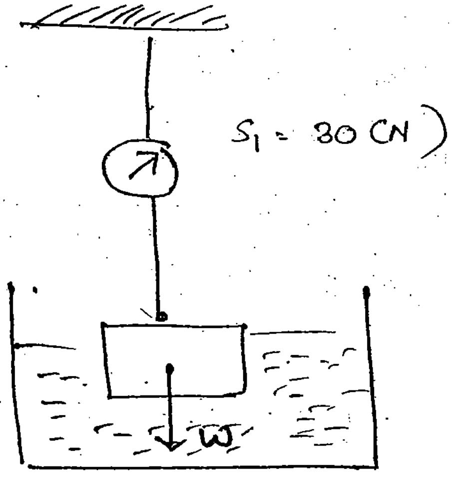
NOTE:

Spring Balance reading (S)

= Tension in string connected to Balance (T)

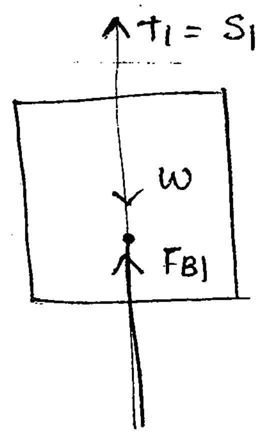
$$\boxed{S = T}$$

Model - 4



Ans:

Case 1 — Free Body diagram



So $F_{B1} + T_1 = W$

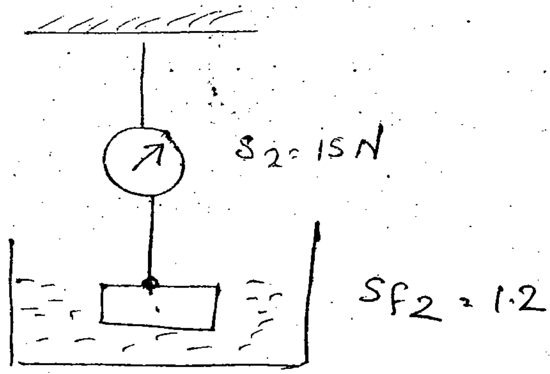
$$\left[\begin{array}{l} \rho_{fluid} \cdot V \cdot g + S_1 = m_{body} \cdot \cancel{g}\, g \\ 8000 \times V \times 10 + 30 = M_{body} \times 10 \end{array}\right]$$

$S_{f1} = 0.8 = \dfrac{\rho_{fluid}}{1000}$

$30 + F_{B1} = W$ —— (1)

$\rho_{fluid} = 800$ kg

same Body



Find (1.) Volume of Body in m^3, in lts

(2.) weight of Body

(3.) ~~s~~ sp ~~gravity~~ wt of Body

(4.) Bouyancy forces offered by fluids.

Case 2

$$15 + F_{B2} = W \quad — \quad (2)$$

$$30 + F_{B1} = W$$
$$15 + F_{B2} = W$$

$$15 + F_{B1} - F_{B2} = 0$$

$$F_{B1} - F_{B2} = -15$$

$$\rho_{fluid1} \times \cancel{A\times} V_{fluid} \cdot g - \rho_{fluid2} \times V_{fluid} \times g = -15$$

$$800 \times \underline{V_{fluid1}} \times 10 - 1200 \times \underline{V_{fluid2}} \times 10 = -15$$

$$V_{fluid} \left[8000 - 12000 \right] = -15$$

$$V_{fluid} = \frac{-15}{-4000} = \underline{3.75 \times 10^{-3}\ m^3}$$

$15 + F_{B2} = W$

$\boxed{\uparrow\uparrow}$

$F_{B1} = \rho_{f_1} \cdot \forall_1 \times g$

$= 800 \times 3.75 \times 10^{-3} \times 10$

$\underline{30N}$

$|||^{ly}$ $F_{B2} = \rho_{f2} \cdot \forall_2 \times g$

$= 1200 \times 3.75 \times 10^{-3} \times 10$

$= \underline{45N}$

[Low density fluid - Low F_B]

$\frac{F_{B2}}{F_{B1}} = \frac{45}{30} = \frac{3}{2}$

$30 + F_{B1} = W$

$30 + 30 = W$

$\underline{W = 60 N}$

(3) Sp wt. $\gamma = \frac{W}{\text{Volume}}$

$\frac{60}{3.75 \times 10^{-3}} = \underline{15.7 \text{ kN/m}^3}$

(4.) Specific gravity $= \frac{W_{body}}{W_{water}} = \frac{15.7 \times 1000}{1000 \times 10} = \underline{1.57}$

Q.) Two ~~spheres~~ Bodies one is solid sphere and another Body is hollow cube which are in equal Surface Area, what is the Ratio of the Bouyancy force by the fluid on the sphere to that on the cube ?

$$\frac{F_B\ \text{sphere}}{F_B\ \text{cube}} = ?$$

Solid sphere

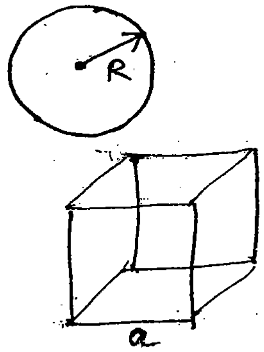
$$\longrightarrow \quad F_B = \rho_{fluid} \cdot A \times h \times g$$

$$F_B \propto \cancel{A}\ h$$

$$\frac{F_B\ \text{sphere}}{F_B\ \text{cube}} = \cancel{\frac{4\pi R^2}{a^2}}\ \frac{h\ \text{sphere}}{h\ \text{cube}}$$

Given

$$A_{\substack{\text{surface}\\ \text{Area}}} = A_{cube}$$



$$4\pi R^2 = 6a^2$$

$$R^2 = \frac{6a^2}{4\pi}$$

$$R = \frac{\sqrt{6}\,a}{2\sqrt{\pi}}$$

$$\frac{F_{B\,sphere}}{F_{B\,cube}} = \frac{W_{fluid\,spher}}{W_{fluid\,cube}} = \frac{M_{f\,sphere}\,\cancel{g}}{M_{f\,cube}\,\cancel{g}}$$

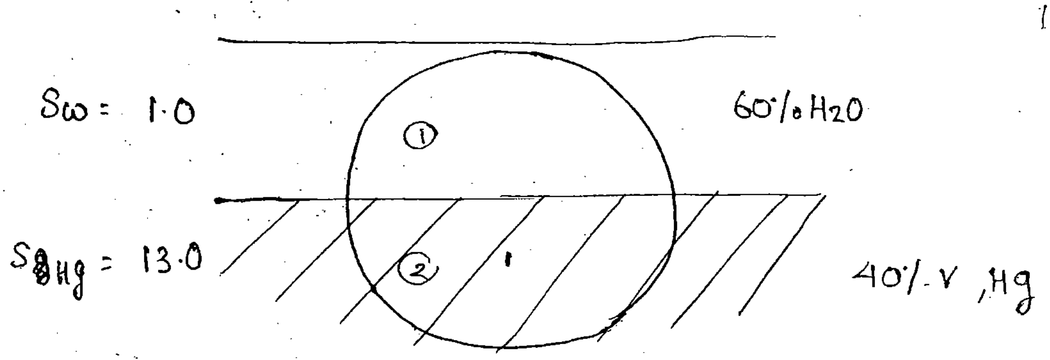
$$= \frac{\cancel{\rho_{fluid}} \times \forall_{sphere}}{\cancel{\rho_{fluid}} \times \forall_{cube}}$$

$$= \frac{\frac{4}{3}\pi R^3}{a^2 . a}$$

$$\frac{\frac{\cancel{4}}{3}\cancel{\pi} R^3}{\frac{\cancel{4\pi} R^2}{6} . \sqrt{\frac{4\pi R^2}{6}}}$$

$$\frac{F_{B\,sphere}}{F_{B\,cube}} = \sqrt{\frac{6}{\pi}}$$

9.) An unknown sp gravity of Body of uniform c/s is allowed to submerge in two fluid system shown in fig. 40% of Body volume submerged in mercury fluid and remaining 60% volume of Body in water. Then specific gravity of Body is ?



$$S_w \cdot h_1 + S_{Hg} \cdot h_2 = S_{solid} \times H .$$

$\cancel{S_w \forall_1}$

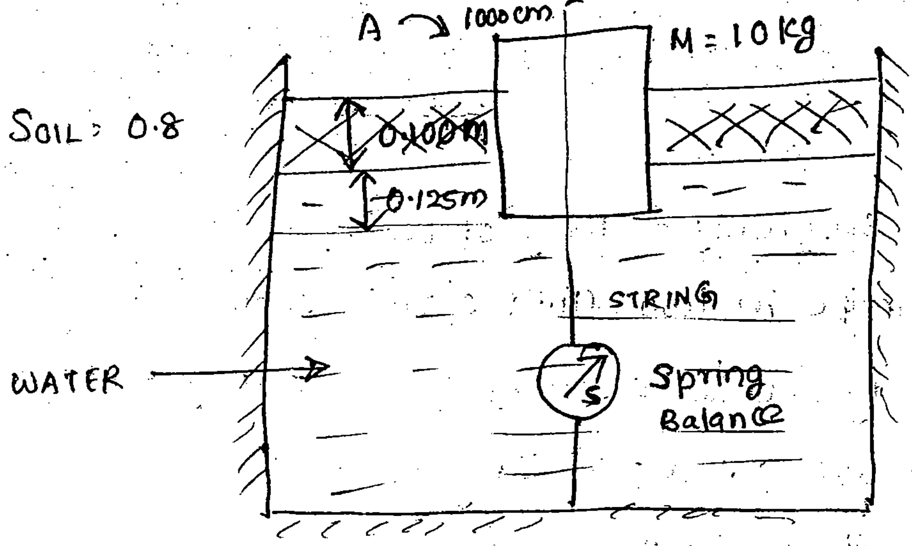
$$S_w \cdot \forall_1 + S_{Hg} \cdot \forall_2 = S_{solid} \cdot \forall$$

$\cancel{13.6 \times 0.4 \forall}$

$$1 \times 0.6\cancel{V} + 13.6 \times 0.4\cancel{V} = S_{solid} \times \cancel{V}$$

$$\underline{S_{solid} = 6.04}$$

Q.) A 10 kg Body; 1000 cm² c/s Area is kept in a Layered fluid system and connected to a string of the spring Balance at the Bottom of the container. The Arrangement shown in figure.

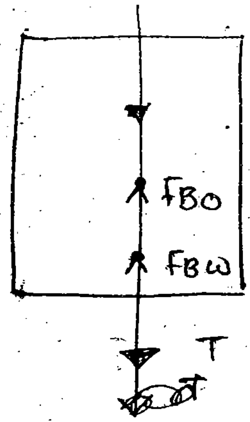


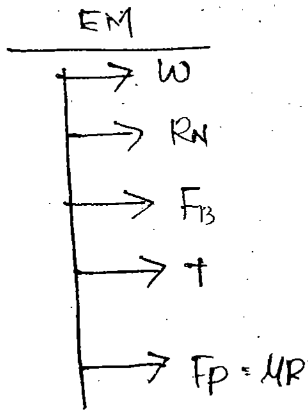
Determine ① spring Balance Reading

② Gauge pressure at the Bottom of Body

③ Ratio of the Bouyancy forces offered by water to that of oil.

Ans: $S_{oil} \times 0.100 + S_{water} \times 0.125 = S_{solid} \times H$





$F_{B1} + F_{B2} = W + T$

$\rho_w \times A \times h_1 g$ ~~$\rho_{\dots}$~~ $\rho_{oil} \times A \times h_2 g =$ ~~$W + T$~~ ~~$W + T$~~ $W + T$

FREE SURFACE OF liquids

↓

NO SHEAR STRESS

$9.81 \times 1000 \times 1000 \times 10^{-4} \times 0.125 + 0.8 \times 1000 \times 1000 \times 10^{-4} \times 0.100 \times 9.81$

$205 = 10 \times 9.81 + T$

$T = 103.0\ N$

(3.) ~~Gauge pressure~~

$$\frac{F_{B\,water}}{F_{B\,oil}} = \frac{\rho_w \times A_w \times h_w \times g}{\rho_{oil} \times A_{oil} \times h_{oil} \times g}$$

$$= \frac{1000 \times .10 \times 0.125 \times 9.81}{0.8 \times 10^3 \times 0.1 \times 0.100 \times 9.81}$$

$= 1.56$

(2.) Gauge pressure

$P = P_{oil} + P_{water}$

$= 1000 \times 9.81 \times 0.125 + 800 \times 9.81 \times 0.1$

$= 2011\ N/m^2$

$= 2.011\ kPa$

(Q) A cubical steel Body [s = 7.85] of each side 10 cm is immersed in liquid mercury. water is added to top of solid Body till submerged. Determine the following

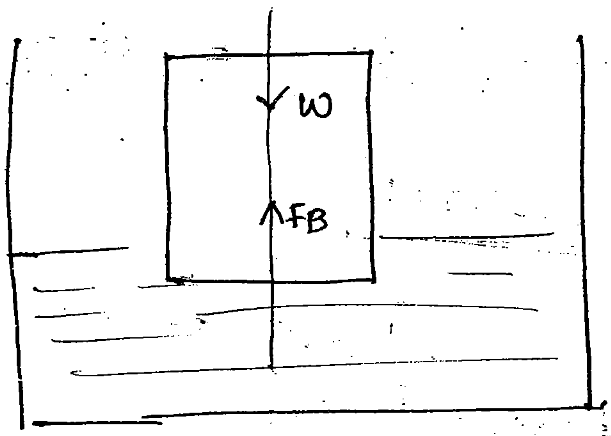
(1) Depth of mercury when water is not included.

(2) Depth of water

Ans: Case I

~~A~~ $S_{steel}: 7.85$

~~S = 7.85~~ $\rho_{steel} = 7850\ kg/m^3$

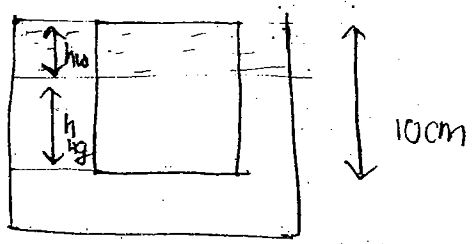


$S_{Hg} \times h_{hg} = S_{solid} \times H$

$13.6 \times h = 7.85 \times 10$

$\cancel{H = }$ $\frac{13.6}{7.85 \times 10}$

After adding water



$\boxed{h_{Hg_1} = \cancel{5.57}\ 5.772\ cm}$

$$S_w \times h_w + S_{Hg} \times h_{Hg} = S_{solid} \times H$$

$$1 \times h_w + 13.6 \times \overset{(10-h_w)}{\cancel{5.772}} = 7.85 \times 10$$

$H = h_g + h_w$

$h_g = H - h_w$

$10 - h_w$

$h_w =$

$$h_w + 136 - 13.6\,h_w = 78.5$$

$$136 - 12.6\,h_w = 78.5$$

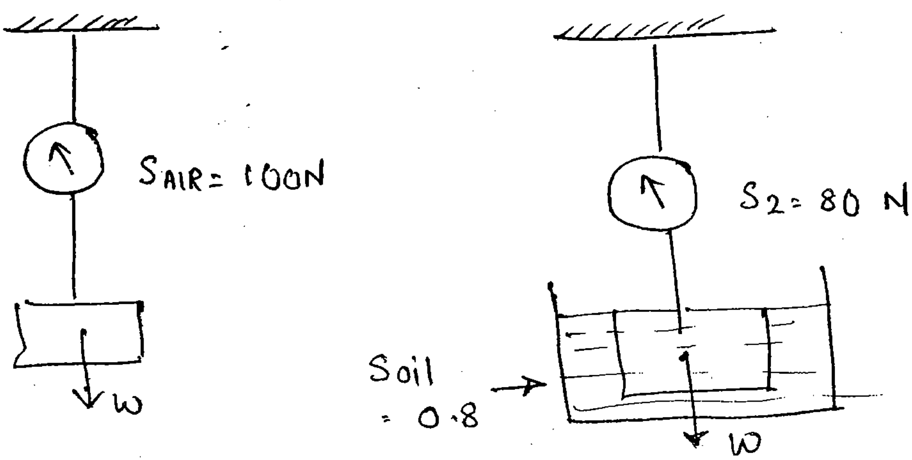
$$h_w = \cancel{7}\ 4.56\ cm$$

$$h_{Hg} = 5.44\ cm$$

NOTE: When H_2O is added to Hg the height of immersion decreases.

Model No: 5

Finding specific gravity of solids and liquids



$$\boxed{S_{SOLID} = \left[\frac{W_{AIR}}{W_{AIR} - W_{OIL}}\right] \times S_{OIL}}$$

$$= \frac{100}{100-80} \times 0.8$$

$$= \frac{100}{20} \times 0.8$$

$$= 5 \times 0.8$$

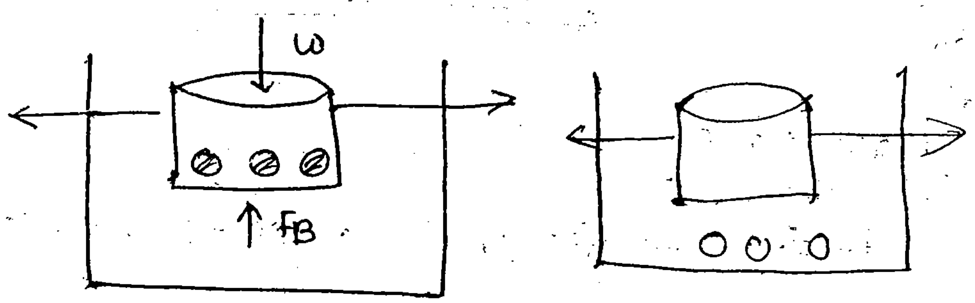
$$= \underline{4}$$

Q.) S ↑ ρ ↑

F_B ↑

$\boxed{\text{So h is less}}$

Q.)



h same

cube having same surface Area

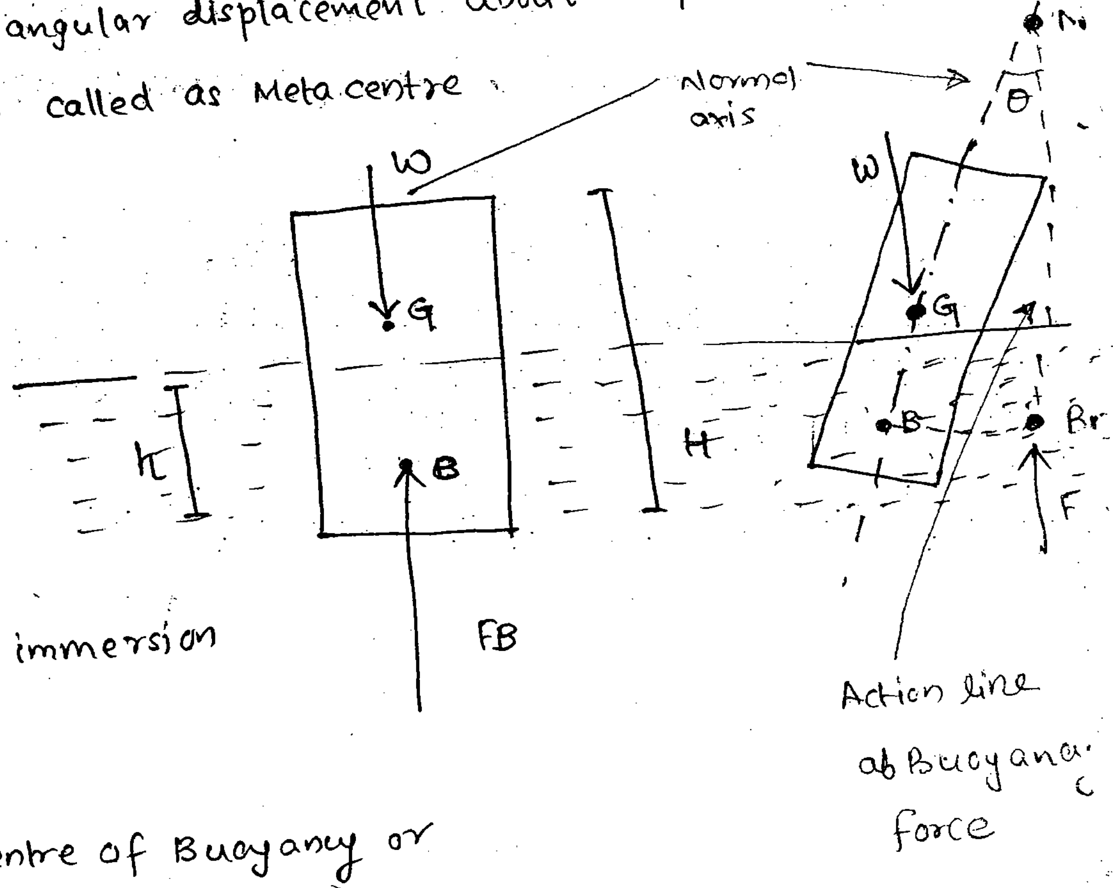
STABILITY OF FLOATING BODIES

when a Body is allowed to float in liquid freely, then two forces are involved

→ self wt of Body

→ Buoyancy force offered by the fluid

when small disturbance is applied to the body causing angular displacement about a particular point is called as Meta centre



h = depth of immersion

B = centre of Buoyancy or centre of gravity of volume of fluid displaced

→ Metacentre point M is obtained by intersecting the Normal axis of the Body ~~with~~ and the action line of Buoyancy force

Conditions of Eqbm

1. stable Eqbm
2. unstable equilibrium
3. Nuetral Equilibrium

Stable Equilibrium

After the floating Body distribed and gaining its initial position without over turning is said to be in stable equilibrium. This happens due to the Restorting Couple of the Bouyancy force against the disturbed couple formed by self wt 'w'.

- It is possible if M is located above 'G' ie

GM = Metacentric Height

$GM > 0$

More ~~ex~~ the Metacentric Height, More the Stability of Floating Body

unstable Equilibrium

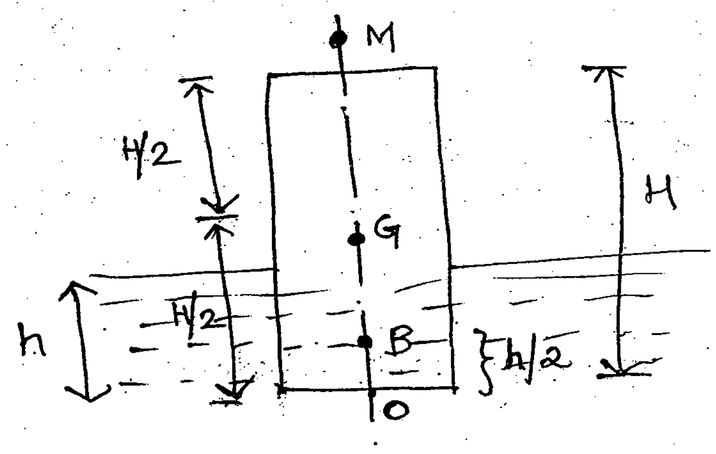
When a floating Body is distribed and the Body can't return its original position, then Body is said to be in unstable Eqbm. The Result is overturning. This Happens if M is located Below 'G'

If M and G coincides, then Metacentric height $GM = 0$, then Body is said to be in Nuetral eqbm. It means than Body will not turn, instead it moves in translational motion ($\theta = 0°$)

Methods to find Metacentric Height (GM)

1. Analytical Method
2. Experimental Method

Analytical Method



$$GM = BM - BG$$

$$= BM - [OG - OB]$$

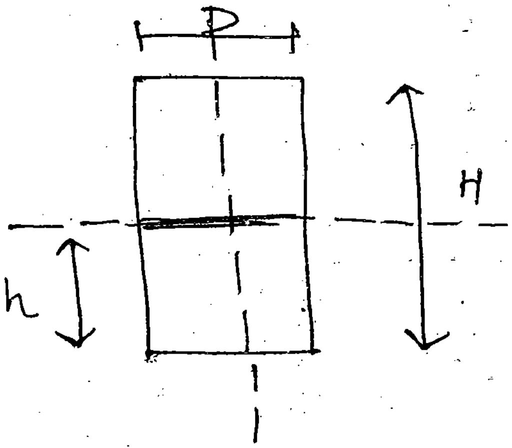
$$BM - [H/2 - h/2]$$

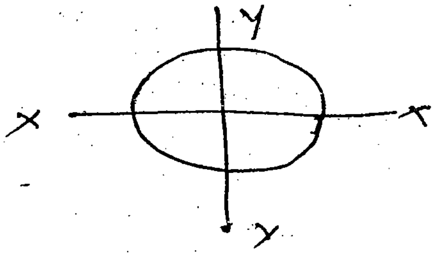
$BM = \frac{I}{\forall}$ → MOI of ~~top A~~ plane Area at Top view (PLAN) of Floating Body at liquid level

wetted volume = A c/s × h

Calculation of I

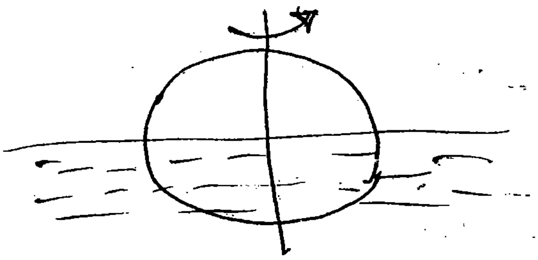
(1.) A cylinder floating vertically

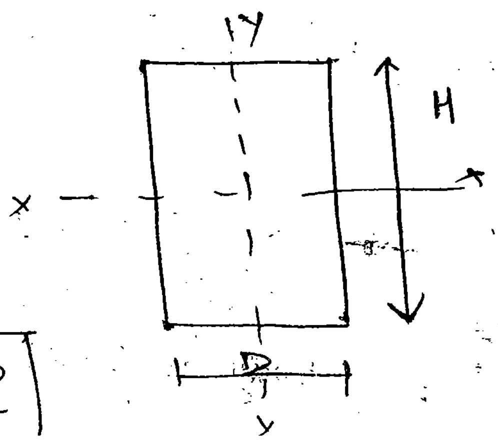




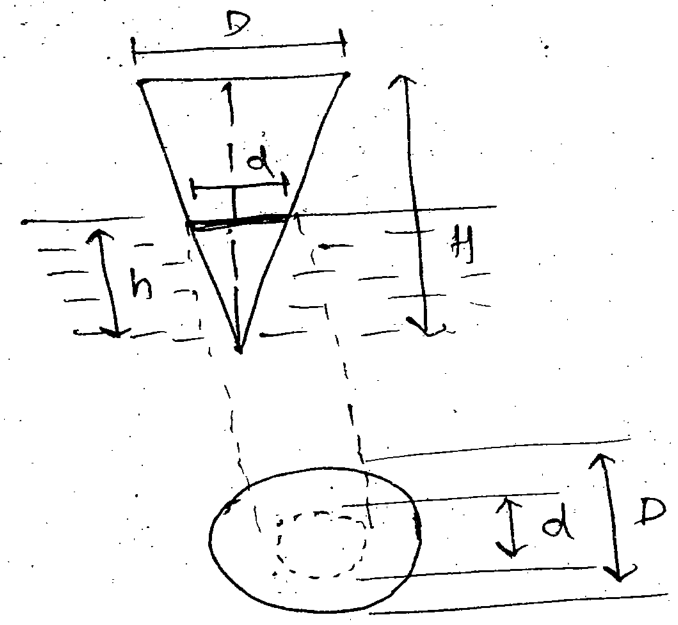
$$I_{yy} \text{ at liquid level} = \frac{\pi}{64} d^4$$

(2.) A cylinder floating Horizontally





$\boxed{I_{yy} = HD^3}$



$$\boxed{\frac{D}{H} = \frac{d}{h}}$$

$$\boxed{I_{yy} = \frac{\pi}{64}\left[d^4\right]}$$

Frequencies of Floating Bodies in Rolling

Imp. eqn

$$\frac{d^2\theta}{dt^2} + \omega_n^2\theta = 0$$

$$\boxed{t_p = 2\pi\sqrt{\frac{k^2}{g \cdot GM}}}$$

$$f_n = \frac{1}{t_p} \text{ c.p.s} \qquad \omega_n = \frac{2\pi}{t_p}\ (\text{rad/sec})$$

$\downarrow$

Hertz

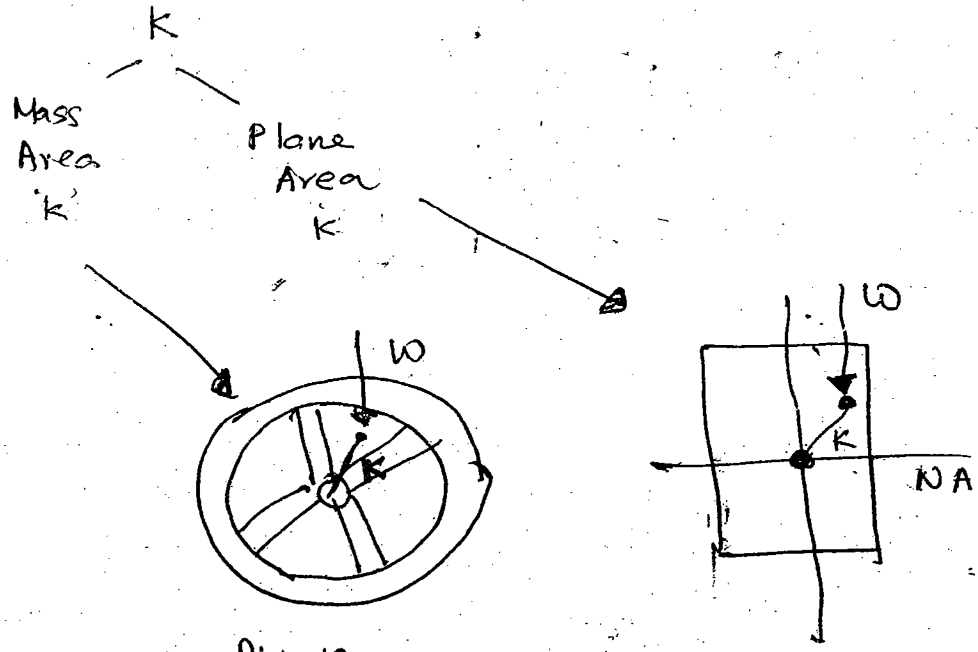
$\downarrow$

Rev/sec

(rps)

$$K = \sqrt{\frac{I}{A}}$$

K: Radius of Gyration



Bicycle wheel

Q) A cylinder made of unknown material of diameter 4m and height also 4cm allowed to float in a fluid of relative density 1.4. It is observed that the depth of immersion is 2.4m. Determine the following.

1. specific gravity of cylinder material
2. Volume of fluid displaced ($\forall$)
3. Metacentric Height
4. Check for stability: [stable or unstable]

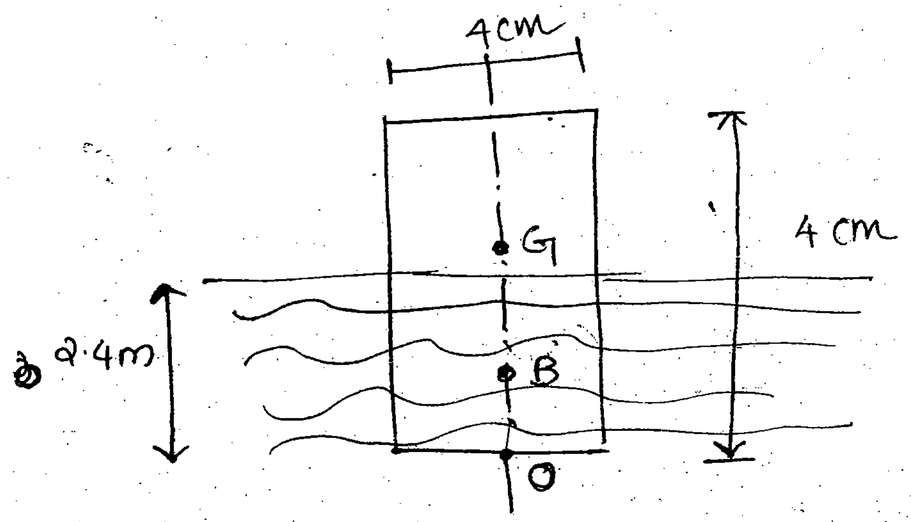
of the cylinder in terms of dia to acheive stability

S_{solid}

$S_{fluid} = 1.4$

$\rho_{fluid} = 1.4 \times 1000 \ \frac{kg}{m3}$

$= \underline{1400 \ kg/m3}$



① $S_{fluid} \cdot h = S_{solid} \times H$

$1.4 \times 2.4 = S_{solid} \times 4$

$S_{solid} = \frac{1.4 \times 2.4}{4} = \underline{0.84}$ Non met-

② $\forall = A \ c/s \times h$

$= \frac{\pi}{4} \times D^2 \times 2.4$

$\frac{\pi}{4} \times (4)^2 \times 2.4$

$\underline{30.16 \ m^3}$

Bouyancy force offered by fluid ($s = 1.4$)

$$F_B = \gamma \times \forall$$

$$(1400 \times 9.81) \times 30.16$$

$$= \underline{\underline{\quad\quad kN}}$$

$$\boxed{\begin{aligned} F_B &= m_{fluid} \cdot g \\ &= \rho_{fluid} \times \forall \cdot g \\ &= \gamma \times \forall \end{aligned}}$$

(3) Metacentric Height (GM_o)

$$GM = BM - BG$$

$$= \frac{I}{\forall} - \left[\frac{H}{2} - \frac{h}{2}\right]$$

$$= \frac{\pi}{64}[4]^4 - \left[\frac{4}{2} - \frac{2.4}{2}\right]$$

$$\underline{\underline{GM = -0.384\,m}}$$

Below 'G'

GM < 0 [means M is below G]

Body is under unstable equilibrium

$$BM - BG > 0$$

$$\frac{I}{\forall} - \left[\frac{H}{2} - \frac{h}{2}\right] > 0$$

$$\frac{I}{\forall} > \frac{H}{2} - \frac{h}{2}$$

$$\frac{\frac{\pi}{64} \times D^4}{\frac{\pi}{4} \times D^2 \times h} > \frac{H}{2} - \frac{h}{2}$$

$$\frac{D^2}{16h} > \frac{H}{2} - \frac{h}{2}$$

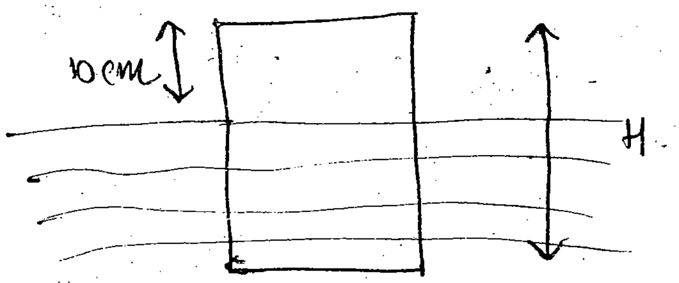
$$\frac{D^2}{8h} > H - h$$

$H =$

Replace $h = \dfrac{S_{solid} \times H}{S_{liquid}} =$

$\therefore$ $H =$

(Q) An ice Block is floating in Seawater shown in figure



Specific gravity of ice = 0.925

Sp gravity of sea water = 1.025. The exposed height is 10 cm. determine Height of ice block ?

Ans:-

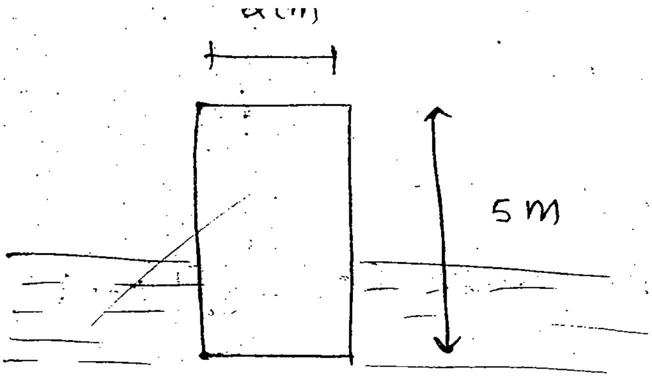
$$S_{ice} \times H = S_{seawater} \times h$$

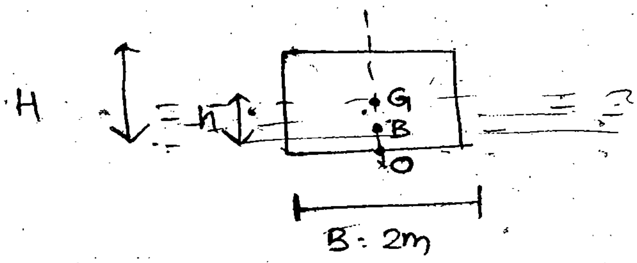
$$0.925 \times H = 1.025 \times (H - 10)$$

$$H = 102.5 \text{ cm}$$

A Rectangular Block 5 m long and 2 m width is floating in water. The water line is 0.4 m above the Bottom and centre of Gravity is 0.8 m from Bottom then the Body metacentric Height will be

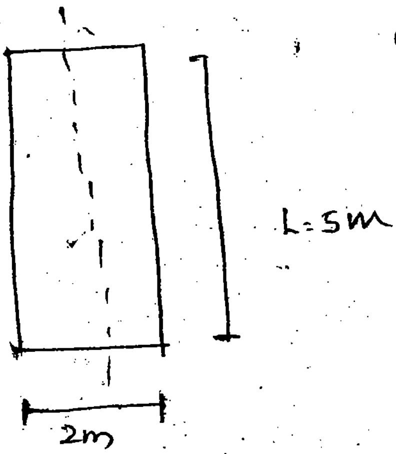
a) 0.83 m b) 0.6 m c) 0.46 m d) 0.23 m





$h = 0.4\ m$

$OG = 0.8\ m$



$$I_{yy} = \frac{5\times(2)^3}{12}$$

$$GM = \frac{I}{\forall} - \left[\frac{H}{2} - \frac{h}{2}\right]$$

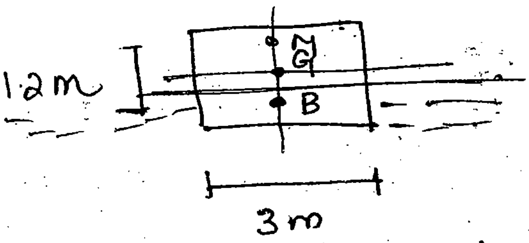
$$\frac{\frac{5\times(2)^3}{12}}{A_{cs}\times h} - \Big[\quad\Big]$$

$$GM \quad \frac{5\times(2)^3}{12} - \left[0.8 - \frac{0.4}{2}\right]$$

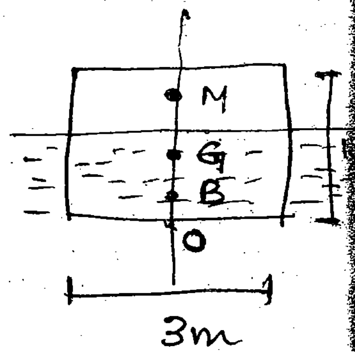
$GM = 0.23\,m$

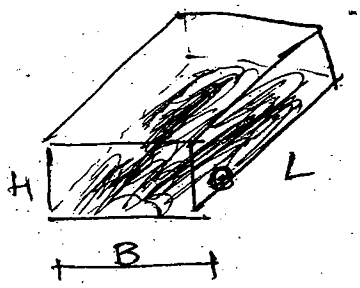
Q.) A Rectangular Block 5m long 3m width and 1.2m depth [s = 2/3] kept horizontally in a liquid [s = 1.025]. State the condition of eqbm.

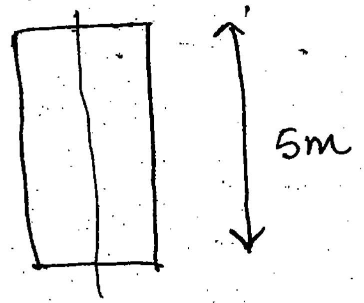
Ans:



$S_{solid} = 0.66$







$$GM = \frac{I}{V} - \left[\frac{H}{2} - \frac{h}{2}\right]$$

$$S_{solid} \times H = S_{fluid} \times h$$

$$h = \frac{0.66 \times 1.2}{1.025} = 0.78$$

$$I_{yy} = \frac{5 \times 3^3}{12} =$$

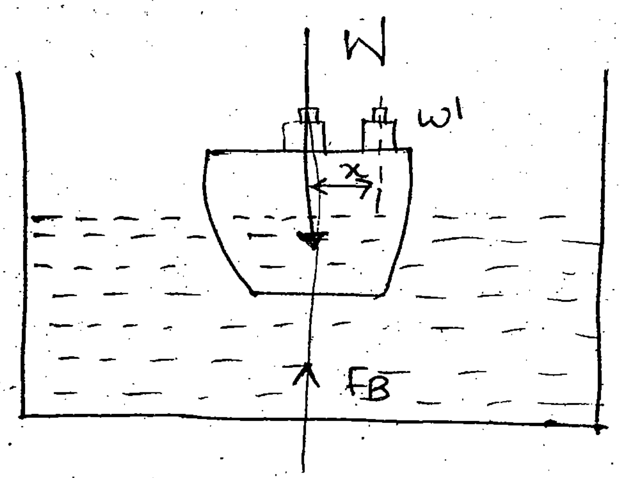
$$GM = \frac{5 \times 3^3}{\ } \left[\quad 0.78 \right]$$

$GM = 0.75 m$ /

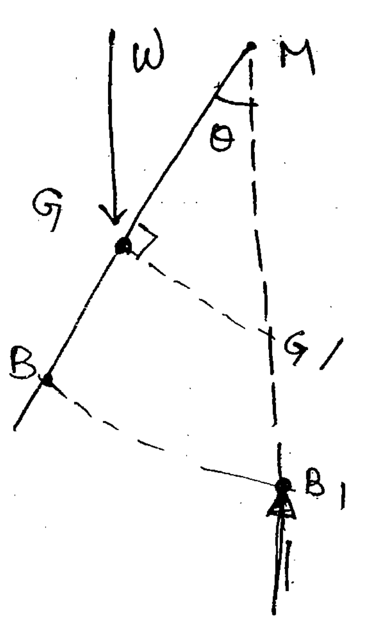
∴ Stable equilibrium

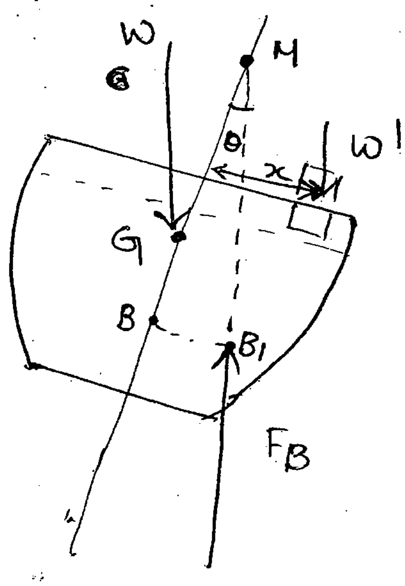
Experimental method to find metacentric Height

In practice like stability of large sized ships and vessels it is easy to check the stability along with the model testing analysis for which the given object is scaled down and applied in ideal conditions of fluid.



Moment about M





$$\tan\theta = \frac{GG'}{GM}$$

$GG' = \tan\theta\, GM$

$$W' \cdot x = W \cdot GG'$$

$$W' \cdot x = W \times GM \tan\theta$$

$$\boxed{GM = \frac{W' \cdot x}{W \tan\theta}}$$

It will give accuracy since θ can't be accurately measured.

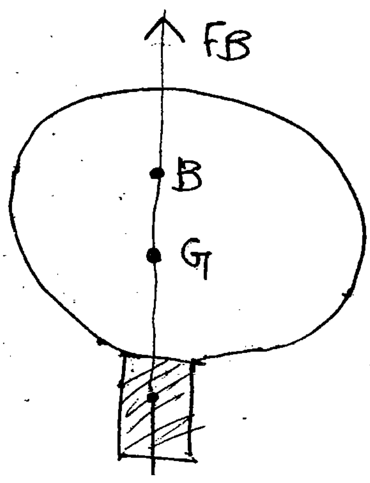
STABILITY OF SUBMERGED BODIES IN FLUIDS

NOTE:-

when a body is submerged fully in any fluid then no meta centre is involved, then the stability is obtained by locating the point B with reference to G.

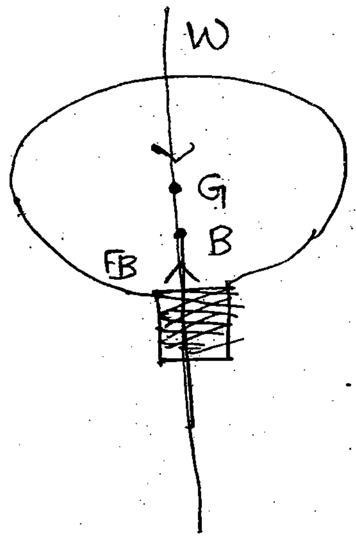
ex:1:- Stable equilibrium for submerged Body

B is above G
it makes body return to its original position



Unstable Equilibrium for submerged Body

B is below G. It cannot Return to its original position



B and G coincides

$\theta = 0$

Moves in Translational motion.

Q.) For a Body Completely submerged in a fluid, the centre of Gravity is 'G' and centre of Bouyancy is 'O' which are known. The Body is considered to be in stable equilibrium if

a) O doesnot coincide with centre of mass of the displaced fluid

b) G coincides with centre of mass of fluid displaced.

c) O lies Below the G

~~d)~~ O lies above the G

Q) A Floating Body with its centre of Gravity at G, centre of Buoyancy at B and metacentre at M is stable when, ~~G lies above~~

a) G lies above B

b) B lies above M

c) B lies below M

d) G lies below M

UNIT II

(3) HYDROSTATIC FORCES

Hydrostatic forces on fluid contact surfaces:-

1. Introduction
2. Hydrostatic force, magnitude and its point of application (Centre of pressure).
3. Hydrostatic force on horizontal plane surface, vertical plane surface, inclined plane surface, curved surface.
4. Moments of Hydrostatic forces.

When a fluid is in contact with any surface

→ pressure is exerted (Normal) which depends on sp wt of fluid and depth of fluid. The Hydrostatic force is the product of Normal pressure and the wetted surface Area.

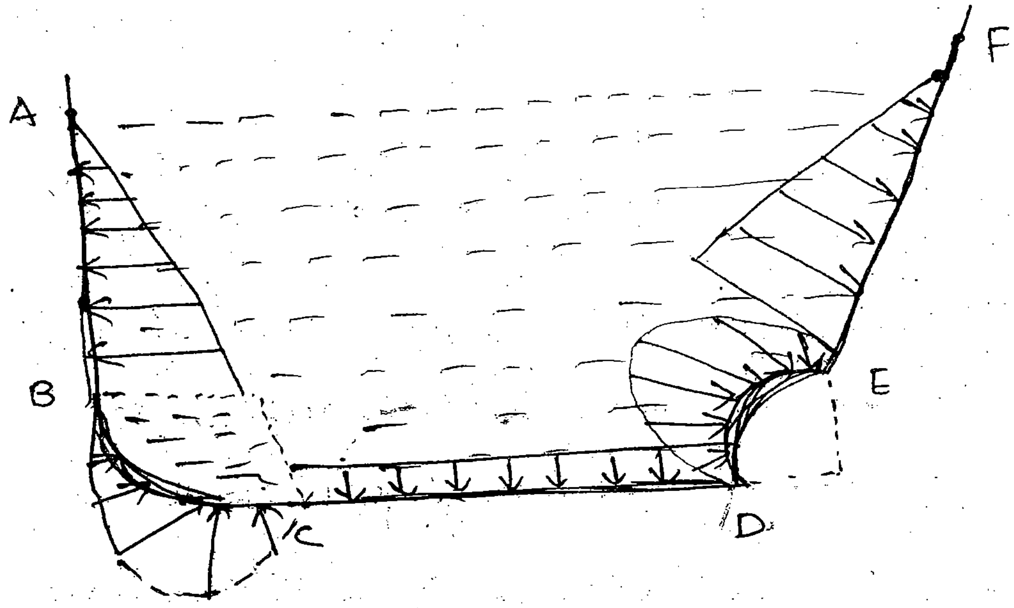
Hydro-static force (F_n)

$$= P \times A$$

$$= \rho g \bar{h} \times A$$

$$= (\gamma \bar{h}) \times A$$

to centroid of the plane surface in contact with fluid.



Horizontal surface = CD

Vertical surface = AB

$$\frac{dp}{dz} = -\gamma$$

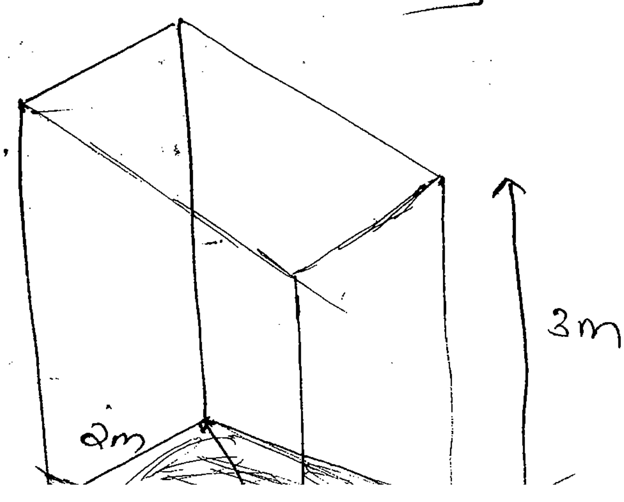
$$\frac{dp}{dh} = -\gamma$$

$P = \gamma \bar{h}$

$\frac{P}{\bar{h}} = \gamma$

[Free surface – free ~~surfa~~ from shear]

<u>Example</u>:-



$F_{horizontal} = \rho g \bar{h} \cdot A$

$= 1000 \times 10 \times 3 \times (2 \times 1)$

$= 60\ kN$

$$F_{vertical} = \rho g \bar{h} \times A$$

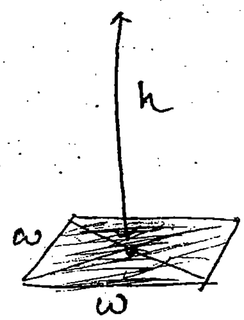
$$= 1000 \times 10 \times \frac{H}{2} \times [3 \times 2]$$

$$1000 \times 10 \times \frac{3}{2} \times [3]$$

$$= \underline{90\ KN}$$

Q.) A tank with 4 equal vertical ~~per~~ planes of width w and depth h is completely filled with liquid. If a hydrostatic force of any vertical plane surface is equal to that force at the Bottom, then Ratio of H/w will be

a) 2 (b) $\sqrt{2}$ (c) 1 (d) 0.5

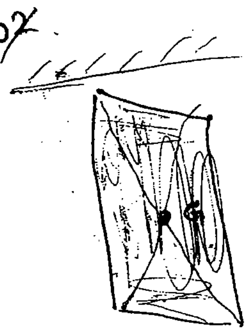


ie

$$F_{vertical} = F_{Horizontal}$$

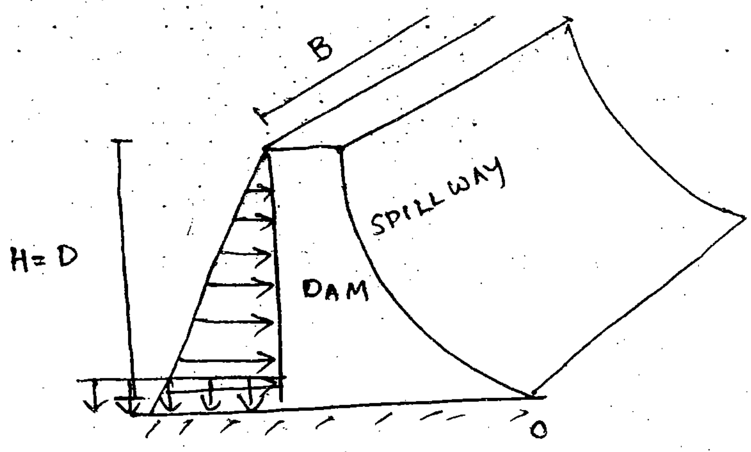
$$\cancel{\rho \times g \times H \times [h \times w] h [w \times h] = \rho \times g \times}$$

$$\cancel{\rho} \times \cancel{g} \times \frac{\cancel{h}}{2} \times h \times \cancel{w} = \cancel{\rho} \times \cancel{g} \times \cancel{h} \times w^{\cancel{2}}$$

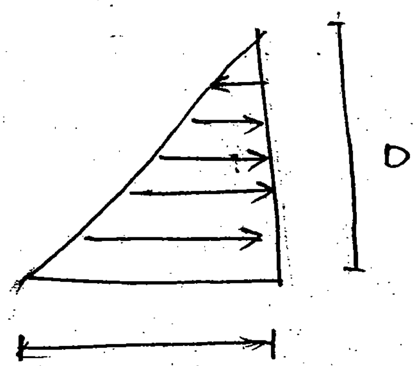


$$h = 2w$$

$$\boxed{\frac{H}{w} = 2}$$



SPILL WAY DAM



$P = \gamma \cdot D$

Case I

$= \rho g \cdot D$

Pressure intensity at Bottom $= \gamma \cdot D$

$= \gamma \cdot H$

$= \rho \cdot g \cdot H$ N/m2

Hydrostatic Force (N)

or

Total Pressure. (N)

$F = \rho \cdot g \cdot H \cdot A_{Horizontal}$

Force normal to horizontal floor

Case 2

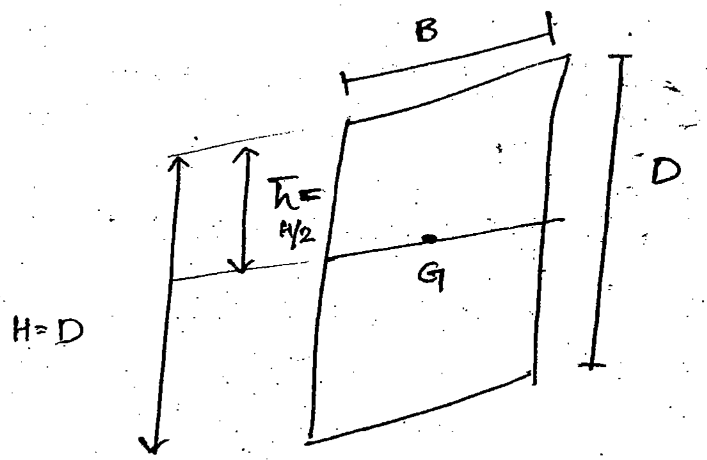
Hydrostatic force on vertical plane surface of dam

$$P = \rho g \bar{h} = \gamma \cdot \bar{h}$$

$$= \gamma \cdot \frac{D}{2}$$

$$\gamma \cdot \frac{H}{2}$$

where $\bar{h}$ = distance from free surface level to centroid of vertical plane surface.

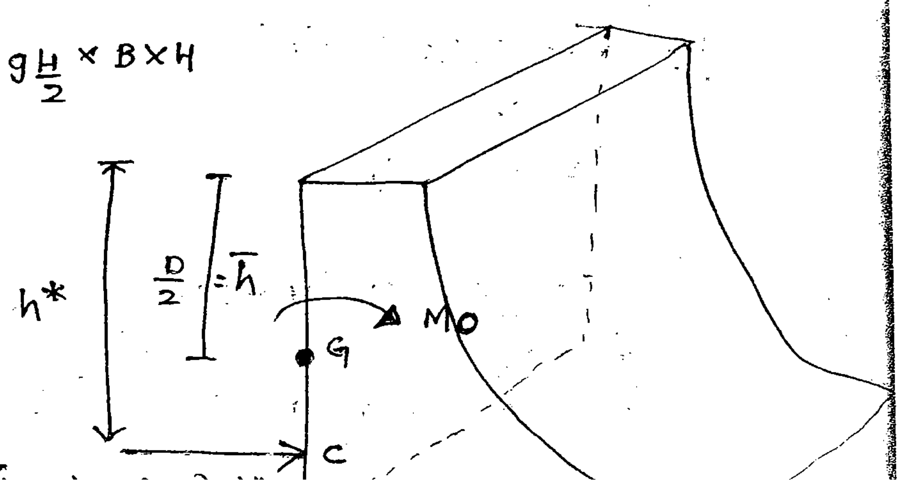


Total pressure force

$$F_H = P \times A_{Normal}$$

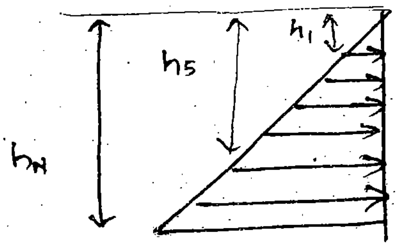
$$= \rho g \bar{h} \times A$$

$$= \rho g \frac{H}{2} \times B \times H$$



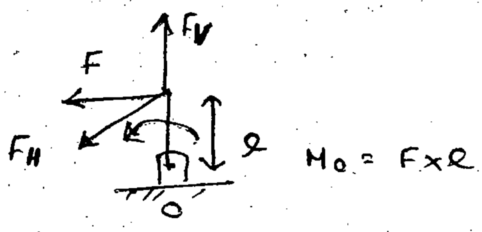
level to point where hydrostatic force acts

h^* calculation



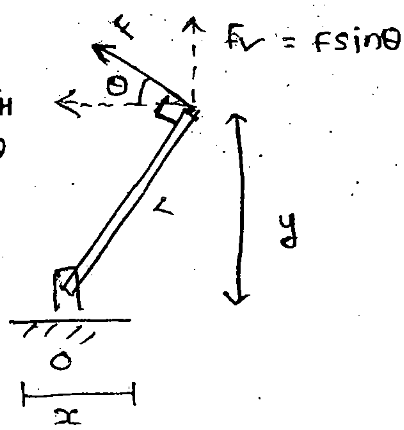
PRESSURE DIAGRAM

Principle of Moments



Moment of Force Components F_V and F_H about O :

$$\boxed{M_o = -F_V \times x = -F_H \times y}$$



$$M_o = F \times L$$

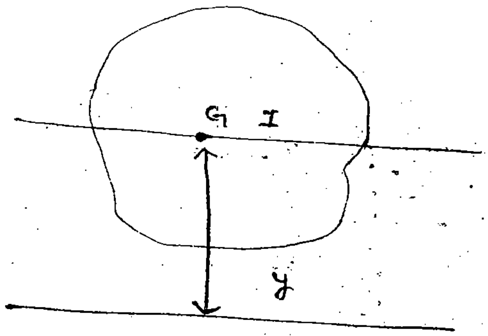
Moment of Hydrostatic forces about free liquid surface

$$\Sigma M_F = F_1 \times h_1 + F_2 \times h_2 + \cdots F_N \cdot H_N$$

$$\Sigma M_F = F_H \times h^*$$

$$h^* = \frac{F_1 \times h_1 + F_2 \times h_2 + \cdots F_n h_n}{\quad}$$

$$\boxed{h^* = \frac{\sum_{i=1}^{N} F_i h}{F_H}}$$



$$\boxed{I_{xx} = I_{CG} + Ay^2}$$

$$\boxed{h^* = \bar{h} + \frac{I_{CG}}{A \cdot \bar{h}}}$$

Lever Arm (Moment Arm) for F_H about

$O = \frac{H}{3}$

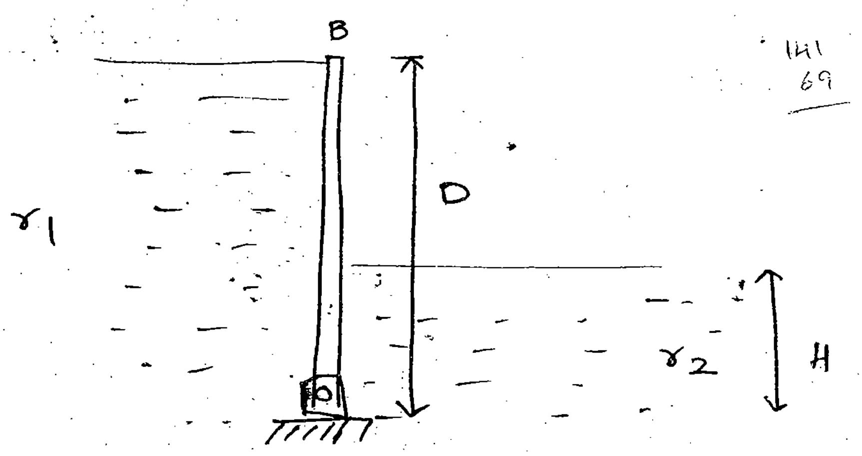
Moment of F_H about O

$$= F_H \times \frac{H}{3}$$

$$= \left[\gamma \times \frac{H}{2} \times \underbrace{\frac{BH}{1}}_{\text{Area}} \right] \times \frac{H}{3}$$

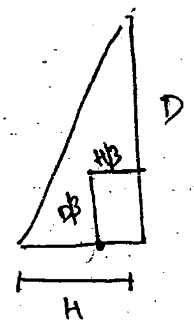
A Rectangular plate of dimensions $B \times D$ (width × Depth) kept Vertically and supporting a fluid of Specific wt γ_1 on one side and supporting another

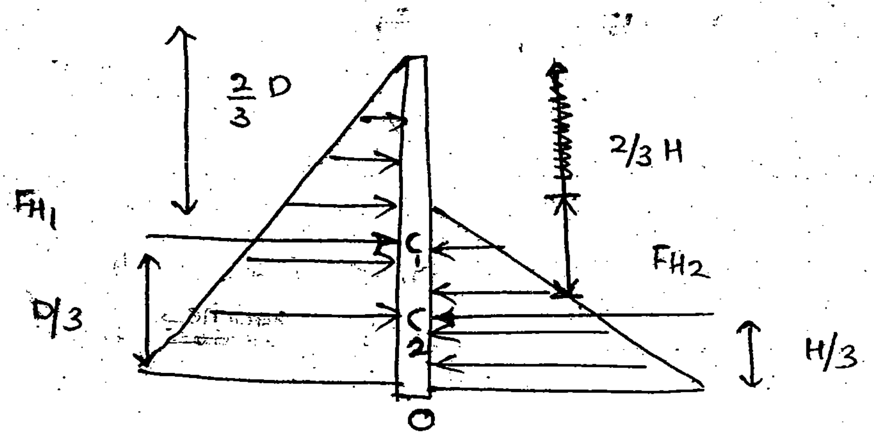
141
69



$$F_{H_1} = \gamma_1 \times \frac{D}{2} \times \underline{(B \cdot D)} \ N \qquad \text{wetted surface Area}$$

$$F_{H_2} = \gamma_2 \times \frac{H}{2} \times (B \cdot H) \ N$$





Moment of forces about O

$$F_{H_1} \times D/3 = F_{H_2} \times \frac{H}{3}$$

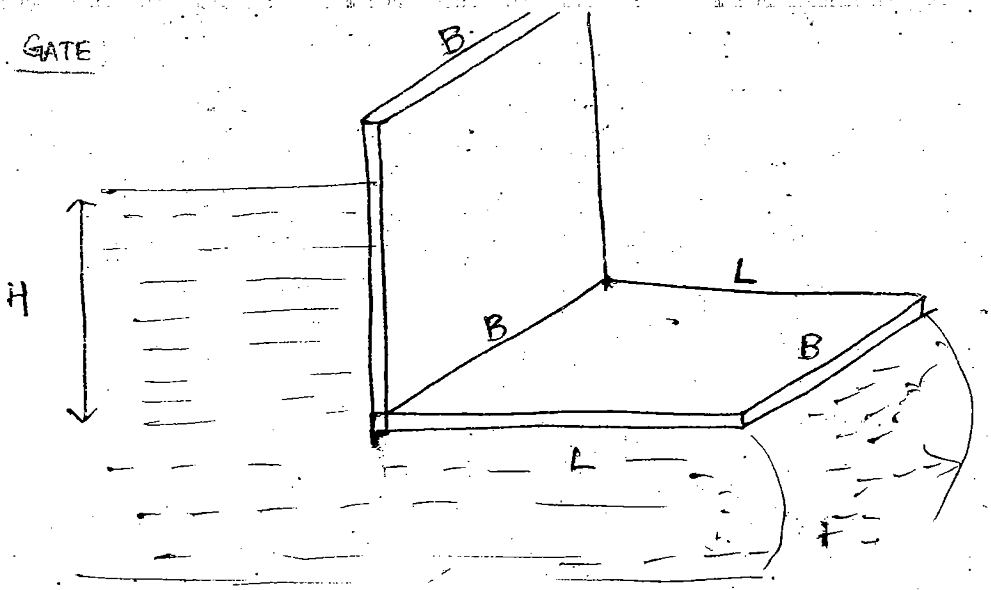
$$\cancel{\gamma_1 \times \frac{H}{3}} \quad \gamma_1 \frac{BD^2}{2} \cdot \frac{D}{3} = \gamma_2 \times \frac{BH^2}{2} \times \frac{H}{3}$$

$$\boxed{\gamma_1 \cdot \frac{BD^3}{6} = \gamma_2 \cdot \frac{BH^3}{6}}$$

$\frac{D}{2} \times BD$

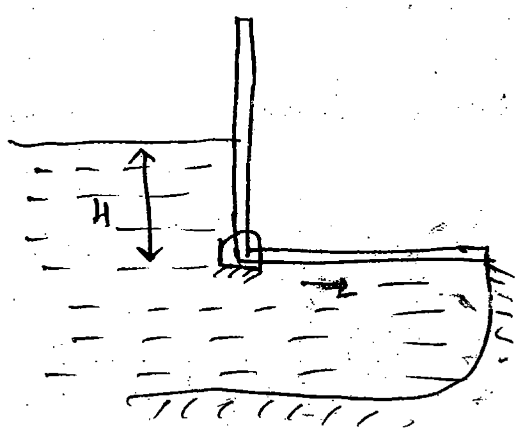
$$F_H = \gamma \times \bar{h} \times \text{Area} \times D/3$$

GATE



At what depth system will Turn

GATE



Find depth of fluid in terms of L such that at particular depth 'H', reaches, then gate turns.

Ans:

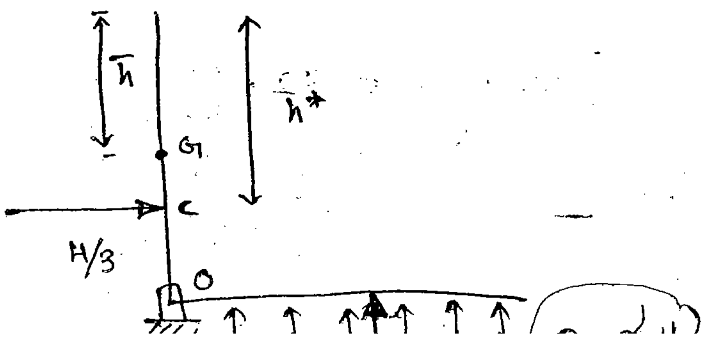
$$F_{H_1} = \gamma \times \frac{BH^3}{6}$$

$F_{H_2} \rightarrow$ Horizontal

$\gamma \cdot \bar{h} \cdot A$

F_H

$= \gamma \cdot \frac{H}{2} \times (B \cdot H)$



Moment of Horizontal = Moment of vertical

$M_H = M_V$

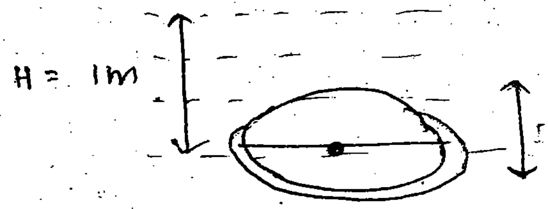
$$\cancel{\gamma}\cdot \cancel{H}\cdot (L\cdot \cancel{B}) \times \frac{L}{\cancel{2}} = \cancel{\gamma}\cdot \frac{H}{\cancel{2}} \times (\cancel{B}\cdot H) \times \frac{H}{3}$$

$$\frac{H^2}{3} = L^2$$

$$\boxed{H = \sqrt{3}\,L}$$

Q.) What is the total pressure on a circular plate of 2m dia is immersed Horizontally in water at a depth of 1m.

a) 10 KN (b) 4 KN (c) 2 KN (d) 31.4 KN



ANS:- $F = \rho \times g \times \bar{h} \times \text{Area}$

$= 1000 \times 10 \times 1 \times \frac{\pi}{4} \times (2)^2$

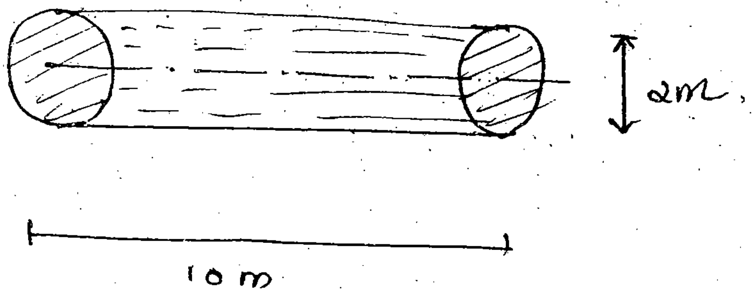
$$\boxed{\bar{h} = H = h^*}$$

$= 10\pi$ KN

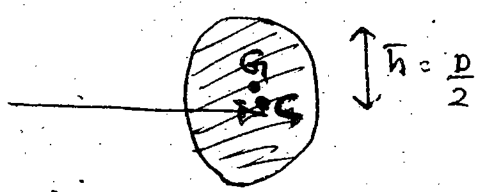
$= 31.4$ KN

Q.) A cylindrical Tank of 2m dia and 10m length is Laid with its axis parallel to the horizontal plane and cylinder is filled with water just to its top what is the Hydrostatic force on one of its end plates

a) 123 KN b) 61.51 KN c) 30.76 KN (d) 19.58 KN.



$F_H = (\gamma \times \bar{h}) \times \text{Area}$



$= \gamma \times \frac{D}{2} \times \frac{\pi}{4} \times (2)^2$

$= 1000 \times 9.81 \times \frac{2}{2} \times \frac{\pi}{4} \times 4$

$\underline{30.819 KN}$

$$h^* = \bar{h} + \frac{I}{A \times \bar{h}}$$

$$= \frac{\frac{\cancel{\pi}}{64} D^4}{\frac{\cancel{\pi}}{4} \cdot D^2 \times \frac{D}{\cancel{2}}} + \frac{D}{2} = \frac{D}{8} + \frac{D}{2} = \frac{D + 4D}{8}$$

$$= \frac{D}{2} + \frac{D}{8} = \underline{\frac{5}{8} D} = \underline{1.25}$$

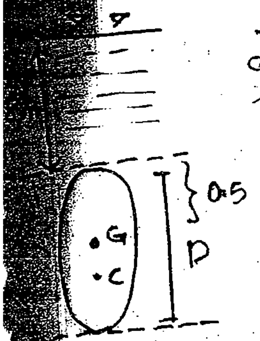
$$= \cancel{\frac{4}{64} D^2}$$

$$h^* = \cancel{\frac{1}{16} D^2} =$$

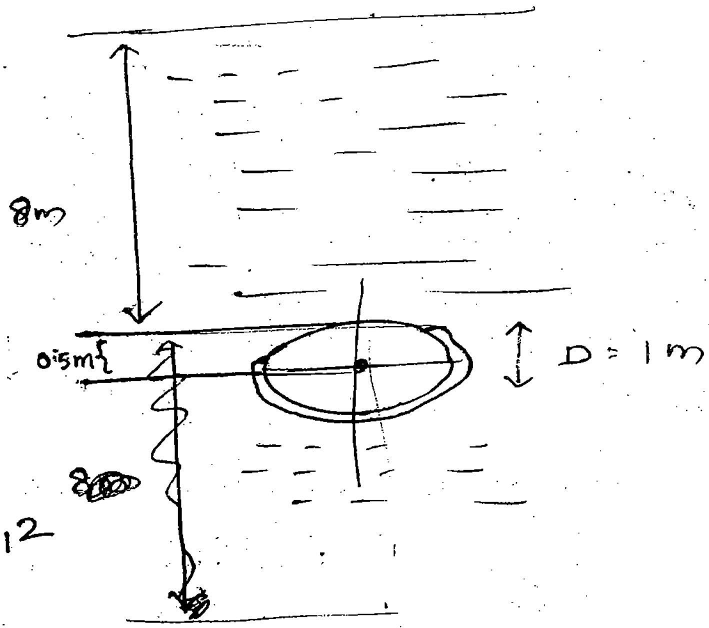
$$= \cancel{\frac{1}{16} \times 2^2}$$

$$= \cancel{\frac{4}{16}} = \frac{1}{4} = \underline{0.25}$$

in water such that its upper edge is 8 m below the free water surface what is the Total hydrostatic pressure force on one side of plate



a) 6.70 KN b) 65.40 KN c) 45.00 KN d) 77 KN



$F_H = (\rho \times g \times \bar{h}) \times$ Area

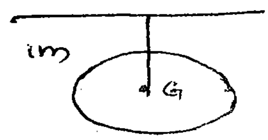
$= 1000 \times 9.81 \times \cancel{85} \times \frac{\pi}{4} \times 1^2$

$= \underline{65.40 \text{ KN}}$

$$\boxed{\begin{aligned} \bar{h} &= 8 + \frac{D}{2} \\ &= 8 + \tfrac{1}{2} = 8.5 \text{ m} \\ & 8 + \tfrac{1}{2} = 8.5 \text{ m} \end{aligned}}$$

The Magnitude of Hydrostatic pressure force on one side of a circular surface, 1 m² Area with its centroid is 1m below the free water surface

(1) depends on the the orientation of the Area of circular plate



(2) is the pdt of spt wt of water, vertical distance from free surface to and its Normal Area. centre ... of pressure

(c) 10 KN

(d) 1000 N

$F_H = 1000 \times 9.81 \times 1 \times \left(\frac{1}{2}\right)$

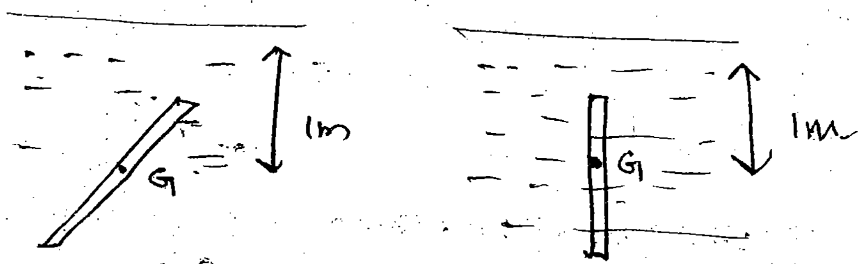
1000×10

$\underline{10000\ N}$

~~981 N~~ $\underline{10\ KN}$

~~1000~~

NOTE:-

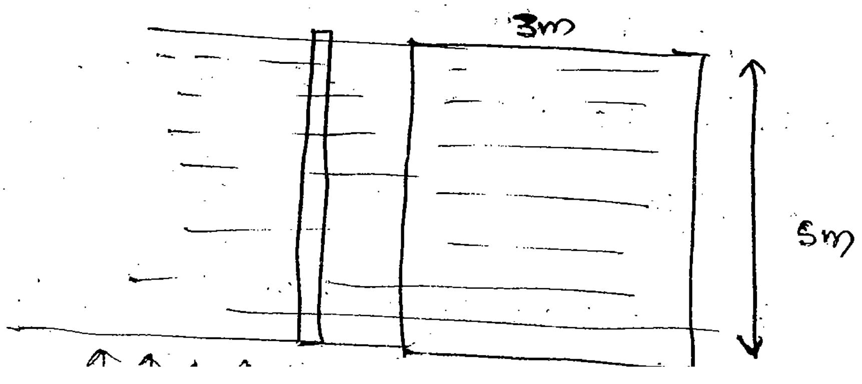


ORIENTATION IS INDEPENDANT

Q) A vertical ~~Guide~~ Gate closes a horizontal tunnel of 3 m width and 5m height Running under full water. the pressure at the Bottom of the Gate is 2×10^4 kg/m². The magnitude of Hydrostatic force on the gate is would be in tonnes

(a) 192 T (b) 250 T (c) 262.5 T

(d) 225.5 T



$F_H = (\rho \times g) \times \bar{h} \times \text{Area}$

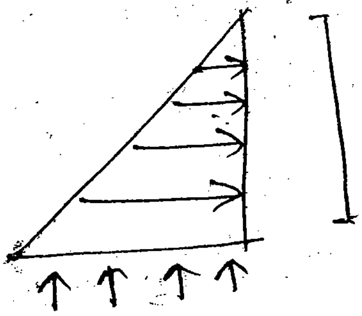
$= 1000 \times 10 \times \frac{H}{2} \times 5 \times 3$

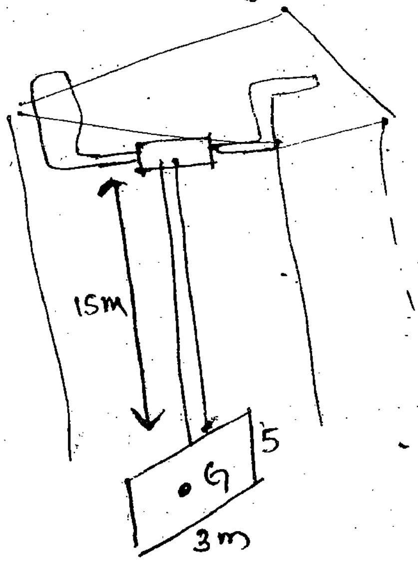
$1000 \times 10 \times \frac{5}{2} \times 15$

$: 75 \times 5000$

$: 375\,000$

$\begin{array}{r} 2 \\ 75 \\ 5 \\ \hline 375 \end{array}$





$P = \cancel{\rho g h}\ \rho g H$

$= 2 \times 10^4\ kg/m^2$

$= 2 \times 10^4 \times 9.81\ \frac{N}{m^2}$

$196,200\ N/m^2$

$196200 = 1000 \times \cancel{10}\ 9.81 \times H$

$H = 20M$

$F_H : 1000 \times 9.81 \times \left[15 + \frac{5}{2}\right] \times (5 \times 3)$

2575125

$\frac{2575125}{9.81} = \cancel{26257}$

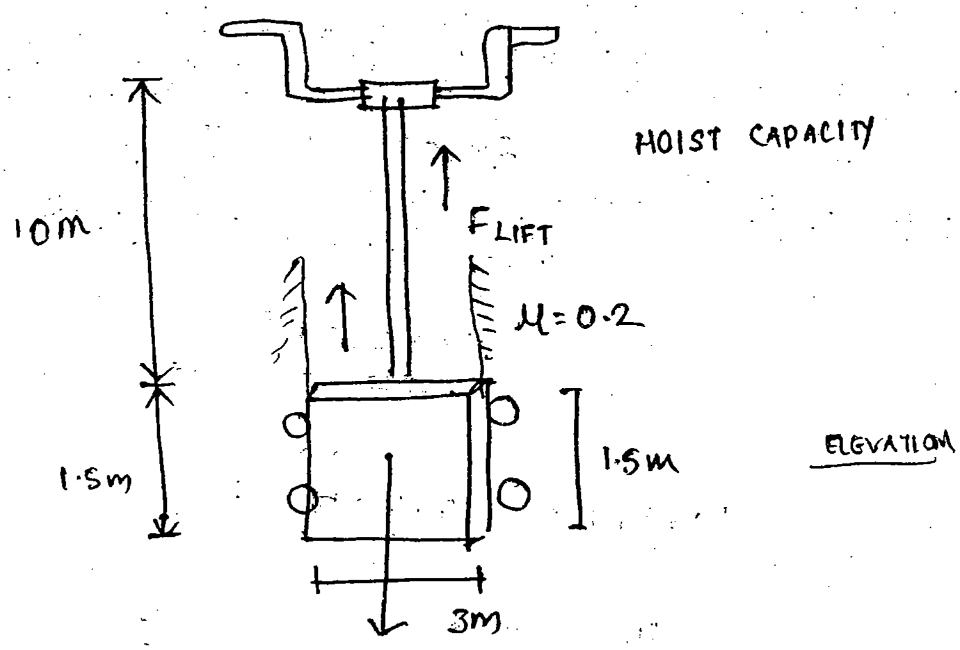
$\frac{262500}{\cancel{9.81}} = kg$

262500 Tonnes

Q:) A sliding gate 3m width and 1.5 m height lies in a vertical plane against the Frictional Surface which has coeff of friction 0.2 b/w. The gate roller and vertical Rubbing surface. The self wt is 2T. (19.6 kN). The upper edge of gate is 10 m below the free water surface. Determine the vertical force Reqd to lift the gate against self wt and frictional force. Neglect Bouyancy effect on the gate.

a) 115 kN ⓑ 494 kN ⓒ 95 kN ⓓ 125 kN

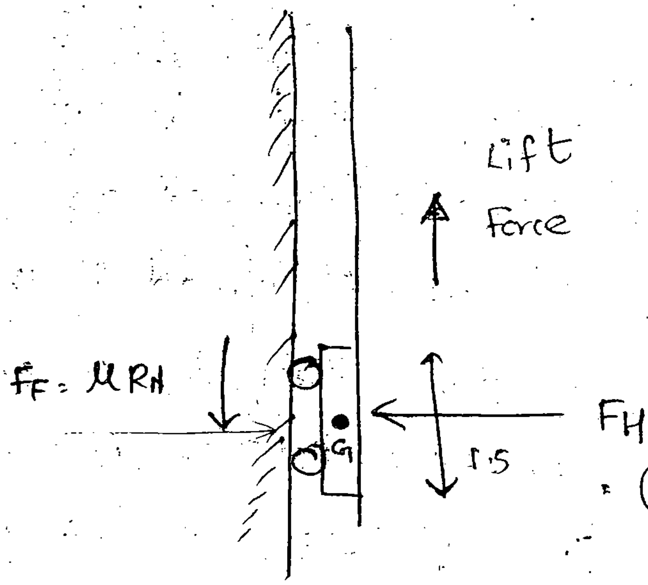
ANS:



2T = 2000 kg

= 19.6 kN

END VIEW



$$= (\rho g)\cdot \bar{h} \times A$$

$$= 1000 \times 9.81 \times \left[10 + \frac{1.5}{2}\right] \times [3 \times 1.5]$$

$$= \underline{474.56 \times 10^3 \text{ N}}$$

$$R_N = F_H$$

$$F_F = 0.2 \times 474.56 \times 10^3$$

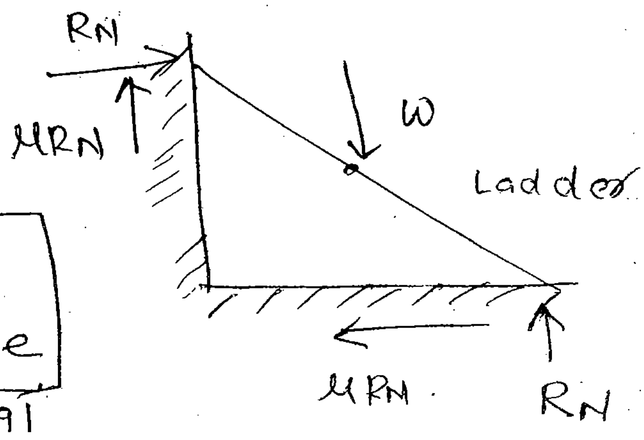
$$= 94911.175 \text{ N}$$

$$= \underline{94.91 \text{ kN}}$$

Rolling frictio[n] less when compared to sliding force

$F_F =$ ~~Force applied~~

$= \underline{94.91 \text{ kN}}$

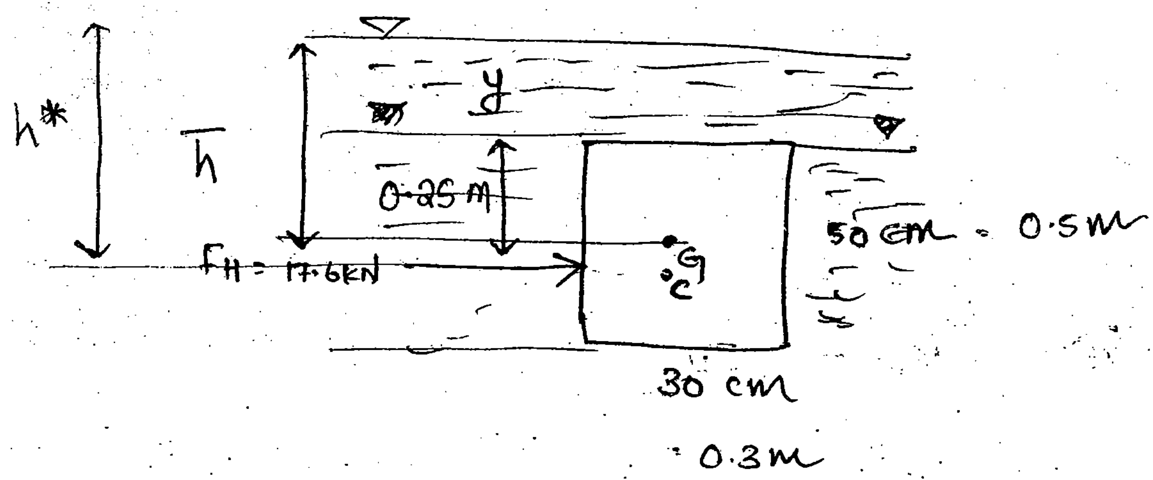


$$\boxed{\text{Lift force} = W_{self} + \text{Friction force}}$$

$= 190 + 94.91$

Q.) A Rectangular plate 30cm × 50cm is immersed vertically in water with its longer side edge vertical. The total hydrostatic force on one side of the plate is estimated as 17.6 KN. Now the plate is turned in the vertical plane about its centroid by 90 degrees and keeping other parameters remains same then the New Total force on one side of the plate would be.

(a) 8.8 KN (b) 15.6 KN (c) 17.6 KN (d) 19.6 KN



$F_N = F_H = 17.6$ kN

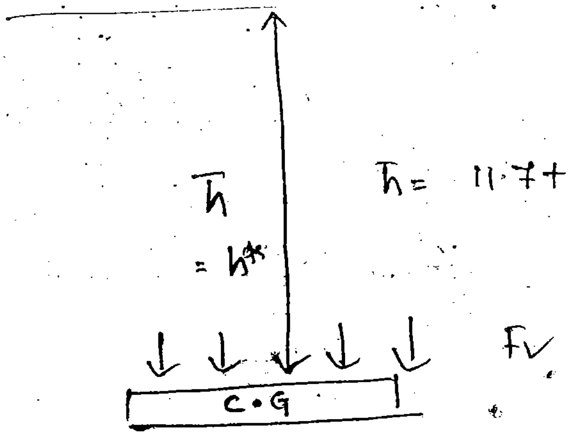
$$= \rho g \cdot \left[y + \frac{D}{2} \right] \times \text{Area}$$

$$= 17.6 \times 10^3 = 1000 \times 9.81 \left[y + \frac{0.5}{2} \right] \times (0.3 \times 0.5)$$

$$y = 11.71 \text{ m}$$

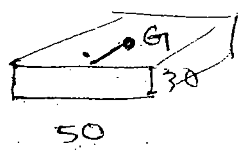
Case 2

Tilting



$\bar{h} = 11.7 + 0.25$

$\frac{50}{2} = 0.25$

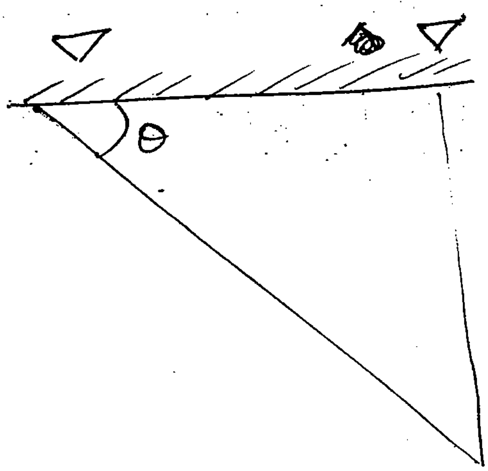


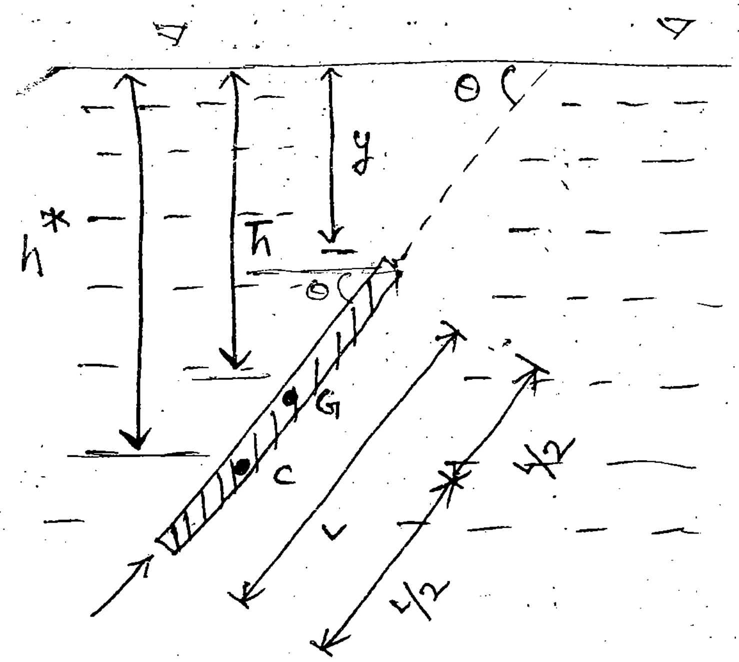
$$F_v = \rho \cdot g \cdot \bar{h} \times \text{Area}$$

$$= 1000 \times 9.81 \times [11.7 + 0.25] \times (0.3 \times \overset{0.5}{\cancel{0.25}})$$

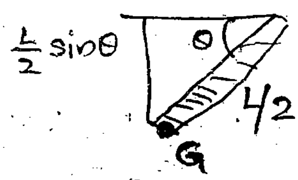
$$= \underline{\underline{17.6 \times 10^3 \text{ N}}}$$

Inclined plane surface





WIDTH



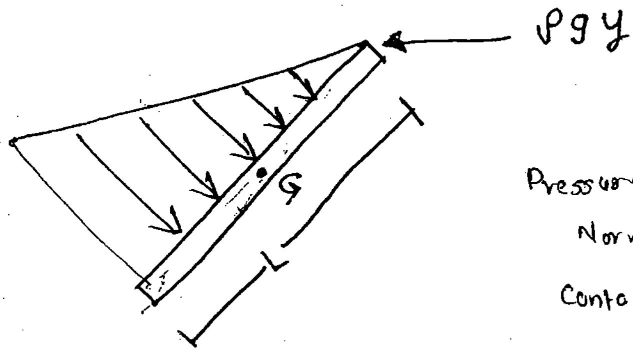
$$P = \rho g \bar{h}$$

$$F_N = \rho \times g \times \bar{h} \times \text{Area}$$

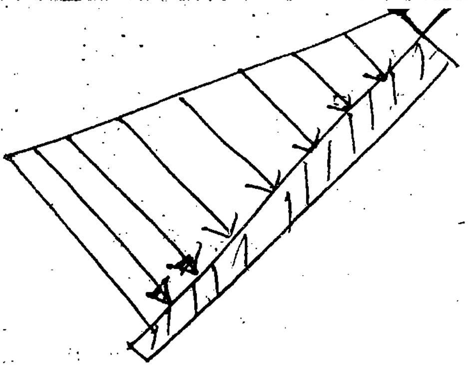
$$\rho \times g \left[y + \frac{L}{2}\sin\theta\right] \times BL$$

$$\boxed{F_N = \rho g \left[y + \frac{L}{2}\sin\theta\right] \times BL}$$

Pressure diagram



Pressure Normal to Contact surface



$$h^* = \bar{h} + \frac{I \cdot \sin^2\theta}{A \cdot \bar{h}}$$

<u>If $y = 0$</u>

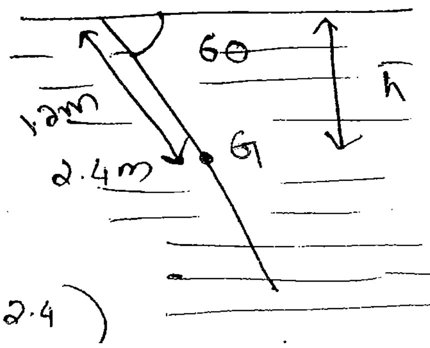
Q.) A Rectangular plate 0.75 m × 2.4 m is immersed in a liquid of relative density 0.8 with its ~~0.75 m~~ 0.75 m side horizontal and just at the liquid surface. The plane of the plate makes an angle 60° with the horizontal. The the total pressure force on one side of plate in kN and centre of pressure in metres are

a) 15.6 kN and 1.39 m

b) 7.8 kN and 1.2 m

c) 24.0 kN and 1.39 m

d) 18.0 kN and 1.2 m



$F_N = \rho \cdot g \cdot \bar{h} \cdot A$

$F_N = 0.8 \times 1000 \times 9.81 \times \left[1.2 \sin 60\right] \times$

$\left[0.75 \times 2.4\right)$

$F = 15.6 \text{ kN}$

$$h^* = \bar{h} + \frac{I_{CG} \times \sin^2\theta}{A \cdot \bar{h}}$$

$$= \left(L/2 \sin 60\right) + \frac{\frac{0.75 \times (2.4)^3}{12} \sin^2 60}{0.75 \times \left(\frac{L}{2} \sin 60\right)}$$

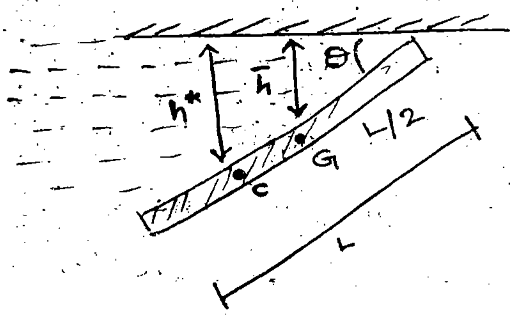
$$= 1.04 + \frac{\frac{0.75 \times (2.4)^3}{12} \times (\sin 60)^2}{0.75 \times 2.4 \times 1.04}$$

$$h^* = 1.39 \text{ m}$$

$$= \left(\frac{2H}{3}\right) \sin 60$$

$$= \frac{2 \times 2.4}{3} \times \sin 60$$

$$h^* = 1.39 \text{ m}$$



$$\sin\theta = \frac{\bar{h}}{L/2}$$

$$\boxed{\bar{h} = L/2 \sin\theta}$$

$$\boxed{h^* = \bar{h} + \frac{I_{CG} \sin^2\theta}{A \cdot \bar{h}}}$$

Q) A Rectangular plate of unit width and length 1m shown in the figure. Determine the Hydrostatic force for unit width. The density of liquid is 'ρ'.

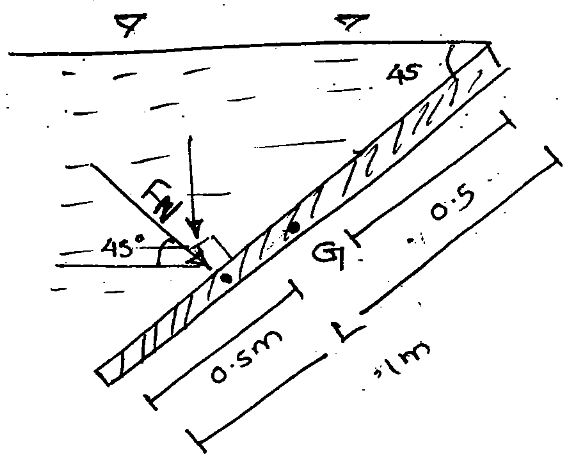
a) $\sqrt{2}\rho g$

b) $\rho g/2$

c) $\frac{\rho g}{2\sqrt{2}}$

d) $\frac{2}{\rho g}$

Ans:



$$\boxed{\bar{h} = \frac{L}{2} \sin\theta}$$

$F_H = F_N \cos 45$

$= \frac{\rho g}{\sqrt{2} \times 2\sqrt{2}}$

$\frac{\rho g}{4}$

$F_H = \rho \times g \times 0.5 \sin 45 \times$ Area

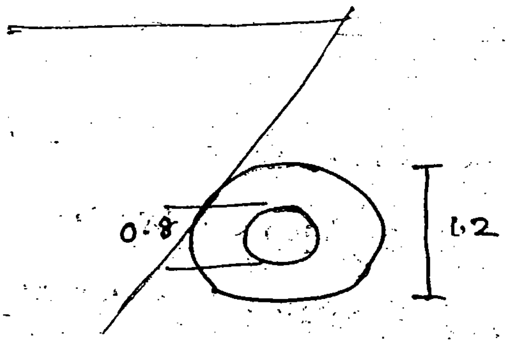
$= \rho \times 9.81 \times 0.5 \times \sin 45 \times [1 \times 1]$

$F_H = \frac{1}{2\sqrt{2}} \rho g$

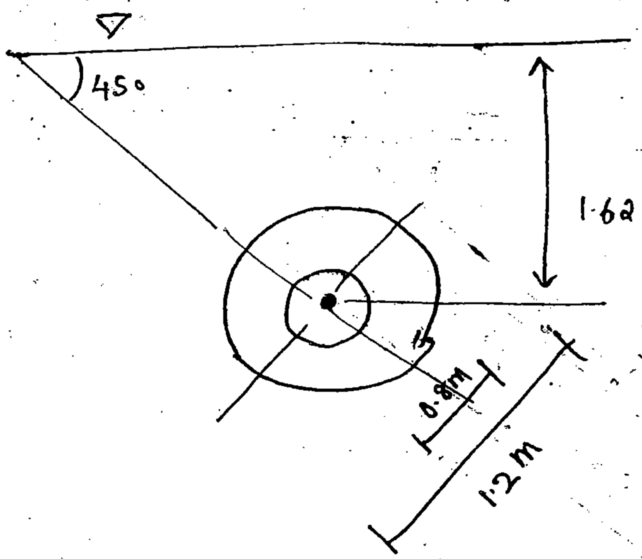
$\boxed{F_H = \frac{1}{2\sqrt{2}} \rho g}$

Q) A circular plate with concentric circles of dia 1.2m and 0.8 m is immersed in water with its plane inclined at an angle 45° with horizontal. If the centre of the circle is 1.625m below the free water surface. Then total pressure force on one side of the circular Ring plate in KN is

a) 70.7 KN $\left(\frac{10}{\sqrt{2}}\right)$
b) 10 KN
c) 14.14 KN $(10\times\sqrt{2})$
d) 18.0 KN



Ans:-

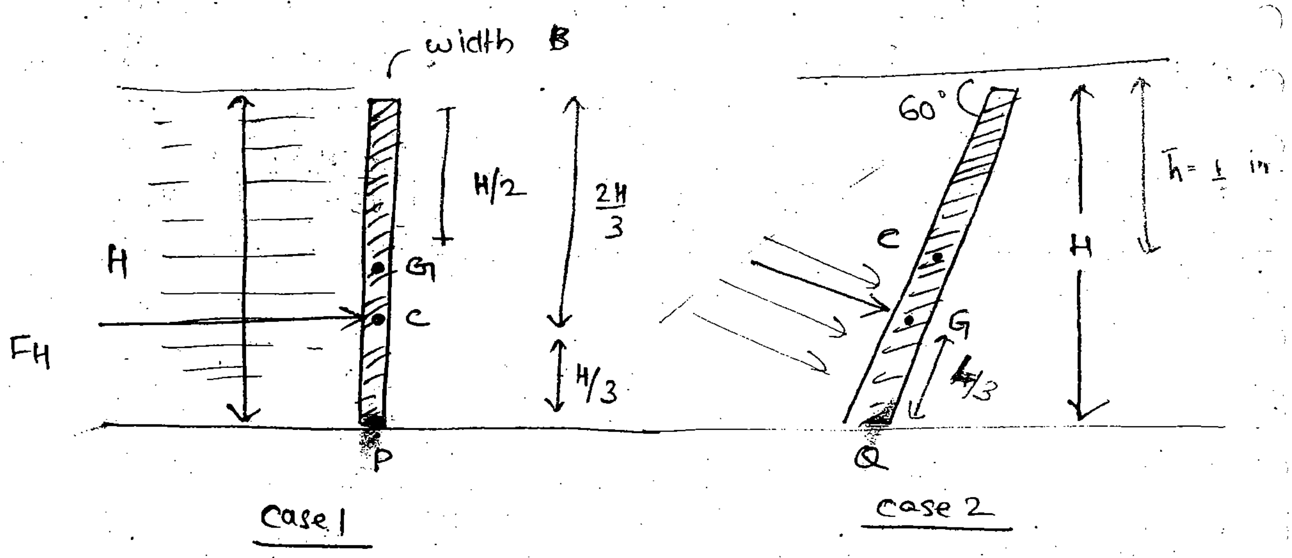


$$F_H = \rho g \bar{h} \cdot A$$

$$= 1000 \times 9.81 \times 1.625 \times \frac{\pi}{4}\left[(1.2)^2 - (0.8)^2\right]$$

$$= 10000\ N$$

$$= \underline{10\ KN}$$



Find moment of Hydrostatic force difference

ie $M_Q - M_P$

$$F_H \times \frac{H}{3} - F_H \times \frac{H}{3}$$

$$\cancel{\rho g \frac{BH^3}{6} = \rho g}$$

$$\rho g \times \bar{h} \times \text{Area} \times \frac{H}{3} - \rho \times g \times \bar{h} \times \frac{H}{3} \times \text{Area}$$

$$\rho g \times \frac{H}{2} \times [B \times H] \times \frac{H}{3} - \left(\rho \times g \times \frac{L}{2} \sin 60\right) \times \frac{L}{3} \times (L \times B)$$

Area

$$\rho g \frac{BH^3}{6} - \rho g \times \frac{BL^3}{6} \times \frac{\sqrt{3}}{2}$$

$$\rho g \frac{BH^3}{6} - \rho g \frac{\sqrt{3}\, BL^3}{12}$$

$$\rho g \cdot \frac{BH^3}{6} - \rho g \frac{\cancel{\sqrt{3}}\, BL^3}{4 \times \sqrt{3} \times \cancel{\sqrt{3}}}$$

$$\rho g \cdot \frac{BH^3}{6} - \rho g \cdot \frac{BL^3}{4\sqrt{3}}$$

$$\rho g \cdot \frac{BH^3}{6} - \rho g \times$$

$\sin 60 = \frac{H}{L}$

$H = L \sin 60$

$L = \frac{H}{\sin 60}$

$L = \frac{2}{\sqrt{3}} H$

$$\frac{\rho g \cdot BH^3}{6} - \rho g \times \frac{L}{2} \sin 60 \times [L \times B] \times \frac{L}{3}$$

$$\frac{\rho g BH^3}{6} - \rho g \times \frac{\cancel{2}}{\sqrt{3}\cancel{2}} H \sin 60 \left[\frac{2}{\sqrt{3}} H \times B\right] \times \frac{2}{\sqrt{3}} H$$

$$\frac{\rho g BH^3}{6} - \rho g \times \frac{H}{\cancel{\sqrt{3}}} \times \frac{\cancel{\sqrt{3}}}{\cancel{2}} \left[\frac{2}{\sqrt{3}} \times BH\right] \times \frac{\cancel{2}}{\sqrt{3} \times 3} H$$

$$\frac{\rho g BH^3}{6} - \frac{2 \rho g BH^3}{3 \times 3}$$

$$= \frac{\cancel{\rho g BH^3 - 4 \rho g BH^3}}{\cancel{6}} \qquad \frac{\rho g BH^3}{6} - \frac{2}{9} \rho g BH^3$$

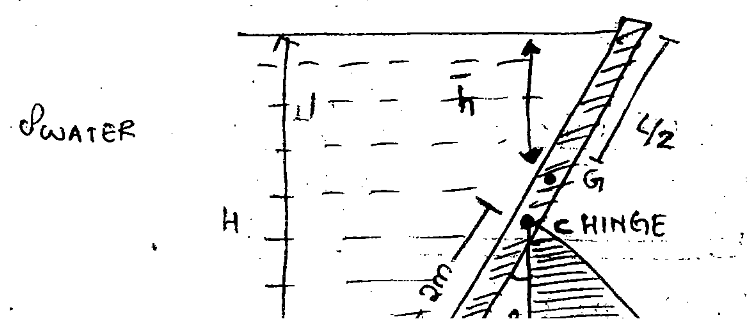
$$= \frac{\cancel{-3 \rho g BH^3}}{\cancel{6}} \qquad = \frac{1}{18} \rho g BH^3$$

$$= \frac{1}{18} \gamma BH^3$$

Q) A plate subjected to Hydrostatic force shown in figure. determine the

① At what depth of water the plate to tilt about hinge point

② What is the hinge reaction



Rectangular flash Board plate

(FLAP GATE)

$\cancel{\rho \times g \times \frac{L}{2} \times [L \times B]}$ $\quad \rho \times g \times \frac{L}{2} \sin 50 \times [B \times L]$

$= \cancel{\frac{1000 \times 9.81 \times BL^2}{2}} = \cancel{4905\ BL^2} \quad 1000 \times 9.81 \times \sin 50 \times \frac{BL^2}{2}$

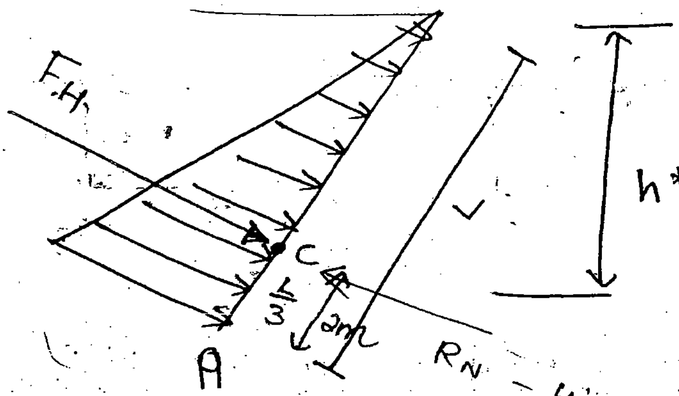
$BC = 2 \sin 50$

$= \underline{1.532}$

$\underline{3757.44\ BL^2}$

R_N is reaction of Hinge

R_N is Balanced by Hydrostatic force



R_N - Hinge Reaction

~~$R_N = F_H$~~ =

Not needed

$$\boxed{\begin{array}{l}\text{Moment About } A \\ F_H \times 2 = R_H \times 2 \\ F_H = R_H\end{array}}$$

$\frac{L}{3} = 2$

$\underline{L = 6\,m}$

$\sin 50 = \frac{H}{L}$

$H = 6 \sin 50$

$= \underline{4.6\,m}$

Take $\underline{B = 1\,m}$

$R_N = F_H$

$= \cancel{\frac{4905 \times B \times 6^2}{2}}$

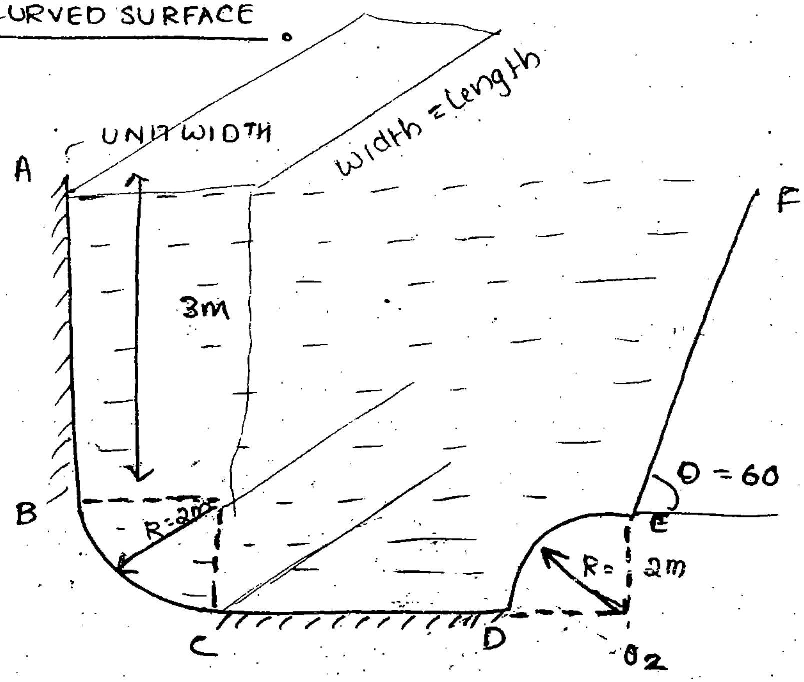
$= 3757.44 \times 1 \times 36$

$\underline{135{,}268.12\ N} = \underline{135.268\,kN}$

A Hinge Support is necessary to withstand wherever Hydrostatic force acts

ie at centre of pressure

CURVED SURFACE



BC and DE are curved Surfaces

NOTE: -

Curved Surface subjected to two hydrostatic components or force_

1. Horizontal component (F_H)
2. vertical Component (F_V)

F_H

It is the Normal force due to hydrostatic pressure

$$F_H = \rho g \bar{h} \times A \quad \text{Newton}$$

where $\bar{h}$ = vertical depth from free water surface to the centroid of the projected plane of

F_v = vertical component

It is the weight of the fluid supported by the curved surface.

$$\boxed{F_v = mg = \rho \times \forall \times g}$$

$$\rho \times (\text{Area} \times w) \times g$$

C/s Area

$$= \rho g (\Sigma A) \times w$$ → width or length

Q.) Determine the following for the curved surface BC and DE shown in the above figure

1. centroid of the curved surfaces
2. Horizontal hydrostatic force
3. vertical component of Hydrostatic force
4. Resultant of Hydrostatic force and its position OR Total pressure
5. Moment of the vertical Hydrostatic force component about the plane at C and D.
6. Moment of horizontal Hydrostatic forces.

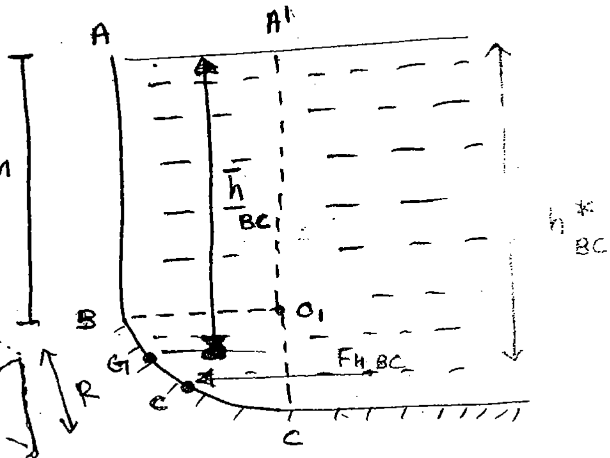
Case1:

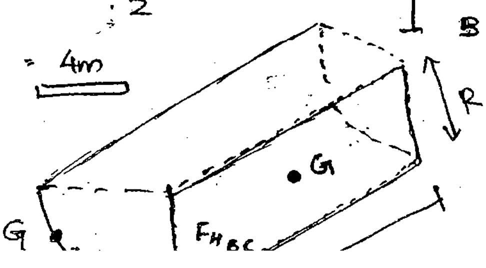
Curved surface BC

$$\bar{h}_{BC} = [y = (AB)] + \frac{2}{2}$$

$$3 + \frac{2}{2}$$

$$= \underline{4m}$$





Projected is

b.) Horizontal force on BC

$$\boxed{F_{H_{BC}} = \rho g \bar{h}_{BC} \cdot A_{projected}}$$

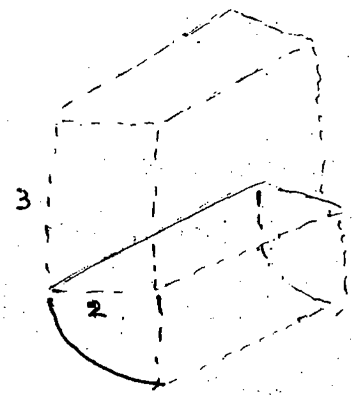
$1000 \times 10 \, [3+1] \cdot [L \times R]$ $\quad g = 10$

$1000 \times 10 \, [4] \times [1 \times 2]$ $\quad w = L = 1$

unit width

$F_{H_{BC}} = \underline{80 \, kN/\text{metre width}}$

c.) $F_V = W_{water}$ supported by BC curved portion



$= Mg = \rho V \cdot g$

$$\left[F_V = \rho g \left[A_{\square_2} + A_{\square} \right] \times L \text{ (or } w) \right]$$

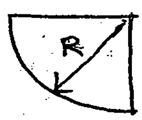
$1000 \times 10 \left[2 \times 3 + \frac{\pi R^2}{4} \right] \times 1$

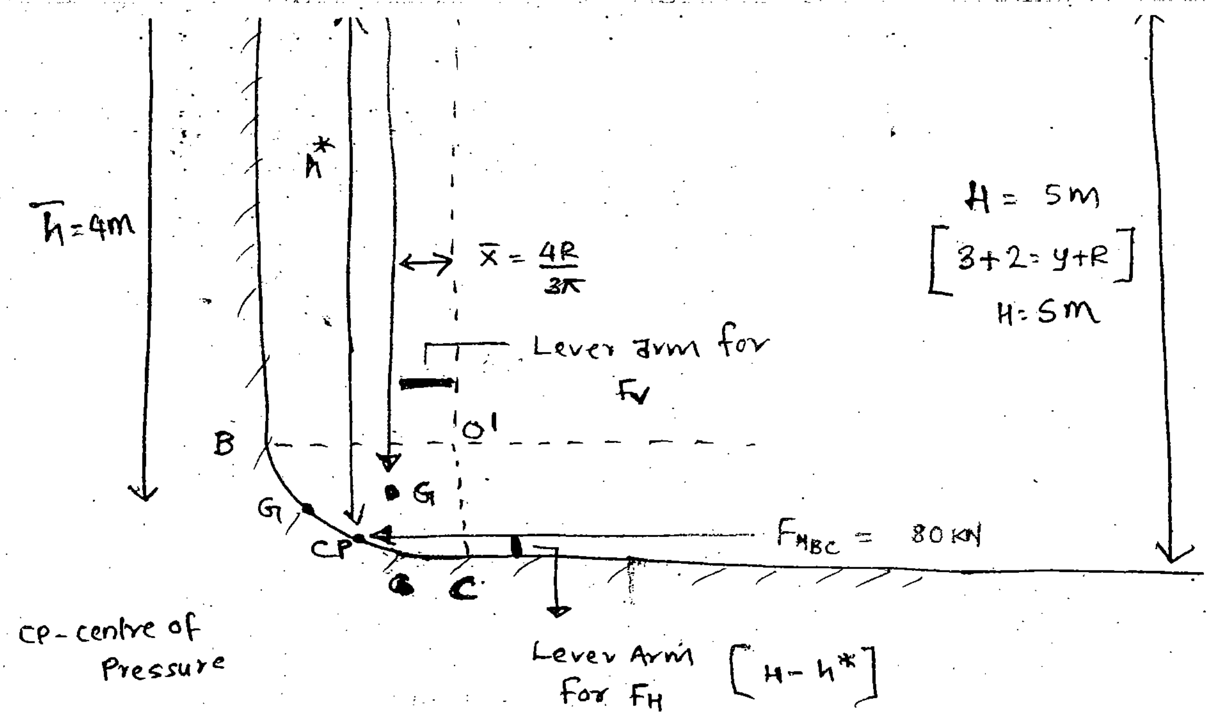
$1000 \times 10 \left[6 + \pi \right]$

$F_V = \underline{91.4 \, kN/\text{metre}}$

Note:

$$\boxed{\text{Area of QUADRANT} = \frac{\pi R^2}{4}}$$





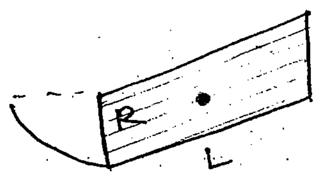
Lever arm for F_v } $\bar{x} = \frac{4R}{3\pi} = \frac{4\times 2}{3\pi} = \frac{8}{3\pi}$ metres

$$\boxed{h^* = \bar{h} + \frac{I}{A \times \bar{h}}}$$

$$= 4 + \frac{\left[\frac{1 \times 2^3}{12}\right]}{(2\times 1)\times 4}$$

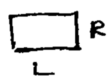
$$= 4 + \frac{1}{12} = \frac{49}{12}$$

$$= 4.08\,m$$



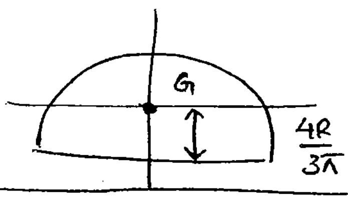
$\frac{LR^3}{12}$

$\left[\frac{BD^3}{12}\right.$ formula



NOTE:-

CENTROID



d) Moment of Hydrostatic force components

$M(F_v) = F_v \times 4R$

$$(91.4)\left(\frac{4\times 2}{3\pi}\right)$$

$= \underline{\quad\quad}$ kN m

Lever Arm

$$\boxed{M_{(F_H)_C} = F_{H_{BC}} \times \cancel{h^*-h}\,[H - h^*]}$$

$$= 80 \times [(3+2) - 4.08]$$

$= \underline{\quad\quad}$ kN m

d.) <u>Resultant Force on curved Surface</u>

$$\boxed{F = \sqrt{F_H^2 + F_V^2}}$$

$$= \sqrt{80^2 + 91.42^2}$$

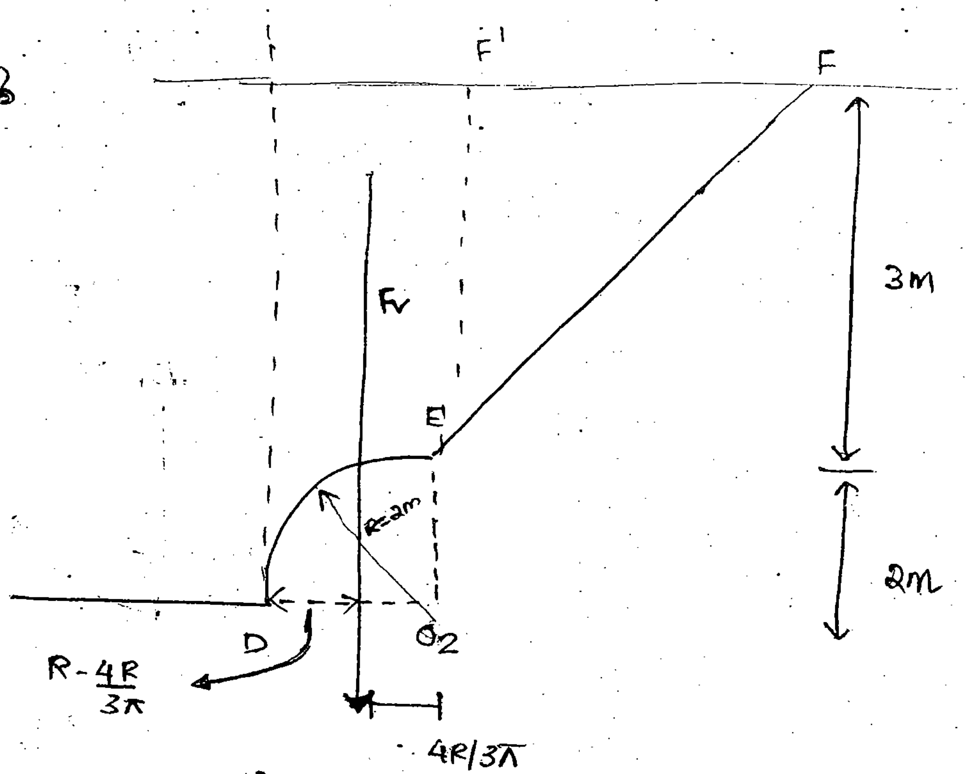
$= \underline{\quad\quad}$ kN

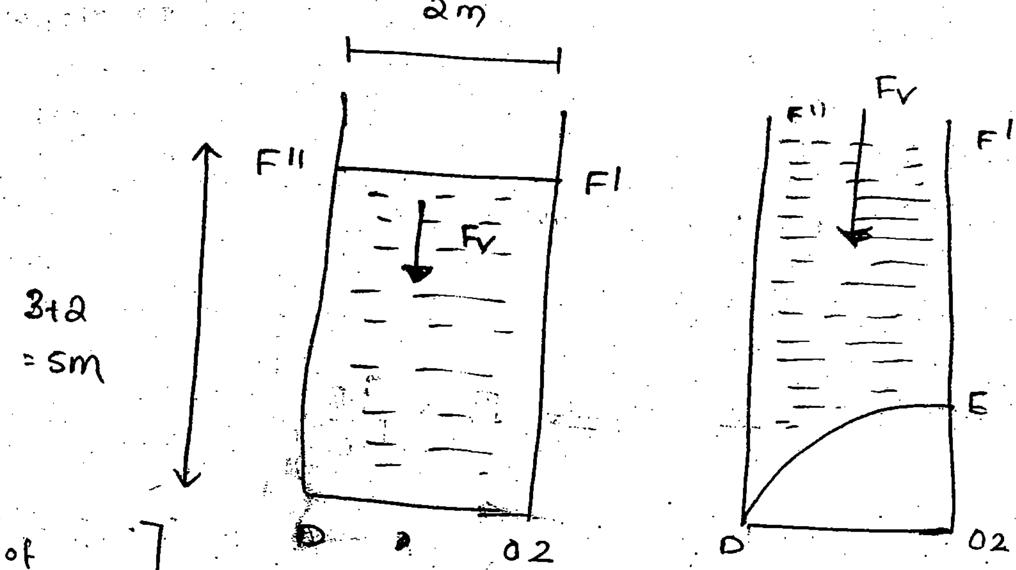
$$\boxed{\begin{aligned} \tan\alpha &= \frac{F_V}{F_H} \\ \alpha &= \tan^{-1}\left[\frac{F_V}{F_H}\right] \end{aligned}}$$

$\boxed{\text{Direction}}$

$= \underline{\quad\quad}$

$F_{V_{DE}} = 3$



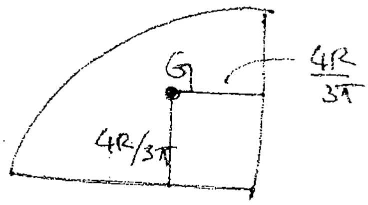


[Fv = weight of fluid supported by curved surface]

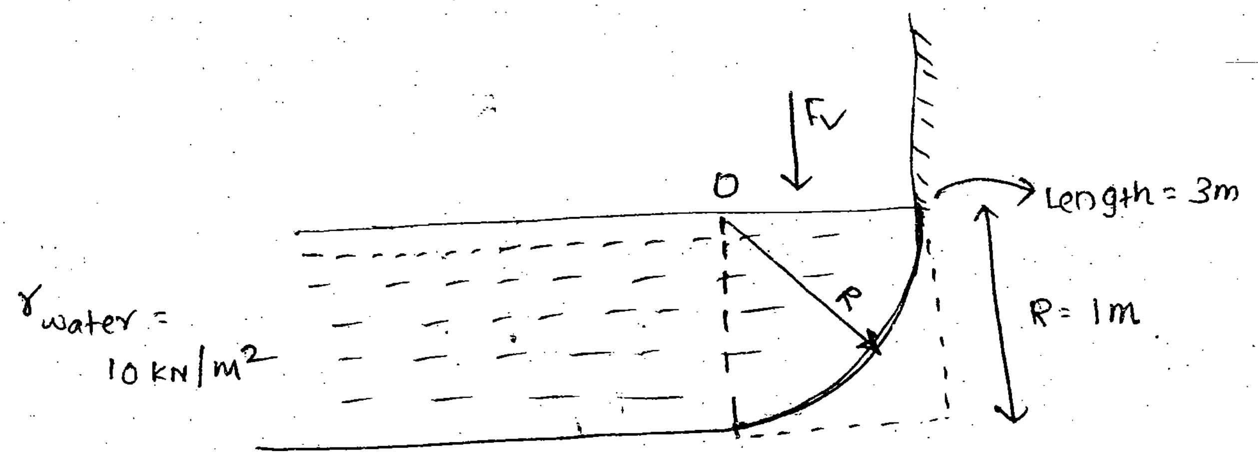
$$F_v = \rho g \left[A_{\square\, 2\times5} - \frac{1}{4} A_{\triangle\, 2} \right] L \qquad L \to \text{unit length}$$

$$= \rho g \left[(2\times 5) - \frac{\pi \times [2]^2}{4} \right] \times 1$$

$$= \underline{\underline{\qquad\qquad}}$$



Q.) Determine moment of vertical component of hydro static thrust on one quadrant of ~~to~~ cylinder as shown in fig



a) 7.854 kN-m

b) 10.000 kN-m

c) 11.781 kN-m

d) 15.708 kN-m

one quadrant of cylinder diameter 2m and 3m Length

Ans:-

$$\boxed{(M)_{F_V} = \cancel{\rho \times g}\ F_V \times \left[\frac{4R}{3\pi}\right]}$$

$$= \gamma \times V \times \frac{4R}{3\pi}$$

$$\boxed{V = \frac{\pi R^2 \times L}{4}}$$

$$\gamma \times \left[\frac{\pi R^2 \times L}{4}\right] \times \frac{4R}{3\pi}$$

$$= \gamma \cdot \left[\frac{\cancel{\pi} R^2 \times \cancel{3}}{\cancel{4}}\right]\left[\frac{\cancel{4} R}{\cancel{3\pi}}\right]$$

$$= \gamma \cdot R^3$$

$$= 10 \times [1]^3$$

$$= 10\ \text{kN-m}$$

~~will~~ fill with water ; then what Happens ?

Ans:

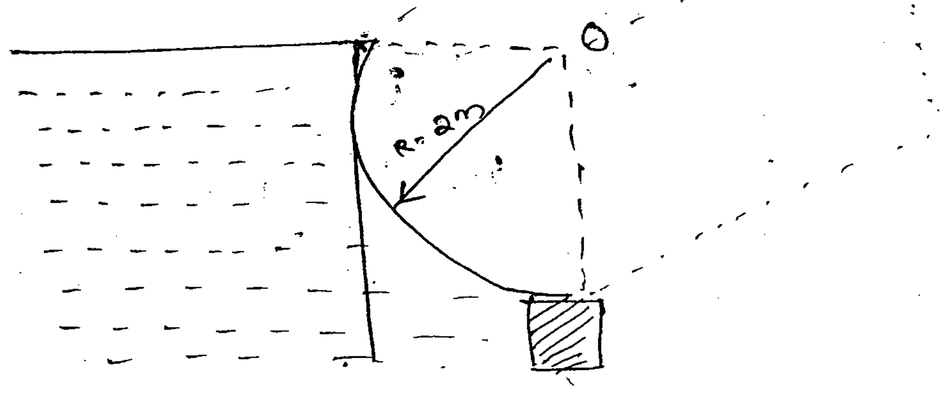
A Quadrant of cylinder diameter = 4m ; width is unity .

find the (i) horizontal hydro static force (KN)

(ii) vertical " " " (KN)

(iii) Resultant " " " (KN) and its orientation.

(iv) Moment of vertical component of water force .



(i) $F_H = \rho \times g \times \bar{h} \times (\text{Area})$

$\cancel{1000 \times 9.81 \times \frac{4 \times 2}{3\pi} \times [2 \times 1]}$

$$1000 \times 9.81 \times \frac{h}{2} \times \left[h \times B \right]$$

$$9810 \times \frac{2}{2} \times \left[2 \times 1 \right]$$

$$= \underline{\underline{19.62\,kN}}$$

__Vertical Force__ (F_v)

F_v = weight of water displaced

$= m \cdot g$

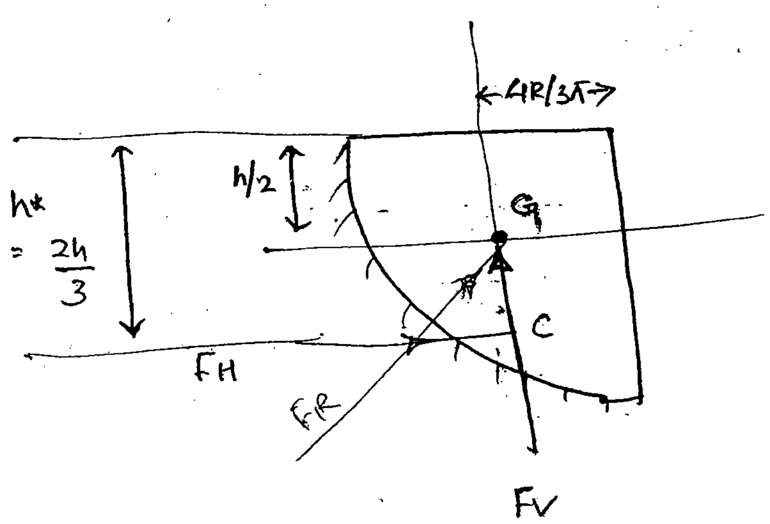
$= \rho_{water} \times \forall \times g$

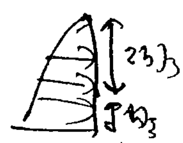
$\rho_{water} \times A_N \times g$

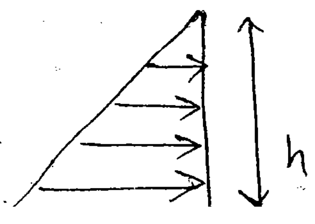
$$1000 \times 9.81 \times \frac{\pi}{4} R^2 \times (1)$$

$$= 1000 \times 9.81 \times \frac{\pi}{4} \times 2^2 \times (1)$$

$$= \underline{\underline{30.81\,kN}}$$







$$F_R = \sqrt{F_H^2 + F_V^2}$$

$$= \underline{\underline{36.5\,kN}}$$

$$\tan\alpha = \frac{F_V}{F_H}$$

$$\underline{\underline{\alpha = 52.5^\circ}}$$

Moment of vertical force about O

$$F_V \times \frac{4R}{3\pi}$$

$$= 30.81 \times \frac{4 \times 2}{3\pi}$$

$$= \underline{\underline{\qquad}}\ Nm$$

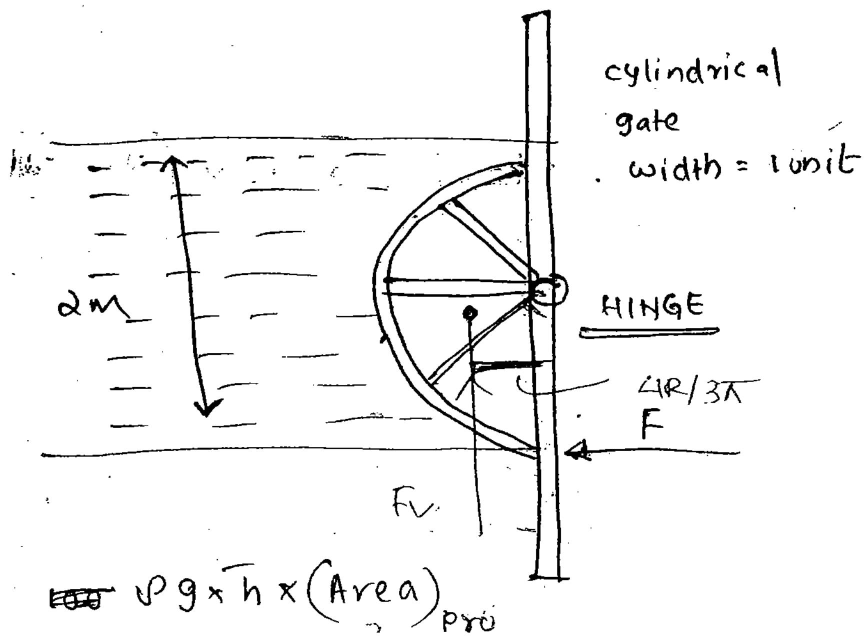
Q) Fig shows a semi circular cylindrical gate supporting water load. Determine the force F read to keep the gate under equilibrium.

a) 0.00 kN

b) 9.81 kN

c) 19.62 kN

d) None of these



Ans

$$F_H = \rho g \times \bar{h} \times (\text{Area})_{pro}$$

~~FMe~~

~~$F_V \times \frac{R}{2}$~~

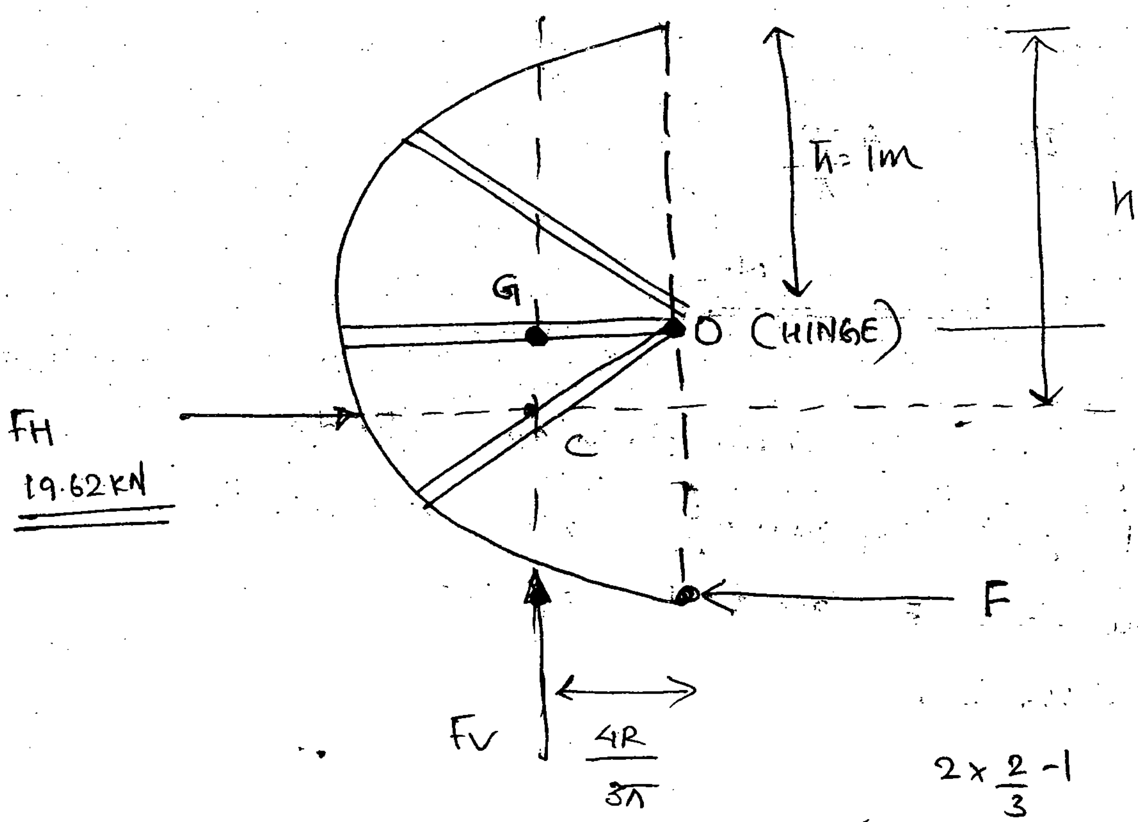
$$F_V = W = Mg$$

$$= \rho \cdot g \cdot A \times h$$

$$= 1000 \times 9.81 \times \left(\frac{\pi R^2}{2}\right) \times (w)$$

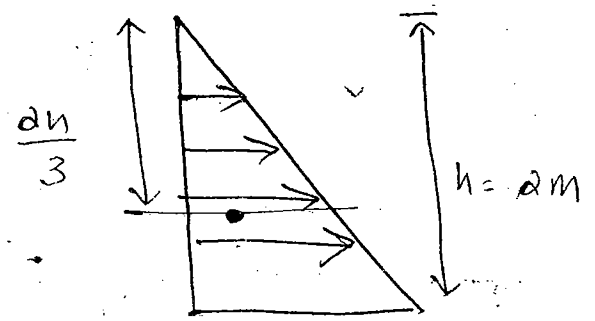
$$= 1000 \times 9.81 \times \frac{\pi (1)^2 \times 1}{2}$$

$$= \frac{9.81 \pi}{2} \text{ kN}$$



$$2 \times \frac{2}{3} - 1$$

$$\cancel{F \times (1) + F_H \times \left[\frac{2 \times 2 - 1}{3}\right] + F_V \times \left[\frac{4 \times R}{3\pi}\right]} = 0$$



$$h^* = \bar{h} + \frac{I}{A \cdot \bar{h}}$$

$$= \frac{h}{2} + \frac{\frac{w \cdot h^3}{12}}{(h \times w) \times \frac{h}{2}}$$

$$= \frac{h}{2} + \frac{h}{6}$$

$$h^* = \frac{2h}{3}$$

$$\cancel{F = 19.62 \times \left[\frac{4-1}{3}\right] - \frac{9.81\pi}{2} \times \frac{4}{3\pi}}$$

~~F = 19.62 kN~~

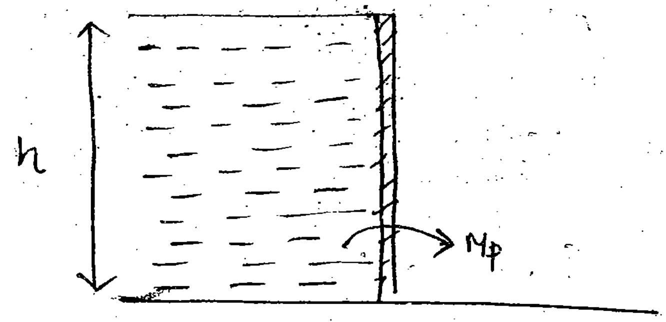
~~F = 6.54~~

$$F \times 1 - F_H \times \left[\frac{2 \times 2}{3} - 1\right] + F_V \times \frac{4R}{3\pi}$$

$$F \times 1 - 19.62 \times \left[\frac{4}{3} - 1\right] + \frac{9.81\pi}{2} \times \frac{4 \times 1}{3\pi}$$

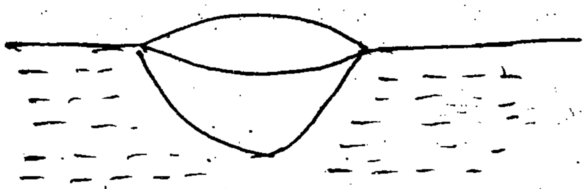
$$F = 0\,N$$

Q. There are 2 ~~bodies~~ Rectangular Blades Subjected to water thrust in two diff cases shown in figure. Determine diff of the moments of the hydrostatic forces about their base points shown ?



Q) A Hollow Hemispherical object of dia 'D' is immersed in water with its plane surface coinciding with the free surface of the water. What is the vertical component of Hydrostatic force on curved surface of the given object ?

(a) $\frac{3}{8}\gamma\pi D^3$ (b) $\frac{1}{12}\gamma\pi D^3$ (c) $\frac{1}{24}\gamma\pi D^3$ (d) 0



$F_V = F_B = W_{fluid\ displaced}$

$= M \cdot g$

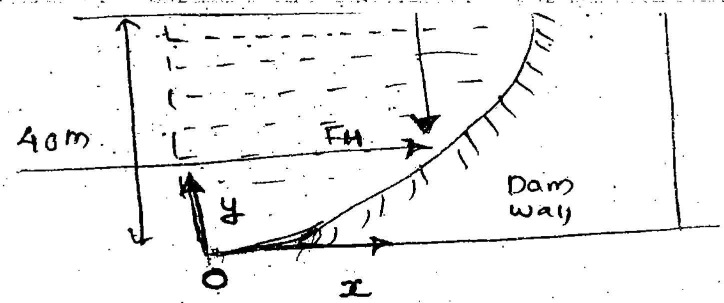
$= \rho \cdot \forall \times g$

$= \gamma \times \frac{4}{3}\pi R^3 / 2$

$\gamma \times \frac{4}{\cancel{6}} \pi \times R^3$

$\gamma \times \frac{4}{6} \times \pi \times \frac{D^3}{8}$

$\gamma \times \pi D^3$



A Reservoir dam spill way shown in fig subjected to water load. Determine

1. eqn of curved surface
2. Horizontal force
3. vertical component of water load
4. Resultant force

Ans:-

~~FH~~ $\cancel{x^2 = 4ay}$

$y = mx^2$

At $y = 40$

$x = 15$

$\therefore 40 = m \times 15^2$

$m = \frac{40}{225}$

$m = \underline{0.177}$

FH

$FH = \rho \times g \times \bar{h} \times (\text{Area})$

$= 1000 \times 9.81 \times \frac{40}{2} \times [40 \times 1]$

$= \cancel{7848}\ldots = 8\,MN$

Fv

$Fv = W$

$m \times g = \rho \times \forall \times g$

$$F_R = \sqrt{F_H^2 + F_v^2}$$

$F_H = 8MN$

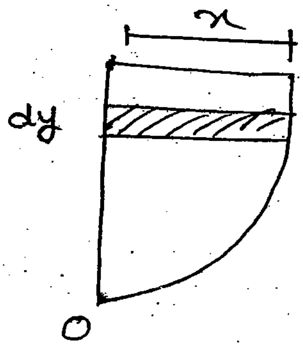
$$F_v = \cancel{\rho \times}\ Mg$$

$$\cancel{1000} = \rho \times V \times g$$

$$\rho g \times [\text{Area} \times w]$$

$$1000 \times 10 \times [\text{Area} \times 1]$$

: A = Area of curved surface



Area = $x \cdot dy$

$$y = 0.18x^2$$

$$\text{Area} = \int_0^{40} \sqrt{\frac{y}{0.18}}\, dy$$

$$= \frac{1}{\sqrt{0.18}} \int_0^{40} y^{1/2} \cdot dy$$

$$= \frac{1}{\sqrt{0.18}} \left[\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_0^{40}$$

$F = 1000 \times 10 \times 397.5 \times 1$

$= \underline{\underline{3895 \text{ KN}}}$

2-5:30 (408)

6-8:30

{ Laplace Transform }

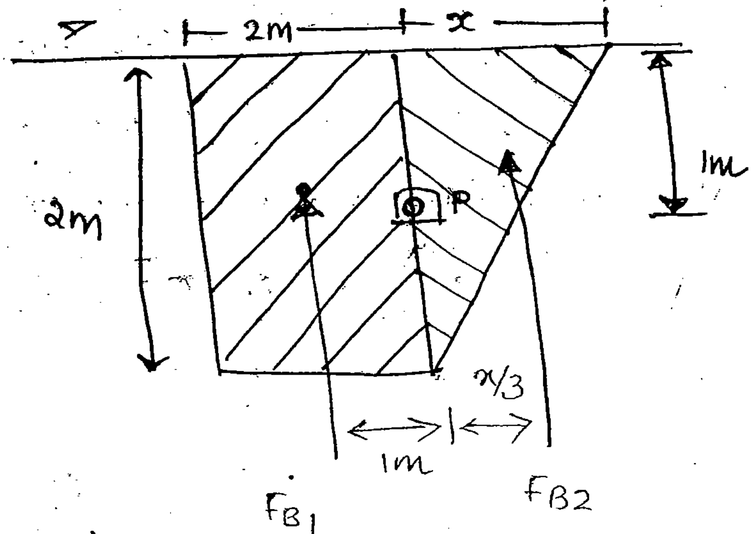
$F_R = \sqrt{F_V^2 + F_H^2}$

$= \underline{\underline{\qquad}}$ KN

Q. A cross section of an object (having same section normal to plane of the paper) is submerged into a fluid consist of a square cross section of each side 2m and a triangle shown in the figure. The submerged object is hinged at point P shown. object is to be kept in that position, then the value of x dimension should be.

a) $2\sqrt{3}$ m (b) $4\sqrt{3}$ m (c) 4m (d) 8m

Ans:



Moment abt P

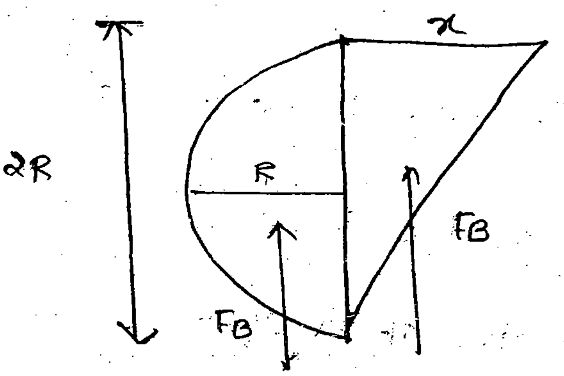
$F_{B\,\square} \times 1 = F_{B\,\triangledown} \times \frac{x}{3}$

$$\cancel{\rho \times g} \times v \times 1 = \cancel{\rho \times g} \cdot v \times \frac{x}{3}$$

$$A \times \cancel{w} \times 1 = A \times \cancel{w} \times 1 \times \frac{x}{3}$$

$$\cancel{1 = \frac{x}{3}} \qquad \underline{x = 2\sqrt{3}}$$

Q.)



$$A_{semicircle} \times \frac{4R}{3\pi} = A_{Triangle} \times \frac{x}{3}$$

$$\frac{\pi R^2}{2} \times \frac{4R}{3\pi} = \frac{1}{2} \times x \times 2R \times \frac{x}{3}$$

$$\underline{x = \sqrt{2} \cdot R}$$

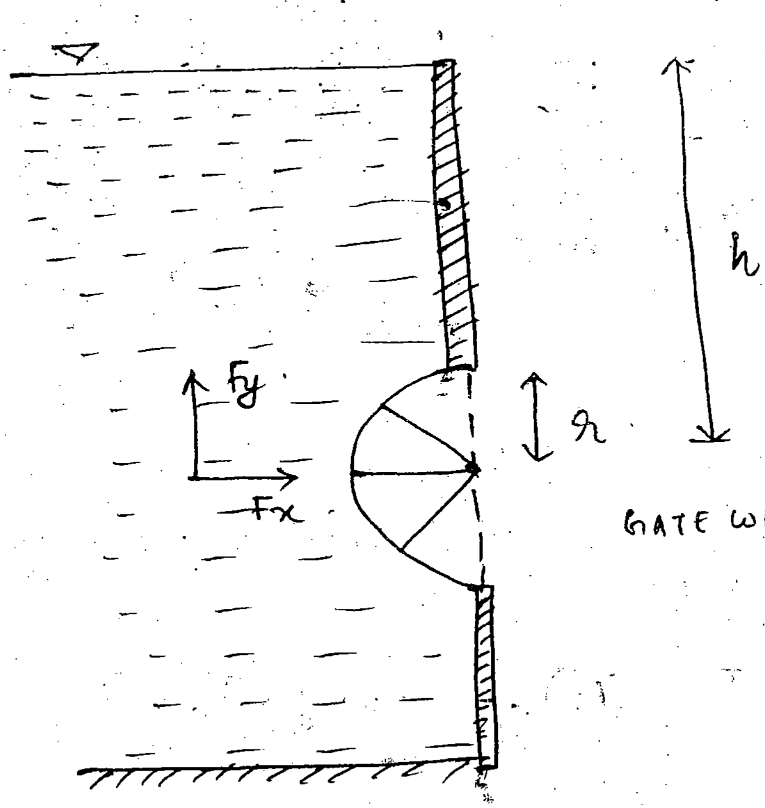
Q.) The ~~horizontal~~ horizontal and vertical forces F_x and F_y on semicircular gate having a width w into ~~the~~ plane of paper are

a) $F_x = \rho g h \cdot r w$ and $F_y = 0$

b) $F_x = 2 \rho g \cdot h \cdot r w$ $\quad F_y = 0$

c) $F_x = 2 \rho g \cdot h \cdot r \cdot w$ $\quad F_y = \rho g \cdot w \cdot r^2$

d) $F_x = 2 \rho g \cdot h \cdot r \cdot w$ $\quad F_y = \pi \rho g w r^2$



$$F_x = (\rho \times g) \times \bar{h} \times \text{Area}$$

$$= \rho \times g \times h \times (r \times 2 \times w)$$

$$= 2 \rho g h r w$$

$$F_v = M \cdot g$$

$$= \rho \times V \times g$$

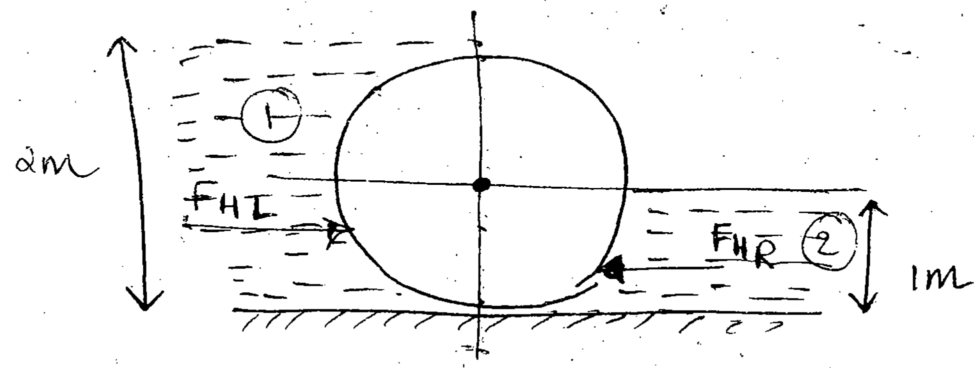
$$= \rho \times (A \times w) \times g$$

$$= \cancel{\rho \times \pi r^2} \ \rho \times g \left(w \times \frac{\pi r^2}{2} \right)$$

$$F_v = \frac{\pi \rho g w r^2}{2}$$

Q) A solid cylinder of dia 2m and length 10m supporting the water shown in fig. Det

(1) Net Horizontal force and Net vertical component.



$\underline{\Sigma F_H}$

$F_{HL} - F_{HR}$

$= \rho_1 \times g \times \bar{h}_1 \times (A_1) - \rho_2 \times g \times \bar{h}_2 \times (A_2)$

~~$\rho_1 \times g \times 1 \times (\pi$~~

$1000 \times 9.81 \times \left[\frac{2}{2}\right] \times \left[2 \times 10\right] -$

$1000 \times 9.81 \times \left(\frac{1}{2}\right) \times \left[1 \times 10\right]$

$= 1000 \times 9.81 \left[1 \times 2 \times 10 - \frac{1}{2} \times 10\right]$

$= 15 \times 9.81 \text{ kN}$

$= \underline{150 \text{ kN}}$

$= \underline{15 \text{ Tonnes}}$ (Towards right)

$\underline{\Sigma F_V}$

$F_{V1} + F_{V2}$

$= W_1 + W_2$

$= m_1 g + m_2 g$

Devices ..

$$1000 \times 9.81 \times 10 \left[\frac{\pi R^2}{2} + \frac{\pi R^2}{4}\right]$$

as Force / unit Area. It is

as many directions

$$= 1000 \times 9.81 \times 10 \left[\frac{\pi \times (1)^2}{2} + \frac{\pi \times (1)^2}{4}\right]$$

(Q.) state true or false

(1) the centre of pressure o... plane surface is

a.) depends upon the type of fluid ~~b) or~~ contact with the surface (x)

$$h^* = \bar{h} + \frac{I}{A \cdot \bar{h}}$$

[depends only on geometric properties] & depth of centroid ($\bar{h}$)

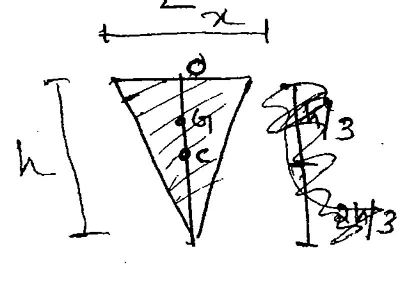
centre of pressure is independant upon type of fluid in contact with surface

(Q) A vertical component of the hydrostatic force on curved surface is equal to weight of the liquid vertically above the curved surface (✓)

(Q.) A triangular plate immersed in water with vertex downwards. the altitude of the vertical ~~o~~ triangular face is 'h'. the centre of pressure is ~~$\frac{h}{2}$~~

$$OC = \frac{h}{2}$$

[True]



$$h^* = \bar{h} + \frac{I}{A \cdot \bar{h}}$$

OC ... $\frac{1}{2} \times$ base $\times$ height^3

$$\frac{h}{3} + \frac{\frac{1}{36} \times x \times h^3}{\frac{1}{2} \times x \times h}$$

$$\frac{h}{3} + \frac{h}{6}$$

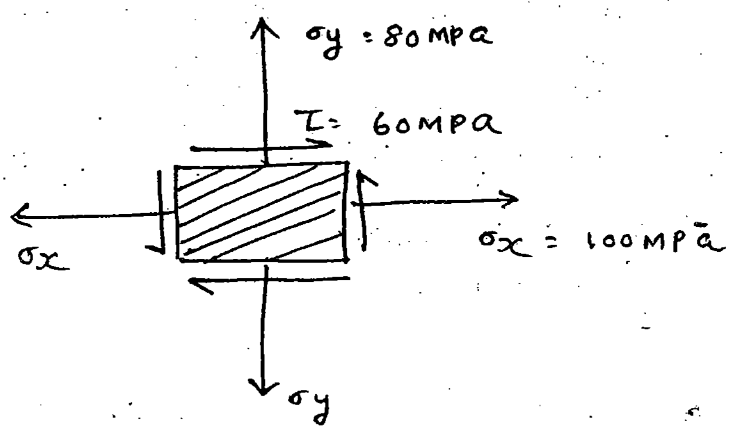
$$= \frac{h}{2}$$

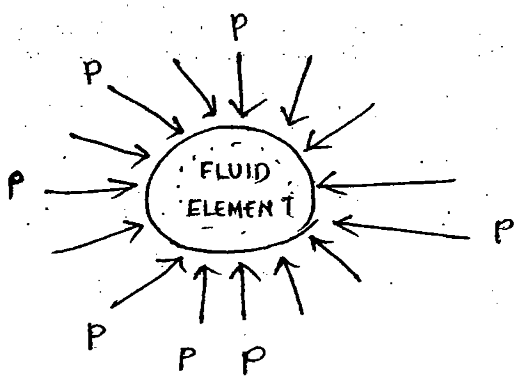
Devices

Pressure is defined as Force / unit Area. It is a Scalar qty. Because it has many directions of action on the given Area.

In FM pressure is due to the fluid contact with the surface. when fluid at Rest no tangential force involved. only Normal forces involved. It can be studied through Mohrs circle concept also.

when fluid element in space subjected to normal pressures which are in compressive nature with same magnitude. ie $P_x = P_y$ . . . .

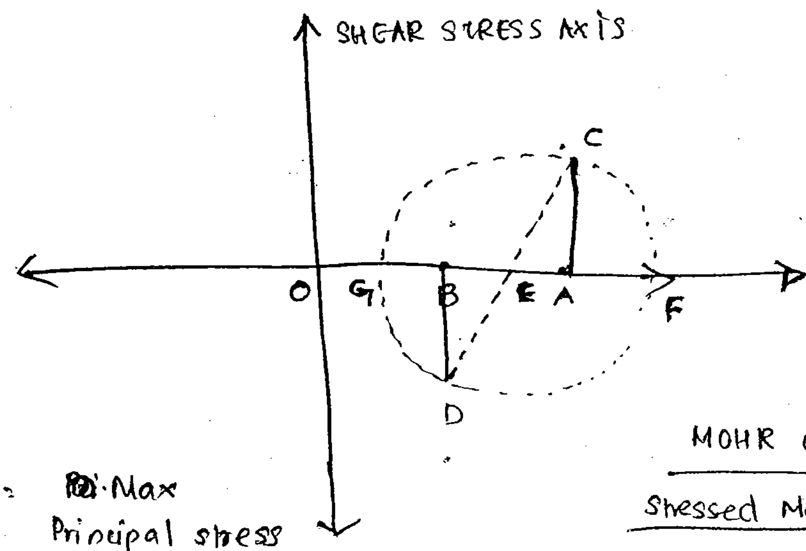




OA = 100 MPa

OB = 80 MPa

AC = BD = 60 MPa



OF = σ_{max} = Max Principal stress

MOHR CIRCLE FOR Stressed Machine element

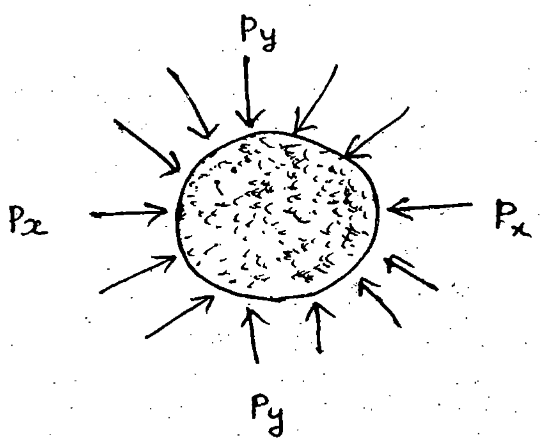
$$OF = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau^2} = \sigma_{max}$$

$$OG = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau^2} = \sigma_{min}$$

EC = ED = Radius of Mohr's circle

$$\tau_{max} = \frac{\sigma_{max}-\sigma_{min}}{2}$$

$$= \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau^2}$$

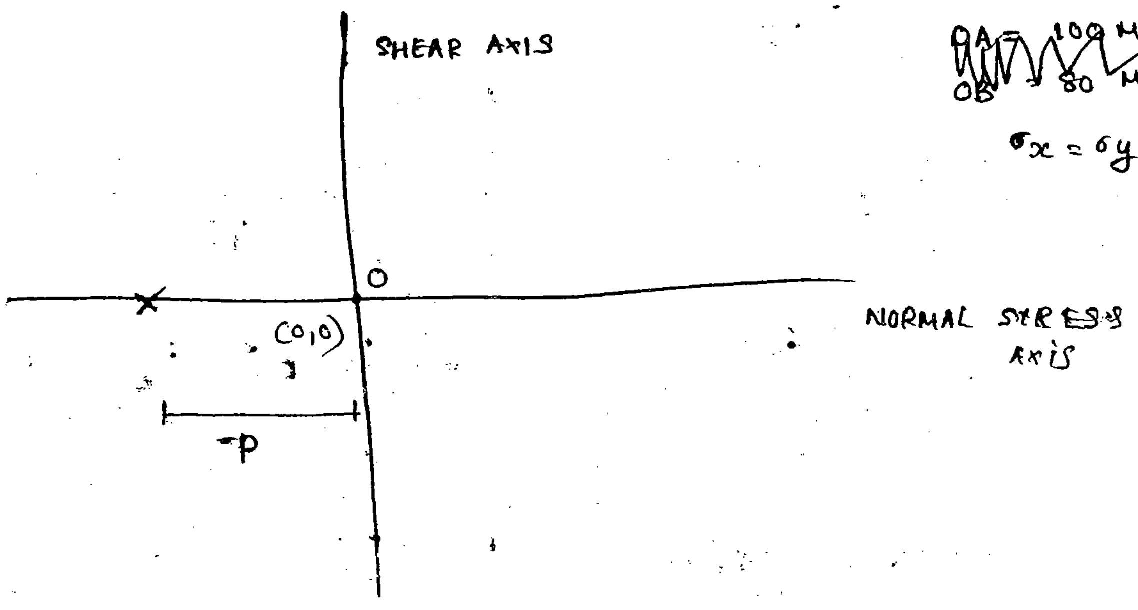


AT REST

$$\boxed{\tau = 0}$$

$P = P_x = P_y$ .... σ_c

$$\left.\begin{aligned}\sigma_x = \sigma_y &= \sigma_c = -P \\ \tau &= 0\end{aligned}\right\}$$ Fluid element At Rest



OA = 100 MPa
OB = 80 MPa

$\sigma_x = \sigma_y = -P$

on the negative normal stress axis.

(3) If a Mohr's circle is drawn for a fluid element which is at Rest then the Mohr's circle would be

a) A circle touching the circle

b) A point on shear stress Axis

c) A circle ~~at~~ not touching the origin

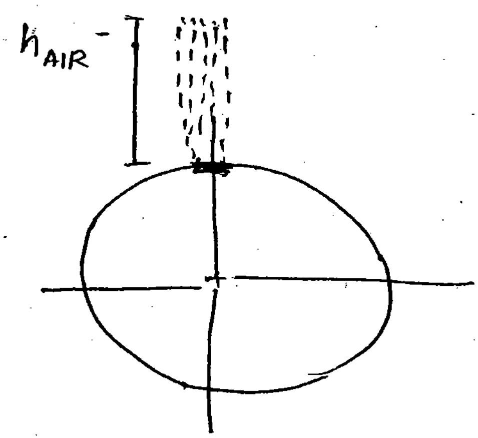
d) A point on the Normal stress Axis

Exactly: A point on -ve normal stress Axis

NOTE:-

(4) The Normal stresses of same magnitude in all directions at a point in a fluid possible when fluid has no shear stress.

PRESSURE INTENSITY AT A POINT DUE TO FLUID



$$P = \frac{F}{A} = \frac{W}{A}$$

$$= \frac{mg}{A}$$

$$= \frac{\rho \times V \cdot g}{A}$$

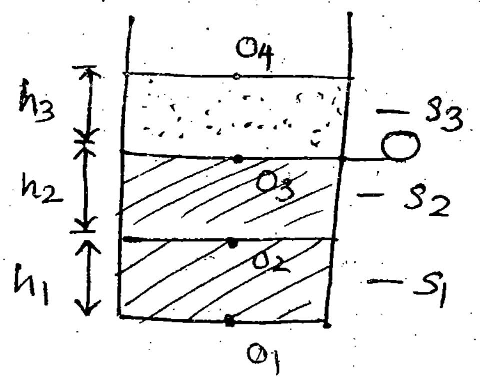
$$= \frac{\rho \times A \times h \cdot g}{A}$$

$$\boxed{P = \rho g h}$$

$$P = \gamma h$$

$$\boxed{\frac{dP}{dh} = \gamma}$$

Pressure Intensity in Multi Fluids:-



$S_1 > S_2$

$P_{O4} = P_{atm} = 101.3\ kPa$

$\approx 100\ kN/m^2 = 10T/m^2$

P_{O3} (guage) $= \rho_3 \cdot g \cdot h_3$

P_{O2} (guage) $= \rho_3 \cdot g \cdot h_3 + \rho_2 \cdot g \cdot h_2$

P_{O1} (gauge) $= \rho_3 g h_3 + \rho_2 g h_2 + \rho_1 g h_1$

$P = \rho_{AIR} \cdot g \cdot h_{AIR} = \rho_{Hg} \cdot g \cdot h_{Hg} = \rho_{H_2O}\, g \cdot H_{H_2O} = \rho_{oil} \cdot g \cdot H_{oil}$

(N/m^2)

Pressure at a point can be Represented in terms of Coloumns. [Air, water, oil . . .]

$P = \frac{F}{A} \left[\frac{N}{m^2}\right]$

$\downarrow$

Pascal (Pa)

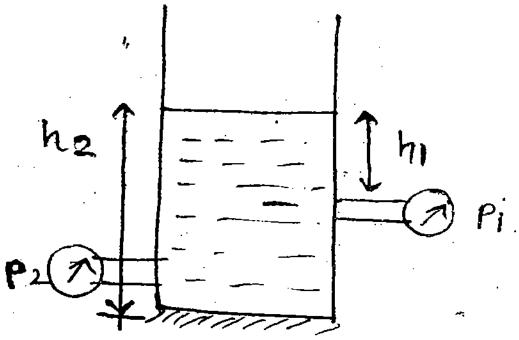
cm^2 $\qquad$ cm → m

P = metres of water or Hg or oil

Difference in Pressure b/w 2 points:-

$P_1 = \rho \cdot g \cdot h_1$

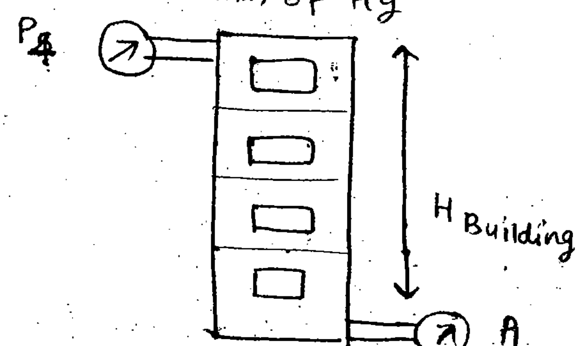
$P_2 = \rho \cdot g \cdot h_2$



$P_2 - P_1 = \rho g (h_2 - h_1)$

$$\boxed{\frac{P_2 - P_1}{h_2 - h_1} = \rho g}$$

745 mm of Hg



$P_1 =$ 755 mm of Hg

$$\frac{P_4 - P_1}{H_2 - H_1} = \rho \times g$$

$\Delta P = P_2 - P_1 = \rho g (h_2 - h_1)$

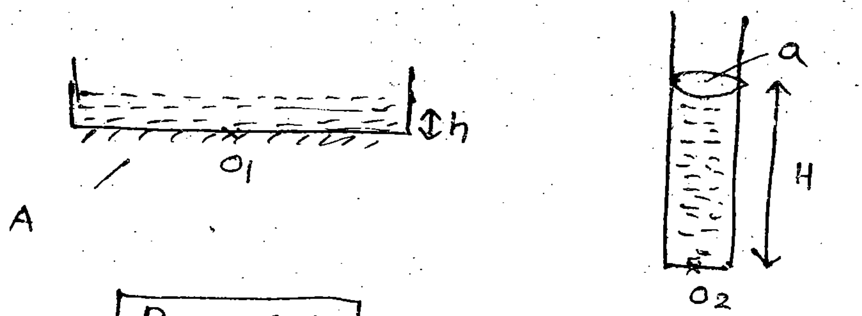
↓

$\rho_{Hg} \cdot g \left[h_{2_{Hg}} - h_{1_{Hg}} \right] = \rho_{AIR} \cdot h_{Building} \, g$

$13600 \times 9.81 \left[0.755 - 0.745 \right] = 1.2 \times 9.81 \times H_{Building}$

$H_{Building} = 120 \text{ m}$

NOTE! — 1 kg of water = 1 lt of water



$$\boxed{P_{O_1} = \rho g h}$$

$$\boxed{P_{O_2} = \rho g H}$$

$P_{O_2} > P_{O_1}$

2011 Pressure intensity is dependent on depth only.

XE

Q) Two tanks A and B with same Height are filled with water till the top. The volume of Tank A is 10 times the vol of Tank B. What can you say about the Pressure P_A and Pressure P_B at the Bottom of the Tanks A and B resp.

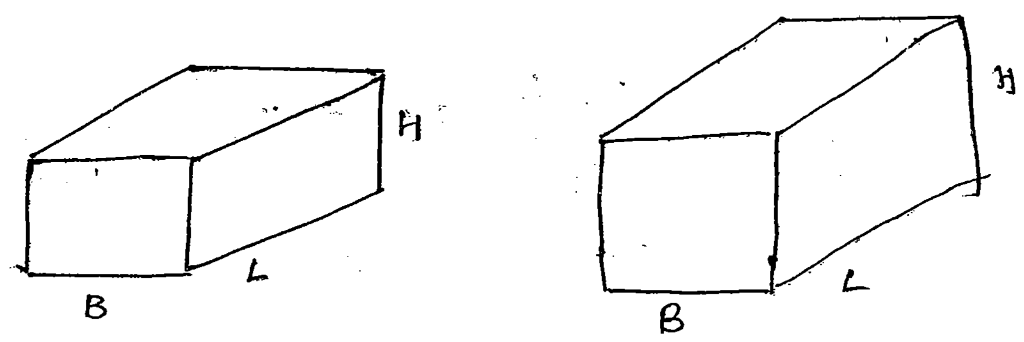
a) $P_A = 10 P_B$

b) $P_B = 10 P_A$

c) $P_A = P_B$

d) Additional data is Reqd to Compare the two pressures.

$H_A = H_B$



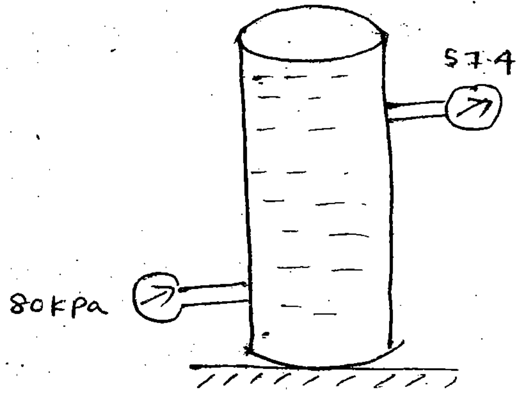
$V_A = 10 V_B$

$A_A \cdot H_A = 10 A_B \cdot H_B$

resp at heights 8m, and 5m resp. Gauges connected on side of the Tank filled with a liquid - what is the specific gravity of that liquid in the Tank.

$P_1 = 57.4\, kPa$ $\quad h_1 = 8\, m$

$P_2 = 80\, kPa$ $\quad h_2 = 5\, m$



$$\cancel{\frac{P_1 - P_2}{h_2 - h_1}} =$$

$$\frac{P_2 - P_1}{h_2 - h_1} = \rho \cdot g$$

$$\frac{10^3 (80 - 57.4)}{8 - 5} = \rho \times 9.81$$

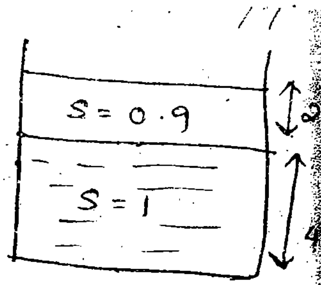
$\rho = \cancel{0.76}\ 768$ $\quad S = \frac{\rho_{liq}}{\rho_{H_2O}} = \frac{768}{1000}$

$$S = 0.76$$

(1.) A 6m deep Tank contains 4m of water at Bottom and 2m of oil (S.R.D = 0.9). Above the water. The Pressure at Bottom of tank in KN/m^2 is $\quad$ unit wt of water = 10 KN/m³

$$P = \rho_w \cdot h_w \cdot g + \rho_{oil} \cdot h_{oil} \cdot g$$

$56.898 \ N/m^2$



$56.898 \ kN/m^2$

OR

$\gamma_w \cdot h_w + \gamma_{oil} \cdot h_{oil}$

$10 \times 4 + (S_{oil} \times \gamma_{water}) \cdot h_{oil}$

$= 10 \times 4 + (0.9 \times 10) 2$

$= \frac{58 \ kN}{m^2}$

Q) Which of the following pressure units represents least pressure units

a) milli bar

b) mm of Hg

c) N/mm^2

d) kgf/cm^2

Millibar = 10^{-3} Bar

$= 10^{-3} \times 10^5 \ Pa = 100 \ Pa$

mm of Hg

760 mm of Hg = 1 bar

mm of Hg = $\frac{1}{760}$ Bar = $\frac{1}{760} \times 10^5 \ Pa$

$= 131.51 \ Pa$

$$P = \rho_{Hg} \cdot g \cdot H_{Hg}$$

$$= 13600 \times 10 (1 \times 10^{-3}) = \underline{136 \text{ Pas}}$$

PUT IN ORDER of Least unit

(A) < (B) < (D) < (C)

2010 ISRO

Given that Specific gravity of mercury = 13.6, Intensity of pressure = 40 kpa. express the Intensity of pressure (gauge) in various units [S.I]

~~40 kPa~~

a) 0.3 bar, 3.077 m of H_2O, 0.15 m of Hg

b) 0.5 bar, 5.077 m of H_2O, 0.339 m of H_2O

c) 0.4 bar, 4.077 m of H_2O, 0.229 m of Hg

d) None of these.

40×10^3 Pa

$$= \frac{40 \times 10^3}{10^5} \text{ bar} = \underline{0.4 \text{ Bar}}$$

$P = \rho g h$

$40 \times 10^3 = 13600 \times 9.81 \times h_{hg}$

$h_{hg} = 0.229$ ~~mer~~ m of Hg

Q) The pressure in metres of oil ($S_{oil} = 0.8$) equivalent to 80 m of water is

ISRO
DRDO

a) 64 m

b) 88 m

c) 80 m

d) 100 m

P = 80 m of water

$\rho_w \cdot h_w \cdot \cancel{g} = \rho_{oil} \cdot h_{oil} \cdot \cancel{g}$

$\cancel{1000} \cdot 80 = 0.8 \times \cancel{1000} \cdot h_{oil}$

$h_{oil} = \frac{80}{0.8} = \underline{100 \text{ m of oil}}$

Q.) Specific Gravity of a liquid is 0.75. what Height of that liquid is needed to provide a pressure difference of 2 bar ?

(a) 27.18 m

(b) 17.98 m

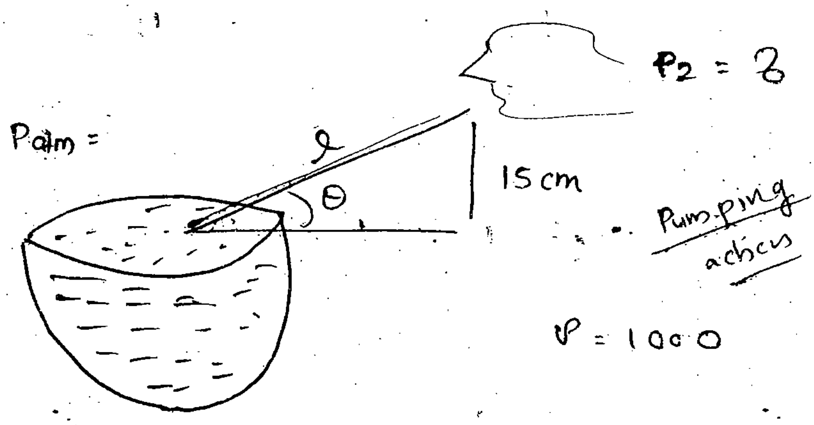
(c) 9.81 m

(d) 15 m

$$2\times 105 = (0.75\times 1000)\times 9.81\times h$$

$$h = \cancel{27}\ 183\,m \text{ of water}$$

(Q.)



How much pressure is to be borne by lungs.

$$\Delta P = \rho g \Delta h$$

$$= 1000\times 9.81\times (0.15)$$

$$= 1.5\times 10^3\ N/m^2$$

$$= 1.5\ kPa$$

$$P_2 - P_1 = -1.5\ kPa$$

$$P_2 = -1.5 + 101$$

$$P_2 = 99.5\ kPa$$

work done $= P.V$

Q) Specific wt of sea water may be taken to vary according to the expression ($\gamma = \gamma_0 + C\sqrt{h}$)

where γ_0 = sp wt of the sea water at datum (free surface)

h = depth below the sea surface.

The pressure at a depth h metres in excess of $\gamma_0 h$ is a) $\frac{1}{3} c h^{3/2}$

b) $\frac{2}{3} c h^{3/2}$

c) $\frac{1}{3} c h^{2/3}$

d) $\frac{1}{2} c h^{2/3}$

$$\frac{dp}{dz} = -\gamma$$

$$\frac{dp}{dh} = \gamma$$

$$dp = \gamma \, dh$$

$$dp = (\gamma_0 + c\sqrt{h})\, dh$$

$\int$

$$p = \int \gamma_0 \, dh + c \int \sqrt{h} \, dh$$

$$= \gamma_0 h + c \frac{h^{\frac{1}{2}+1}}{\frac{1}{2}+1}$$

$$= \gamma_0 h + c \cdot \frac{2}{3} h^{3/2}$$

$P = \rho g h$

$\dfrac{P}{h} = \rho g$

$\boxed{\dfrac{P}{h} = \gamma}$

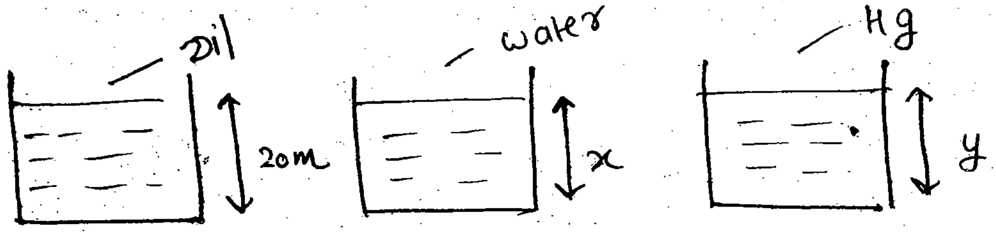
$\boxed{\dfrac{dp}{dh} = -\gamma}$

$h \uparrow \; P \uparrow$

$\dfrac{dP}{dh} = \gamma$

Q) Figure shows an open Tank of diff Fluids shown 3 tanks at same height

$S = 0.8$



$h = 20m$

$P = \rho_{oil} \cdot g \cdot h_{oil} = \rho_w \cdot g_w \cdot h_w = \rho_{Hg} \cdot h_{Hg} \cdot g$

$\cancel{P = 0.8 \times 10^3 \times 10 \times 20}$ 9.81

$= \cancel{8000 \times 20}$

$= \cancel{156,960}$

$\rho_{oil} \cdot \cancel{g} \cdot h_{oil} = \rho_w \cdot \cancel{g} \cdot h_w$

$\cancel{10^3} \times 0.8 \times 20 = \cancel{10^3} \times x$

$x = 20 \times 0.8$

$= \underline{16m}$

$\rho_{oil} \cdot \cancel{g} \cdot h_{oil} = \rho_{Hg} \cdot h_{Hg} \cdot \cancel{g}$

$0.8 \times \cancel{10^3} \times 20 = 13.6 \times \cancel{10^3} \cdot y$

Q) Match list 1 and list 2

List 1 [Pressure due to a column of 0.4 m]	List 2 [Pressure Intensity (units)]
	0.353 N/cm²
a) Water	5.337 N/cm²
b) Mercury	0.392 N/cm²
c) Air	4.709 N/m²
d) oil (S = 0.9)	

$P_w = \rho_w \cdot h_w \cdot g$

$= 10^3 \cdot 0.4 \times 9.81$

$= \underline{0.392}$

$P_{Hg} = \rho_{Hg} \cdot h_{Hg} \cdot g$

$= 13600 \times 0.4 \times 9.81$

$= \underline{5.337}$

$P_{AIR} = \rho_{AIR} \cdot h_{AIR} \cdot g$

$= 1.2 \times 0.4 \times 9.81$

$= \underline{4.709\ N/m^2}$

$P_{oil} = (0.9 \times 10^3) \times 0.4 \times 9.$

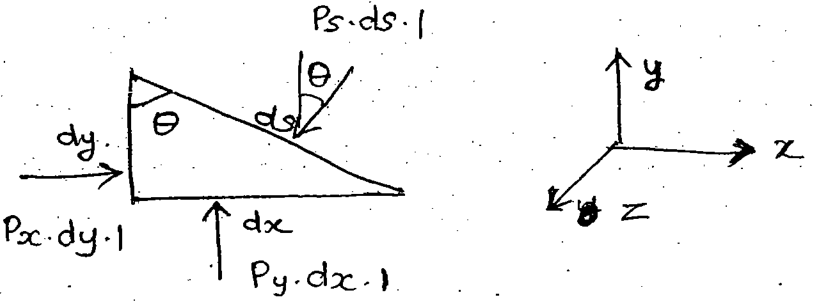
$= \underline{0.35\ N/cm^2}$

1. Barometer
2. Piezometer
3. Simple Manometer
4. U tube Manometer
5. Differencial Manometer
6. Inverted Differencial Manometer
7. Bourden gauge

PASCALS LAW

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all the directions.

Fluid element of dimensions dx, dy, ds.

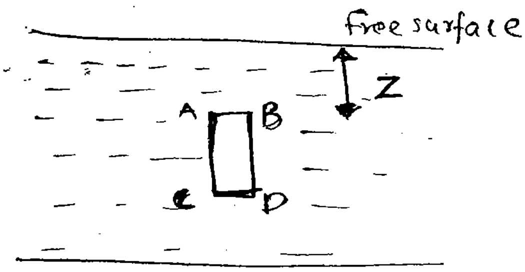


According to pascals Law

$$\boxed{P_x = P_y = P_z}$$

HYDROSTATIC LAW

It states that the increase in pressure of a fluid in vertical direction is equal to the weight density of the fluid at that point.



$$\frac{\partial P}{dz} = \rho g = \gamma$$

Sing $dP = \rho g \, dz$

$P = \rho g z$

$\boxed{P = \rho g h}$ $\boxed{h = \frac{P}{\rho g}}$

MEASUREMENT OF PRESSURE:-

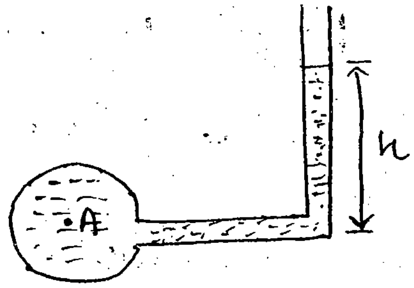
1. Manometers → [Measure Pressure at a point by balancing coloumn of fluid by same or another coloumn of fluid]
2. Mechanical Gauges
   - Diaphragm pressure Gauge
   - Bourden Tube Pressure Gauge
   - Dead wt Pressure gauge
   - Bellows pressure gauge

Manometers :-

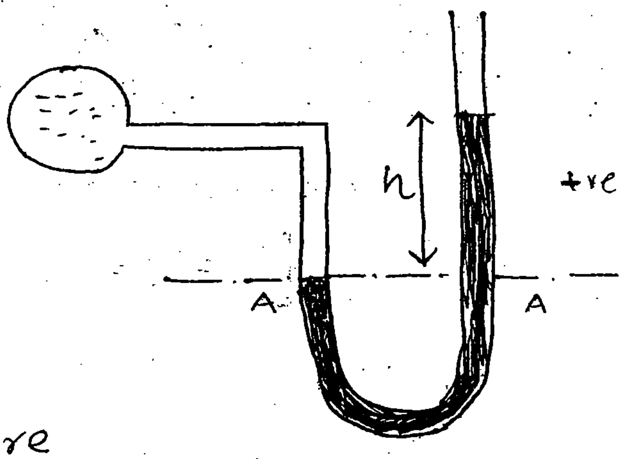
1. Simple Manometer :-

$P_A = \rho g h \ \frac{N}{m^2}$

PIEZOMETER →

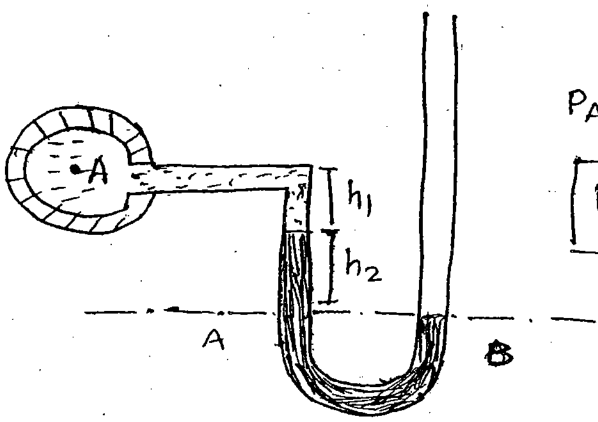


U tube Manometer showing Negative pressure or Vaccum pressure.



+ve pressure or Gauge pressure

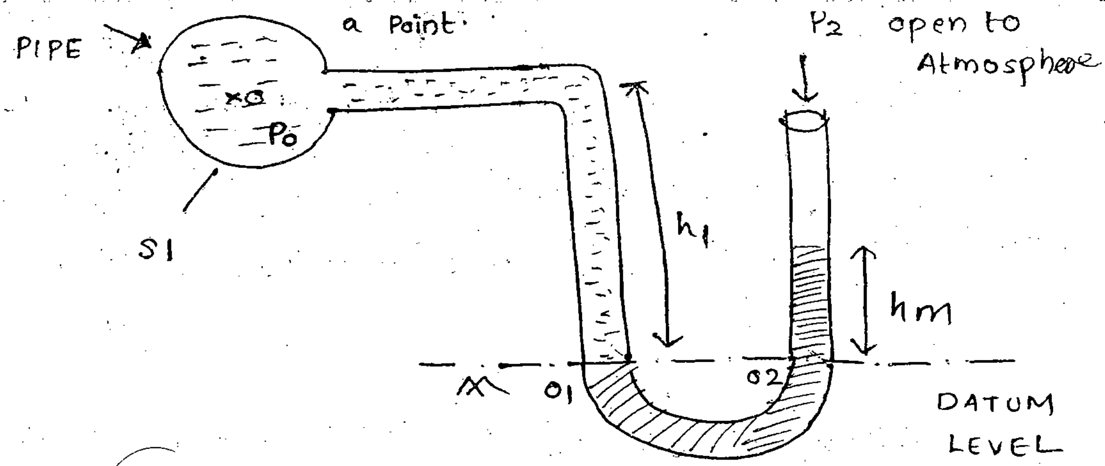
Vaccum pressure



$P_A + \rho_m g h_2 + \rho g h_1 = 0$

$$\boxed{P_A = -(\rho_m g h_2 + \rho g h_1)}$$

N/m^2



Mercury - Manometric fluid

According to principle of Manometry (Pascals Law)

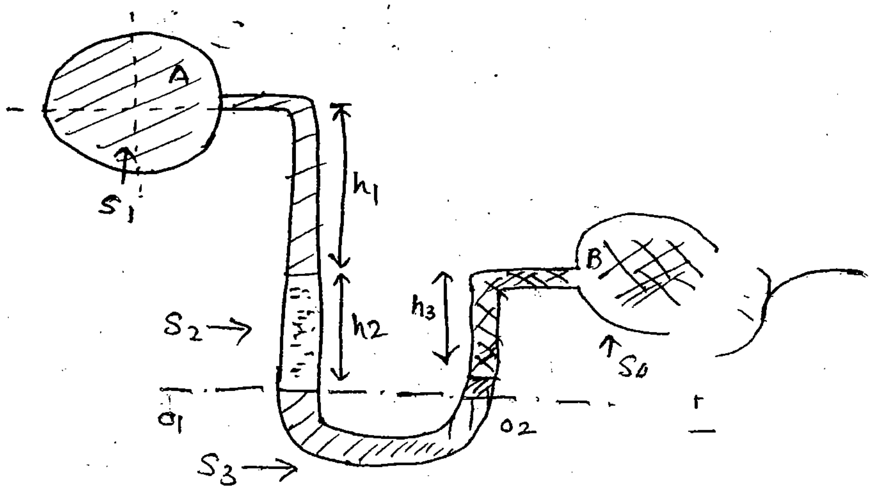
$$P_{o1} = P_{o2}$$

$$\boxed{(P_o)_{gauge} + \rho \cdot g \cdot h_1 = \rho_m g h_m}$$

$$\boxed{(P_o)_{gauge} = \rho_m g \cdot h_m - \rho \cdot g \cdot h_1} \quad N/m^2$$

4) Differencial Manometer

It measures pressure b/w 2 points of the fluids under pressure.

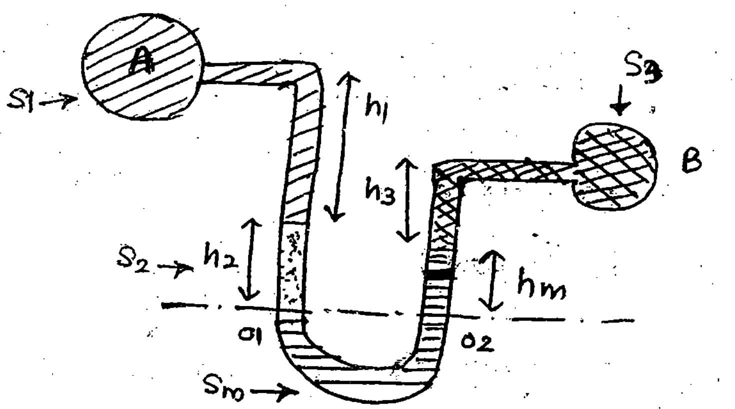


Continous flow

$P_{o1} = P_{o2}$

$$P_A + \rho_1 g h_1 + \rho_2 g h_2 = \rho_m g h_m + \rho_3 g h_3 + P_B$$

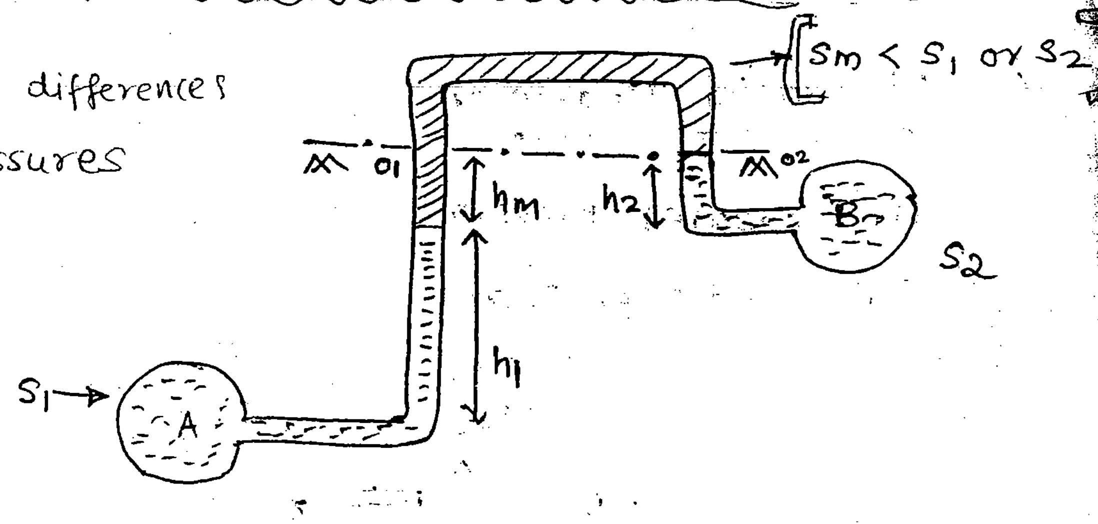
$$\boxed{P_A - P_B = \rho_m g h_m + \rho_3 g h_3 - \rho_1 g h_1 - \rho_2 g h_2}$$



Inverted U-tube Manometer

This Meter consists of Manometric fluid of specific Gravity Lower than the fluids in pipes where Pressure diff is tobe Measured. this meter is used to measure Pressure differences in fractions

measures differences of low pressures

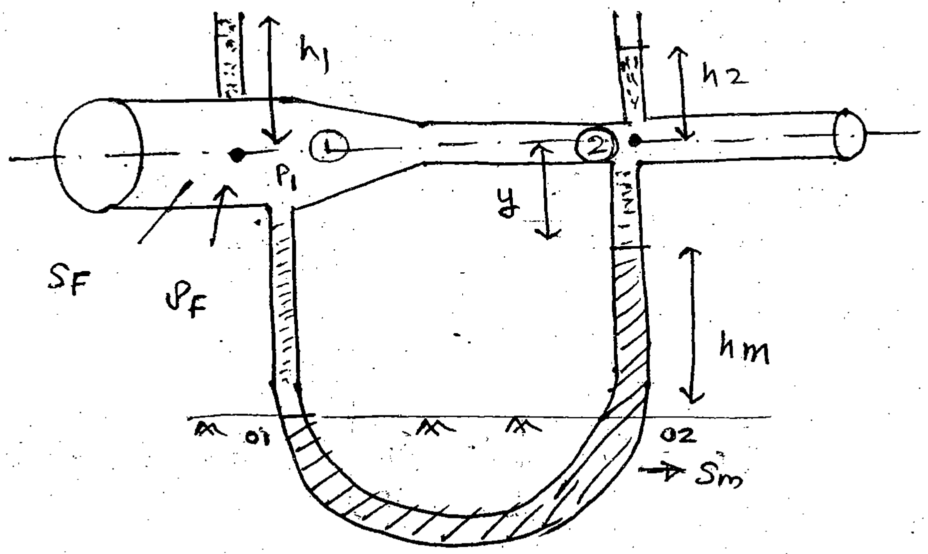


Identification

$\boxed{S_m < S_1 \text{ or } S_2}$

$$\boxed{P_A - P_B = -\rho_2 g h_2 + \rho_1 g h_1 + \rho_m g h_m}$$

VENTURIMETER : To measure flow Rate



Derive an expression of pressure difference $(P_1 - P_2)$ or Pressure difference head $(h_1 - h_2 = \Delta h)$ in terms of (S_F, S_m, h_m)

$$\Delta h = h_1 - h_2 = h_F = \frac{P_1 - P_2}{\rho_F \cdot g} = h_m\left[\frac{S_m}{S_F} - 1\right]$$

According to Principle of Manometer

$$P_{01} = P_{02}$$

$$P_1 + \rho_1 g\,[y + h_m] = P_2 + \rho_m g h_m + \rho_1 g y$$

$$P_1 - P_2 = \rho_m g h_m + \cancel{\rho_1 g y} - \cancel{\rho_1 g y} - \rho_1 g h_m$$

$$P_1 - P_2 = \rho_m g \cdot h_m - \rho_1 g h_m$$

$$P_1 - P_2 = [\rho_m - \rho_1]\, g h_m$$

$$\frac{P_1 - P_2}{\rho_1 g} = h_m \left[\frac{\rho_m}{\rho_1} - 1 \right]$$

$$h_1 - h_2 = h_m \left[\frac{\rho_m / \rho_{water}}{\rho_1 / \rho_{water}} - 1 \right]$$

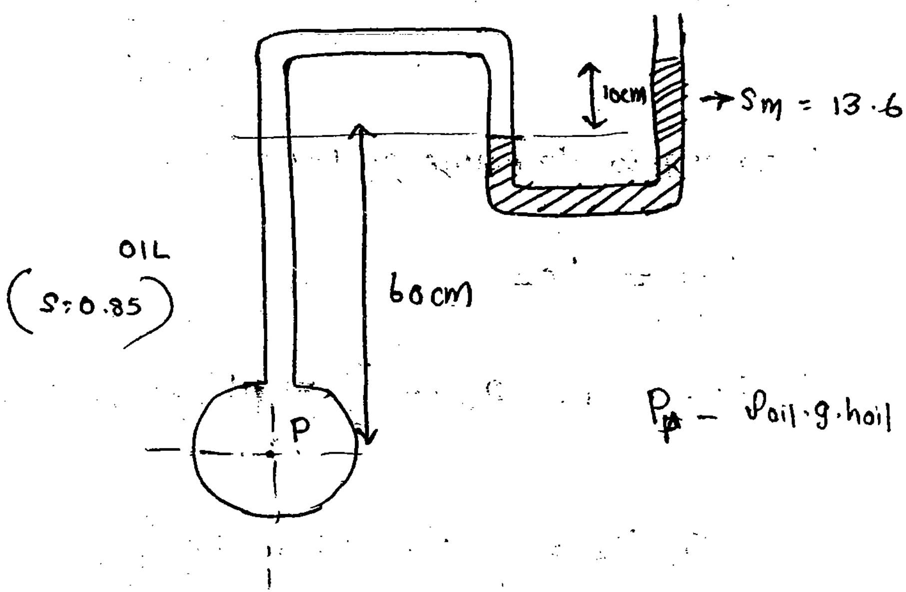
$$h_1 - h_2 = h_m \left[\frac{S_m}{S_F} - 1 \right]$$

$$\boxed{\Delta h_F = h_m \left[\frac{S_m}{S_F} - 1 \right]} \qquad S_m > S_F$$

Head in terms of 'm' of water

$$\boxed{(\Delta h)_F = h_m \left[1 - \frac{S_m}{S_F} \right] \quad \text{if} \quad S_m < S_F}$$

Q.) Fig shows pressure measurement Arrangement. Determine Pressure at the Point 'P' (gauge Pressure).



$P_P - \rho_{oil} \cdot g \cdot h_{oil} = \rho_m g h$

$P_P = \rho_m \cdot g \cdot h_m + \rho_{oil} \cdot g \cdot h_{oil}$

$\ldots \times 10^{-2} + 0.85 \times 10^3 \times (10) \times 60 \times 10 \ldots$

$P_P = \rho_{oil} \cdot g \cdot H_{oiL}$

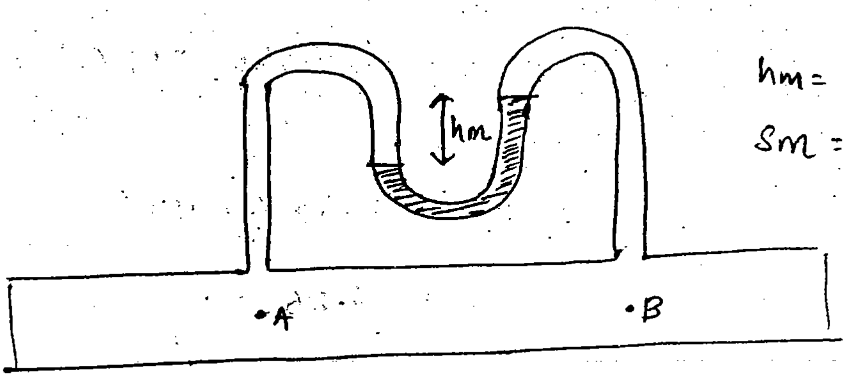
$18344.7 = 850 \times 9.81 \times H_{oil}$

$H_{oil} = 2.2m$

A pipe carrying water, a diff manometer connected to pipe at 2 points A and B and the deflection of the mercury level is 60 cm and the unit weight of mercury is 13.6 g/ml.

$[\rho_{Hg} = 13.6 \, g/ml = 13.6 \, g/cm^3 = 13600 \, kg/m3]$

$[g/ml \rightarrow \frac{g}{cm^3}]$



$h_m = 60$ cm

$S_m = 13.6 \, g/ml$

find $P_A - P_B$ in terms of m of water

(a) 7.56 m of H_2O

(b) 75.76

(c) 8.16

(d) 16.32

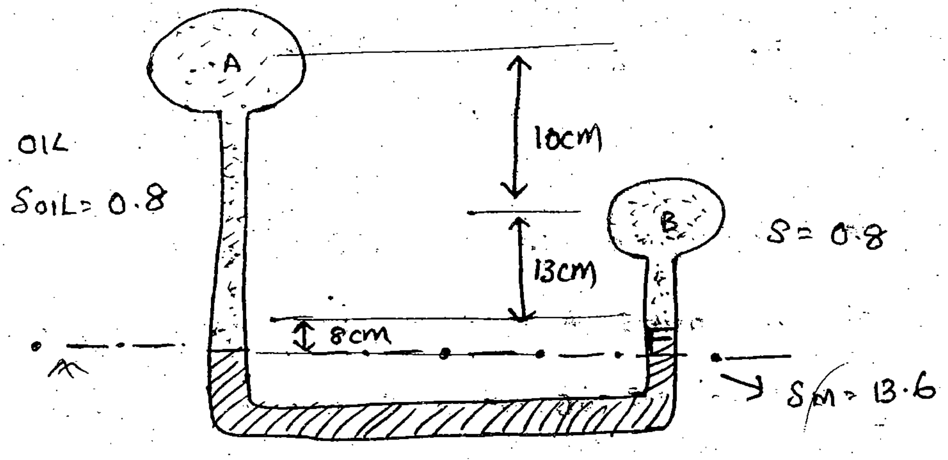
$$h_A - h_B = \underset{\text{water}}{(\Delta h)_{AB}} = h_m \left[\frac{S_M}{S_{water}} - 1 \right]$$

$$= 0.6 \left[\frac{13.6}{1.0} - 1 \right]$$

$= 0.6 \times 12.6$

$= 7.56$ metres of water

Q.) what is the Pressure diff b/w Points A and B shown in the figure



$$P_A + \rho_{oil} \cdot g \cdot H_{oil} = P_B + \rho_m \cdot g \cdot h_m + \rho_{oil} \cdot g \cdot h_{oil}$$

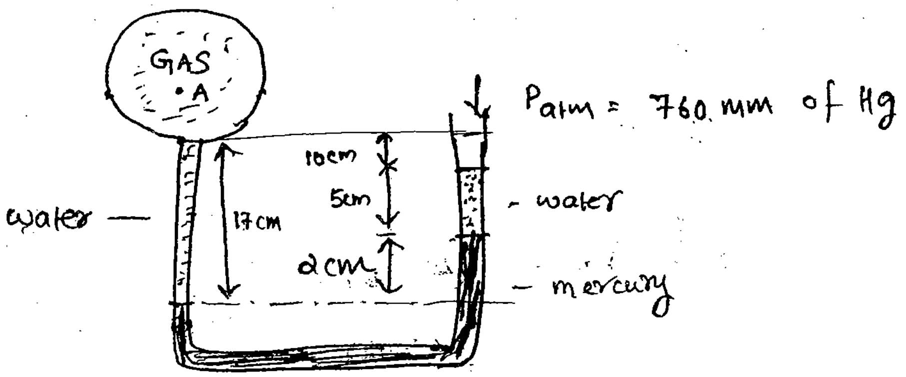
$$P_A - P_B = (13.6 \times 1000) \times 9.81 \times 0.08 + 800 \times 9.81 \times 0.13$$

$$800 \times 9.81 \times 0.13$$

$$9260.64 \ N/m^2$$

$$= 9.26 \ kPa$$

Q.) Refer to the figure the Absolute pressure of the Gas in the pipe at 'A' is



$$P_A + \rho_w \cdot g \cdot h_w = \rho_m \cdot g \cdot h_m + \rho_w \cdot g \cdot h_w + \cancel{\rho_g \cdot g \cdot h_g}$$

$$(P_A)_{gauge} = 13600 \times 9.81 \times 0.02 + 1000 \times 9.81 \times 0.05 + \cancel{1.2 \times 9.81 \times 0.1} - 1000 \times 9.81 \times 0.17$$

$$(P_A)_{gauge} = \cancel{3157.99}\ \frac{N}{m^2} = \frac{3157.99}{10^5} = 0.031\ mm\ \text{of Hg}$$

$$(P_{abs}) = (P_A)_{gauge} + P_{atm}$$

$$= \cancel{3157.99}$$

$$3157.99\ \frac{N}{m^2}$$

$$1\ bar = 760\ mm\ \text{of Hg}$$

$$10^5\ Pa = 760$$

$$1\ Pa = \frac{760}{10^5}\ mm$$

$$P_{abs} = 760 + 0.031$$

$$= 760.031$$

$$(P_A)_{gauge} = \underline{1491.12\ N/m^2}$$

$$(P_A)_{gauge} = \rho_{Hg} \cdot g \cdot H_{Hg}$$

$$1491.12 = 13600 \times 9.81 \times H_{Hg}$$

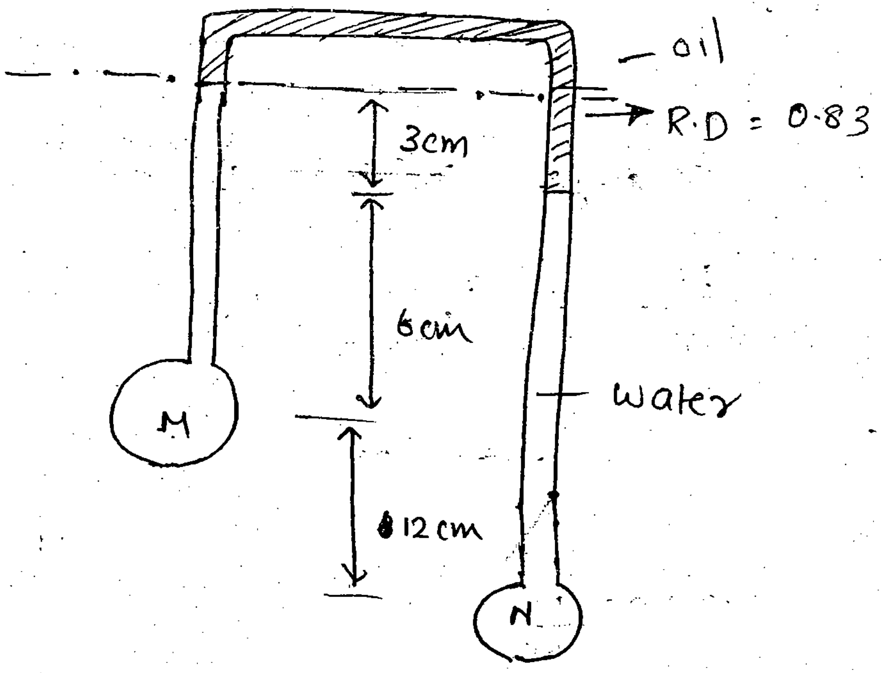
$$H_{Hg} = \frac{1491.12}{13600 \times 9.81} = \underline{0.0117\ m}\ \text{of Hg}$$

$$= \underline{11.7\ mm\ \text{of Hg}}$$

$$\boxed{P_{abs} = P_{Atm} + P_{gauge}}$$

$$= 760\ mm + 11.7\ mm = \cancel{771}$$

c) Pressure diff b/w M and N for the manometer shown in fig in terms of cm of ~~Hg~~ oil



$$P_M - \rho_w \cdot g \cdot h_{w_1} = P_N - \rho_{fluid} \cdot h_{fl} \cdot g - \rho_w \cdot h_w \cdot g$$

$$P_M - P_N = \rho_w \cdot g \cdot h_{w_1} - \rho_{fl} \cdot h_{fl} \cdot g - \rho_w \cdot h_{w_2} \cdot g$$

$$1000 \times 9.81 \times 0.09 - 830 \times 9.81 \times 0.03 - 1000 \times 0.18 \times 9.81$$

$P_M - P_N = -1127.169 \ N/m^2$

$$(P_M - P_N) = \rho_{\cancel{Hg}\ oil} \cdot h_{\cancel{Hg}\ oil} \cdot g$$

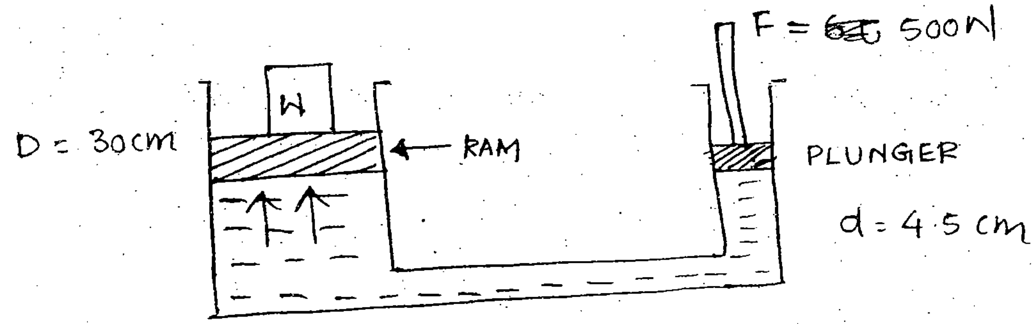
$$1127.169 = \cancel{13600}\ 830 \cdot h_{\cancel{Hg}\ oil} \cdot 9.81$$

~~$h_{Hg} = 8.44 \times 10^{-3}$ m of Hg~~

$= \cancel{0.84}$

PROBLEMS BASED ON HYDROSTATIC LAW

(Q.1) ~~221~~ A hydraulic press has a ram of 30 cm dia



$$P = \frac{F}{A} = \frac{500}{\frac{\pi}{4}\times(0.045)^2} = 314465.4\ \frac{N}{m^2}$$

$$P_{RAM} = P_{PLUNGER}$$

$$\frac{W}{\frac{\pi}{4}\times(0.3)^2} = \cancel{314464}\ 314465.4$$

$$W = \underline{\underline{22.222\,kN}}$$

V =

(22)

D = 20 cm = 0.2 m

d = 3 cm = 0.03 m

W = 30 kN

F_p = ?

$$P_{RAM} = P_{PLUNGER}$$

~~F_{RAM} = $F_{PLUNGER}$~~

$$\cancel{\frac{\pi}{4}\times D^2\times W = \frac{\pi}{4}\times d^2\times F_p}$$

$$\cancel{0.2^2\times 30\times10^3 = (0.03)^2\times F_p}$$

$$\frac{W}{\frac{\cancel{\pi}}{\cancel{4}}\times D^2} = \frac{F_p}{\frac{\cancel{\pi}}{\cancel{4}}\times d^2} \qquad F_p =$$

$$F_p = \frac{30\times10^3\times(0.03)^2}{\ } = \underline{\underline{675\,N}}$$

(23) $P = ?$ 0.3 m of water

$P = \rho_w \times g \times h_w$

$= 1000 \times 9.81 \times 0.3$

$= \underline{2943\ N/m^2}$

due to oil

$S = 0.8$

$P = 0.8 \times 10^3 \times 9.81 \times 0.3$

$= \underline{2354.4}$

Mercury

$S = 13.6$

$P = 13600 \times 9.81 \times 0.3$

$= \underline{40024.8}\ N/m^2$

(24) $P = 3.924\ N/cm^2$ $\frac{3.924\ N}{10^{-4}\ m^2}$

$h = ?$ (a) water
(b) oil of $S = 0.9$

$3.924 = \rho_w \times g \cdot h_w$

$3.924 \times 10^4 = 1000 \times 9.81 \times h_w$

$\underline{h_w = 4\ m}$

(b) oil

$3.924 \times 10^4 = 900 \times 9.81 \times h_{oil}$

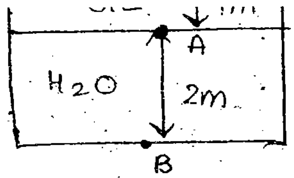
$\underline{h_{oil} = 4.44\ m\ \text{of oil}}$

(25) $S = 0.9$

$h_{oil} = 40\ m$ $h_w = ?$

$P_{oil} = 0.9 \times 1000 \times 9.81 \times 40$

$= \underline{353160\ N/m^2}$



$P_A = \rho_{oil} \times g \times h_{oil}$

$P_A = 0.9 \times 1000 \times 9.81 \times 1m$

$= \underline{\underline{8829 \, N/m^2}}$

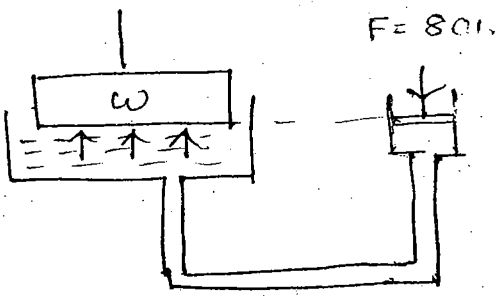
$P_B = \rho_{oil} \times g \cdot h_{oil} + \rho_w \cdot g \cdot h_w$

$= 88.29 + 1000 \times 9.81 \times 2$

$= \underline{\underline{28449 \, N/m^2}}$

(2.7) $d = 3cm$

$D = 10cm$



Pistons at same level.

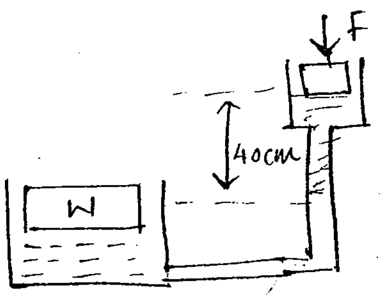
$P_{SMALL} = P_{LARGE}$

$$\frac{80}{\cancel{\frac{\pi}{4}} \times (0.03)^2} = \frac{W}{\cancel{\frac{\pi}{4}} \times (0.1)^2}$$

$W = \underline{\underline{888.89 \, N}}$

Case 2:

Small piston 40 cm above large piston



$P_{Large} = P_{small} + P_{due \ to \ 40 \ cm}$

$$\frac{W}{\pi/4 \times (0.1)^2} = \frac{80}{\frac{\pi}{4} \times (0.03)^2} + \rho_w \cdot h_w \cdot g$$

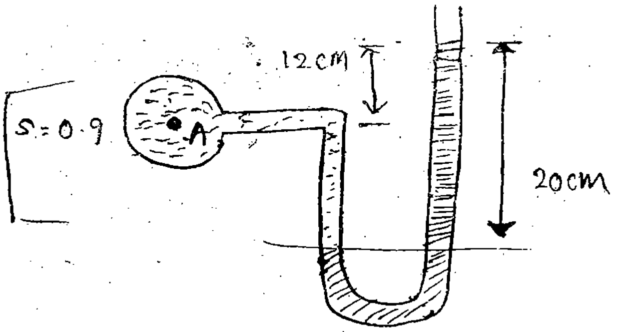
$$\frac{80}{\pi/4 \times 0.1^2} + 1000 \times 0.4 \times 9.81$$

Text

P 43

2.9

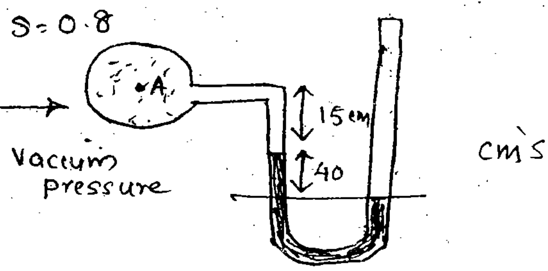
$$P_A = \rho_m \cdot g \cdot h_m - \rho_{oil}\, g \cdot h_{oil}$$



$$13600 \times 9.81 \times 0.2 -$$
$$900 \times 9.81 \times 0.18$$
$$\underline{25093.98 \ N/m^2}$$

2.10

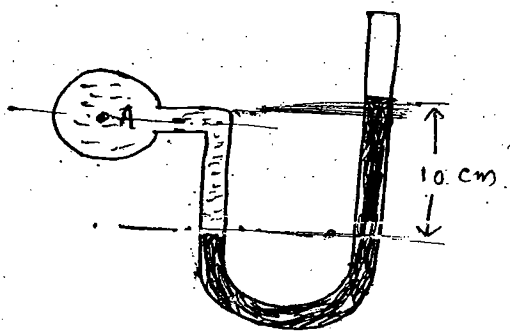
Case of vaccum pressure PIPE →



$$P_A + 13600 \times 9.81 \times 0.4 +$$
$$800 \times 9.81 \times 0.15 = 0$$
$$P_A = -[54543.6]\ N/m^2$$
$$\underline{-\ 5.45\ N/cm^2}$$

2.11

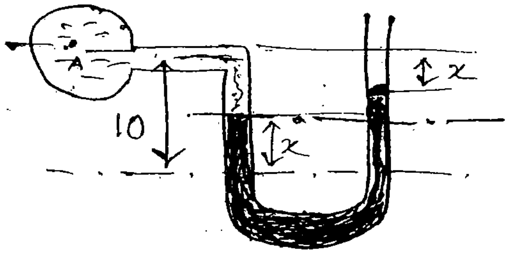
Case 1:



$$P_A + 1000 \times 9.81 \times 0.10$$
$$= 13600 \times 9.81 \times 0.10$$
$$\underline{P_A = 12360.6\ N/m^2}$$

Case 2:

$$P_A = 9810\ N/m^2$$



$$9810 + 1000 \times 9.81 \times \left(\frac{10-x}{100}\right)$$

~~1000 x 9.81 x x~~

$$= 13600 \times 9.81 \times \left(\frac{10-2x}{100}\right)$$

$$9810 + 9810\,(10-x) = \cancel{13600}\ 133416\,[10-2x]$$

$$-257022x = 981000 + 98100 - 1334160$$

$$-257022x = -255060$$

$$x = 0.99 \simeq \underline{1m}$$

Diff in mercury level $= 10 - 2x$

$$= 10 - 2 \times 0.99$$

$$= \underline{8.02m}$$

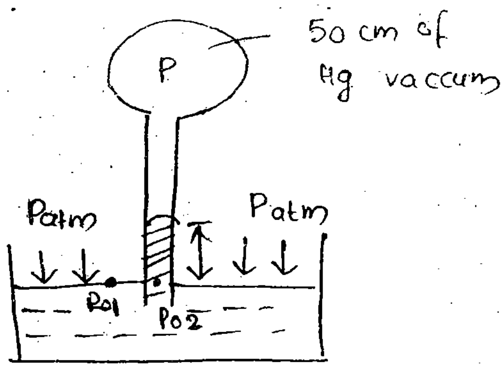
Q) Figure the pressure Reading Find H

a) 50 cm

b) 76 cm

c) 126 cm

d) None of these



$$(P_{o1})_{gauge} = (P_{o2})_{gauge}$$

$$0 = \rho_{Hg} \cdot g \cdot H + P$$

$$= \rho_{Hg} \cdot g \cdot H + \rho_{Hg} \cdot g \cdot 50 \text{ cm of vaccum}$$

$$0 = \rho_{Hg} \cdot g \, [H - 50]$$

$$H - 50 = 0$$

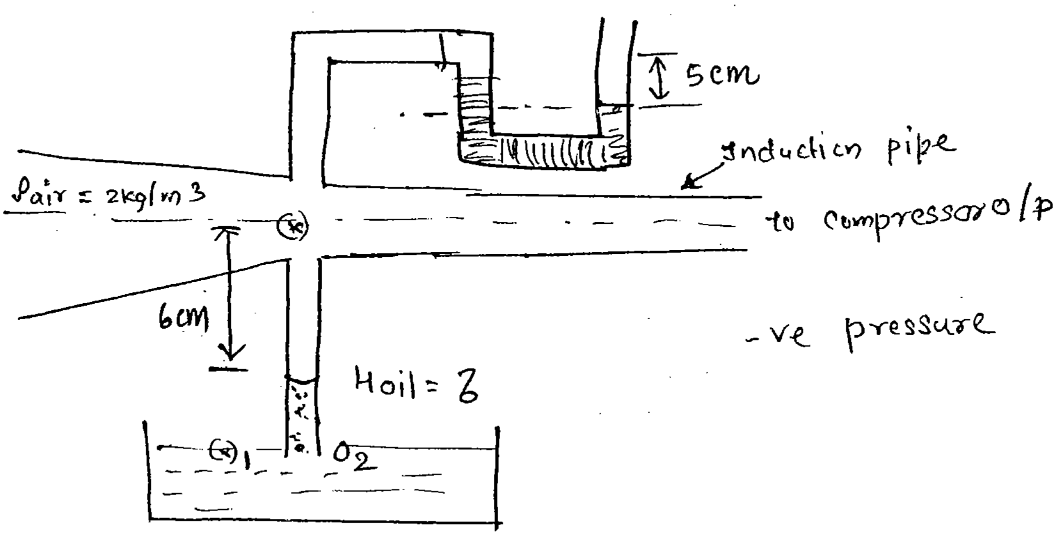
$$H = 50 \text{ cm}$$

Q) An Induction pipe to a Compressor (vaccum cleaner is connected) to a pressure gauge show in fig Determine the height of oil Rised

from atmos →

$S_{Hg} = 13.6$

$H_{Hg} = 5$ cm

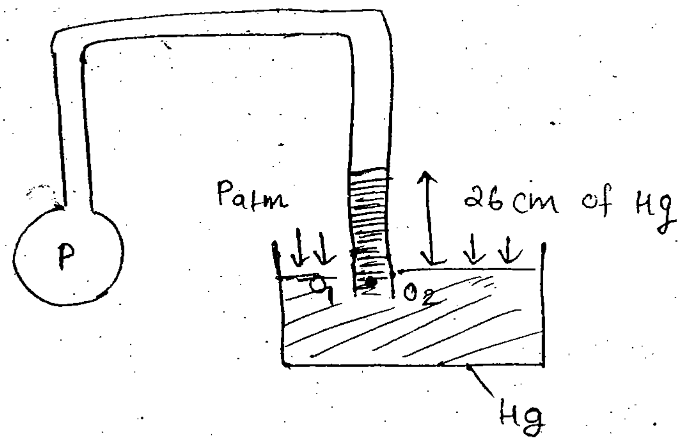


$$(P_{o1})_{gauge} = (P_{o2})_{gauge}$$

$$0 = \rho_{oil} \times g \times h_{oil} + \rho_{air} \cdot g \cdot H_{air} + P$$

$H_{oil} = 0.835$ m of oil

Q.)



a) 26 cm of Hg (abs)

b) 50 cm of Hg (abs)

c) 76 " "

d) 102 " "

What is Pressure at P ?

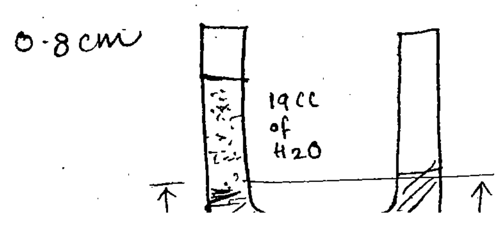
$(P_{O1})_{gauge} = (P_{O2})_{gauge}$

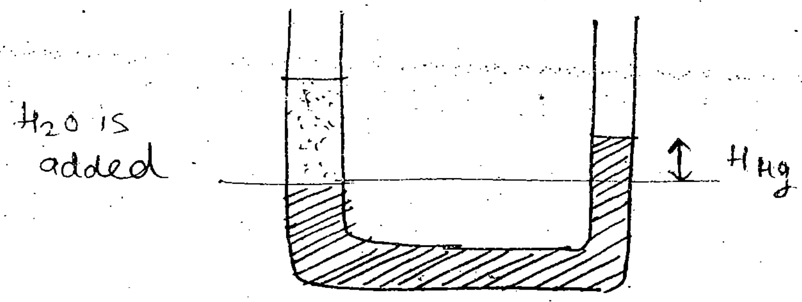
$$0 = \underbrace{\rho_{Hg} \cdot g \cdot h_{Hg}}_{26 \text{ cm of Hg}} + \text{Gas coloum neglected} + P_P$$

$$P_P = -26 \text{ cm of Hg}$$

$$(P_P)_{abs} = -26 + 76 \text{ cm of Hg}$$
$$= 50 \text{ cm of Hg}$$

Q) A U bend tube Both ends are open and filled with Hg and observed 10 cm above Horizontal plane. The inner dia of glass Tube is 8 mm, if 19 cc of H_2O is added to one side of U bend then what is the diff in Hg level ~~of~~ at eqbm ?





$$\rho_w \times \cancel{g} \times h_w = \rho_{Hg} \cdot \cancel{g} \cdot H_{Hg}$$

$$H_{Hg} = \frac{\rho_w}{\rho_{Hg}} \times h_w$$

$$= 1000 \times$$

$\underline{h_w = ?}$

$$V = A \times h$$

$$19\ cm^3 = \frac{\pi}{4} \times d^2 \times h$$

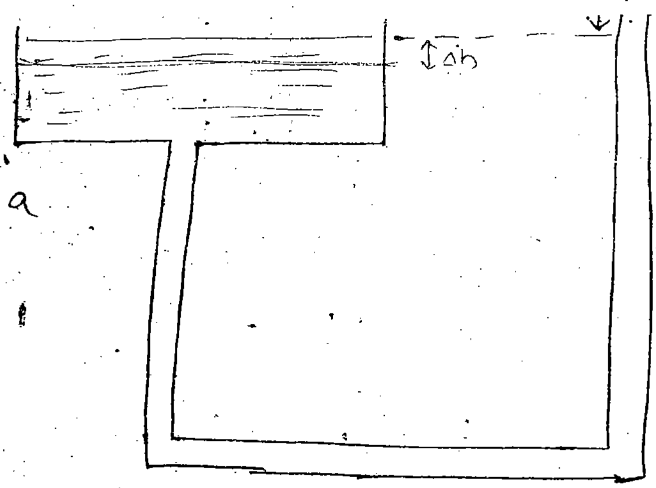
$$19 = \frac{\pi}{4} \times (0.8)^2 \times h$$

$$\underline{h = 37.79\ cm}$$

$$H_{Hg} = \frac{1000}{13600} \times 37.79$$

$$\underline{= 2.77\ cm\ of\ Hg}$$

Q.) The c/s Area of manometer shown in figure made 500 times larger area than narrow tube Then reading in figure Det % error in measurement.



A = 500a

Law of conservation of mass

mass remains const.

$$M = m$$

$$\cancel{\rho} \times V = \cancel{\rho} \times v$$

$$A \times \Delta h = a \times H$$

$$a\,500 \times \Delta h = a \times H$$

$$500 \times \Delta h = H$$

$$\frac{\Delta h}{H} = \frac{1}{500} = 2 \times 10^{-3}$$

$$= \boxed{0.2\%}$$

Mass Risen in tube = Mass fallen in Tank when Area is increased

Single Coloumn Manometer

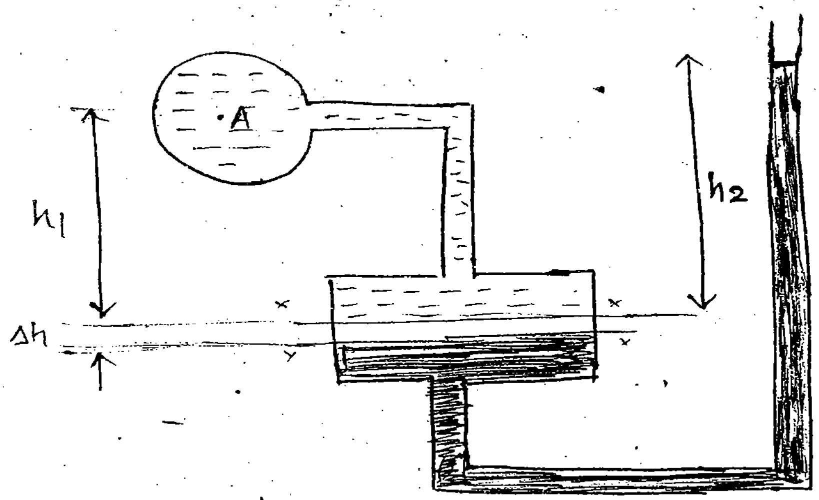
- Modified version of u tube Manometer
- large c/s Area (100 times) the c/s of tube
  Reservior

Reserviour having large area

- Types

1. vertical single coloumn Manometer
2. Inclined single coloumn Manometer

VERTICAL SINGLE COLOUMN MANOMETER



Fall of Heavy liquid in the Reserviour = Rise of Light liquid in the Tube

$$A \times \Delta h = a \times h_2$$

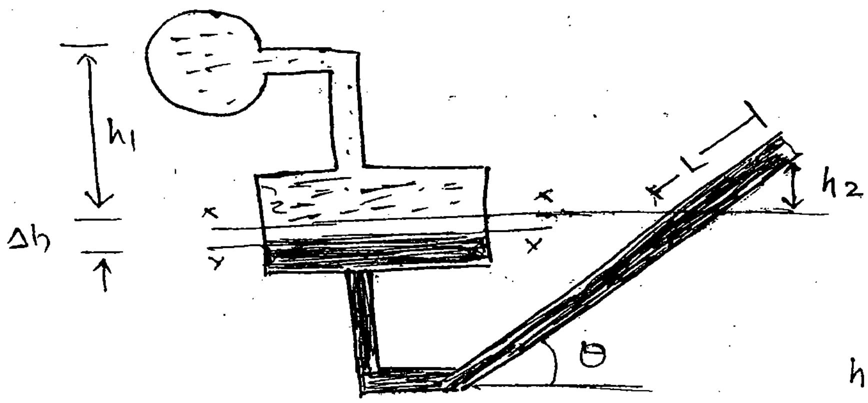
$$\boxed{\Delta h = \frac{a \times h_2}{A}}$$

$$\boxed{P_A = \Delta h \cdot [\rho_2 g - \rho_1 g] + \rho_2 g h_2 - \rho_1 g h_1}$$

If A is very Large compared to a [$\frac{a}{A}$ can be neglected]

then $\boxed{P_A = \rho_2 g h_2 - \rho_1 g h_1}$

Inclined Single coloumn Manometer



$h_2 = L \sin\theta$

$$P_A = h_2 \rho_2 g - h_1 \rho_1 g$$

$$\boxed{P_A = L\sin\theta \, \rho_2 g - h_1 \rho_1 g}$$

- Forces involved in fluid motion
- Assumptions in fluid dynamics
- Fluid motion equations
- Derivation of Euler's eqn of motion
- Statement and eqn of energy for ~~ft~~ a fluid under motion (Bernoulli's eqn)
- Applications of Bernoulli's eqn
- 5 models
- Momentum theorem (Impulse - Momentum eqn) and its application (4 models).

Fluid dynamics is a Branch of fluid science which deals motion of the fluid under influence of external forces. Fluid dynamics focus on flow analysis through pipes, design of turbines, pumps and forces involved in fluid motion.

Diff forces on fluid dynamics

The following are the three forces involved in fluid dynamics.

1. Body force (Self wt of fluid matter)
2. Normal forces (due to pressure of the fluid)
3. External forces acting on the fluid matter

4.

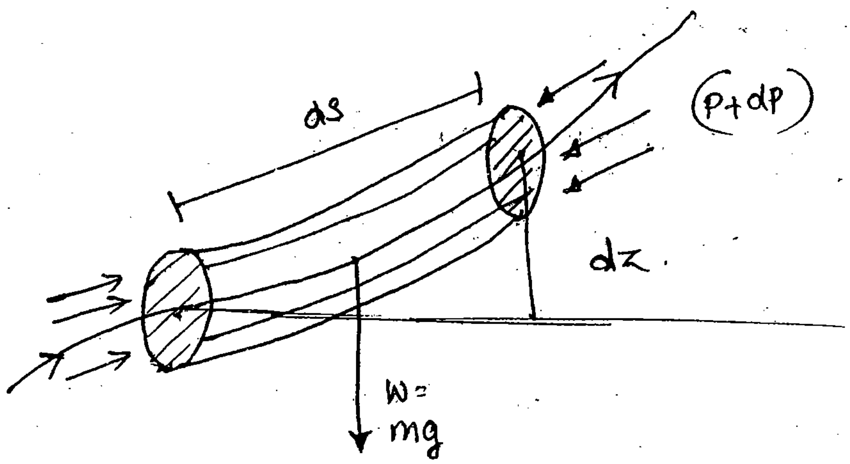
Equation of motion.

1. Euler's eqn Based on law of conservation of mass
2. Energy eqn (Bernoulis eqn)
3. Momentum eqn

Assumptions:-

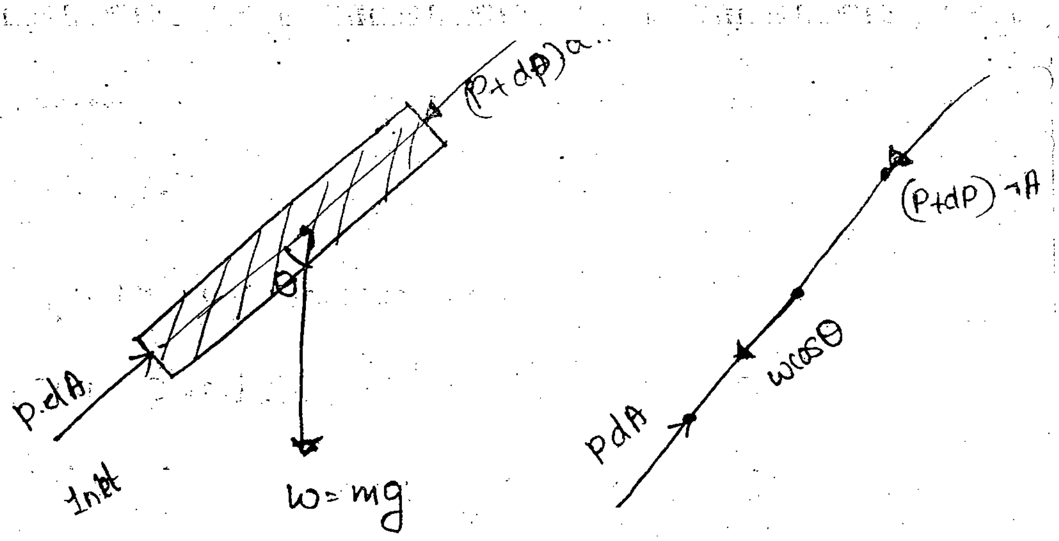
1. fluid flow is steady
2. fluid flow is non uniform
3. fluid is non-viscous
4. flow is through streamlines
5. flow is continous
6. forces ~~like~~ due to surface Tension, Bulkmodulus (due to Compressibility) due to turbulency and influence of magnetism etc are ignored.

Expression for Euler's motion eqn



Consider a fluid element of length ds and normal Area dA (one dimensional flow).

In Actual process decrease in pressure occurs.

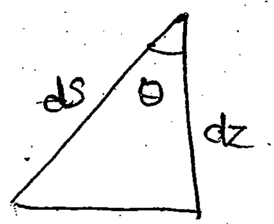


ΣForces = mass × Accln

$$P.dA - W\cos\theta - (p+dp)dA = m \times \frac{dv}{dt}$$

$$\cancel{P dA} - W\cos\theta - \cancel{P dA} - dp\,dA = m \times \frac{dv}{ds} \times \frac{ds}{dt}$$

$$mg\cos\theta - dP\,dA = m \cdot \frac{dv}{ds} \cdot V$$



$$mg \cdot \frac{dz}{ds} - dP \cdot dA = \rho \cdot dA\,ds \cdot \frac{dv}{ds} \cdot V$$

$$\rho \cdot \cancel{dA}\,\cancel{ds} \cdot \frac{dz}{\cancel{ds}} \cdot g - dP\,\cancel{dA} = \rho\,\cancel{dA}\,\cancel{ds} \cdot \frac{dv}{\cancel{ds}} \cdot V$$

$$-\rho g\,dz - dP = \rho\, V \cdot dV$$

$$dP + \rho g\,dz + \rho\, V dV = 0$$

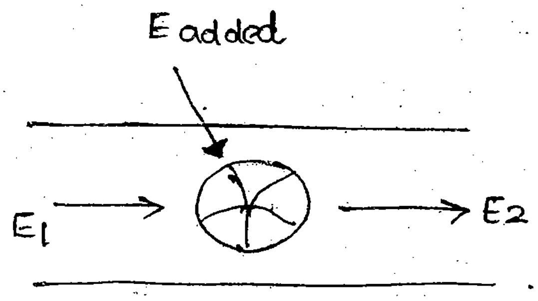
Divide by ρ

$$\boxed{\frac{dP}{\rho} + g\,dz + V dV = 0}$$ EULER'S EQN

Apply Bernoulis equation

$$\frac{P_1}{\rho g} + z_1 + \frac{v_1^2}{2g} - h_L + \underbrace{E_{added}}_{\text{Head Rise}} = \frac{P_2}{\rho g} + z_2 + \frac{v_2^2}{2g}$$

- Turbine :



(J) ← E

$$E_1 + E_{added} = E_2$$

E added
kwhr

$Q = 50\ L/s$

$P = 7.5\ kW$

$H = ?$

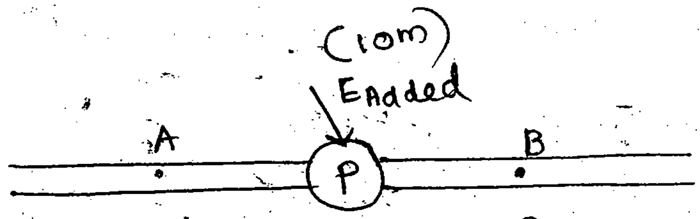
$1m^3 = 1000\ l$

$$\eta = \frac{O/P}{I/P} = \frac{\dot{m} g H}{P_{shaft}}$$

$$100 = \frac{\rho \cdot Q \cdot g \cdot H}{P_{shaft}} \cdot \frac{1000 \times 50 \times 10^{-3} (m^3/s) \times}{7.5 \times 10^3}$$

$$H = \underline{15.32\ m}$$

Q7.)



$h_L = 3.0$

$P_A = ?$

$P_B = 120\ kPa$

$H = 10m$

$V_A = V_B$

$$\cancel{\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B}$$

$$\frac{P_A}{\rho g} + \cancel{\frac{V_A^2}{2g}} + \cancel{Z_A} - h_L + E_{added} = \frac{P_B}{\rho g} + \cancel{\frac{V_B^2}{2g}} + \cancel{Z_B}$$

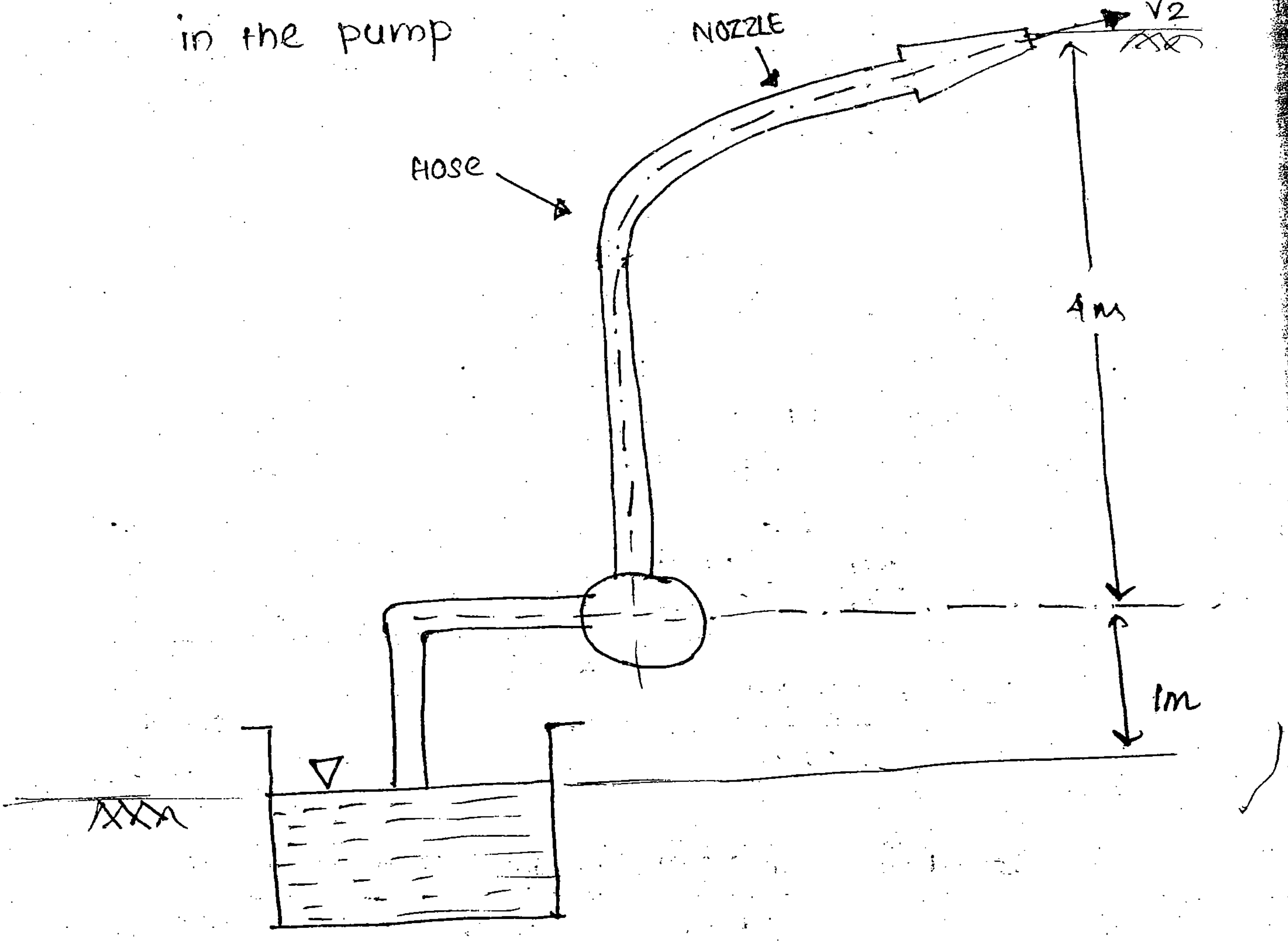
$$\frac{P_A}{\rho g} + (-3) + 10 = \frac{120 \times 10^3}{\rho g}$$

$$\frac{P_A}{\rho g} + 7 = \frac{120 \times 10^3}{1000 \times 9.81}$$

$$\frac{P_A}{\rho g} = \frac{120 \times 10^3}{1000 \times 9.81} - 7$$

(b) Power Reqd to Run the pump.

Assume there are no losses in the pipeline and in the pump



$P_2 = P_{atm}$

$P_{gauge} = 0$

$V_2 = ?$

$z_2 = 5m$

$z_1 = 0$

$P_1 = 0$

$\frac{V_1^2}{2g} = 0$

$$0 + 0 + 0 - 0 + E_{added} = 0 + 5 + \frac{V_2^2}{2g}$$

$Q = A_2 V_2$

$$3000 \frac{\cancel{cm^3}}{mt} = \frac{\pi}{4} \times (50 \times 10^{-3})^2 \cdot V_2$$

$$\frac{(m^3)}{sec} \quad \frac{3000 \times 10^{-3}}{60} = \frac{\pi}{4} \times (50 \times 10^{-3})^2 \times V_2$$

$$E_{added} = \frac{(25.46)^2}{2 \times 10}$$

H developed by pump $= \underline{37.42 m}$ of water

(b) $P = ?$

$$\cancel{\eta = \frac{O/P}{I/P} = \frac{\dot{m} g H}{P_{shaft}}}$$

$$\boxed{\text{Power} = \dot{m} g H}$$

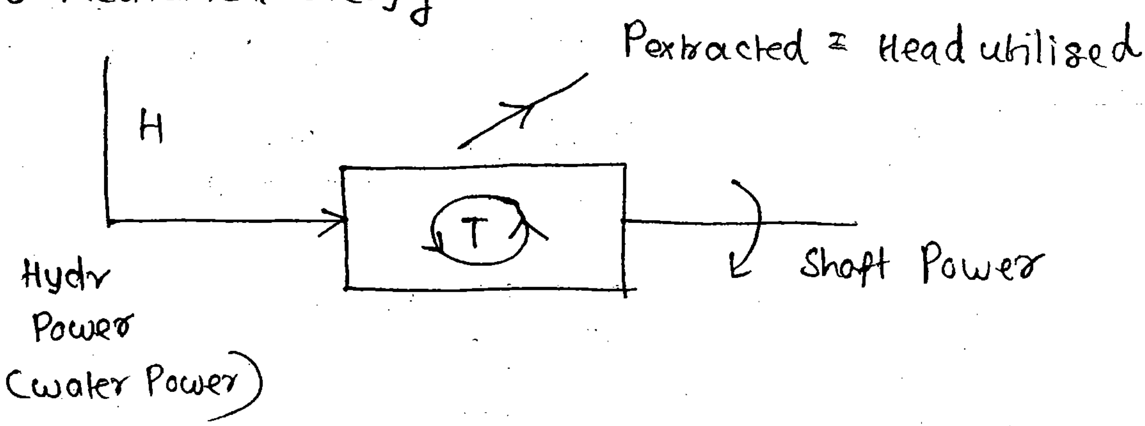
$$= \rho Q g H$$

$$= 1000 \times 10 \times \left(\frac{3300 \times 10^{-3}}{60}\right) \times 37.42$$

$$= \quad kW$$

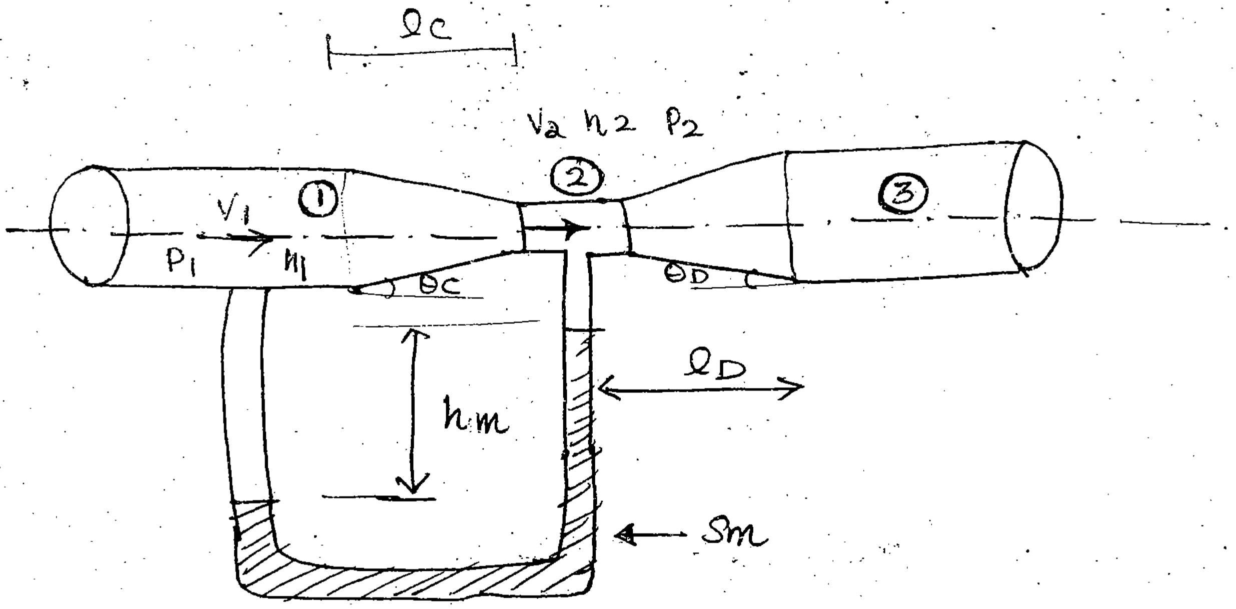
TURBINE.

It is a machine which converts Hydraulic energy into mechanical energy



$$\eta = \frac{O/P}{I/P} = \frac{P_{shaft}}{P_{water}} = \frac{P_{shaft}}{\dot{m} g \cdot H}$$

$$\boxed{\eta = \frac{P_{shaft}}{\rho \cdot g \cdot Q \cdot H}}$$



1. $l_D \gg l_c \rightarrow$ To avoid flow Seperation

2. $\theta_D < \theta_c$

3. $\dfrac{d_{throat}}{d_{pipe}} = \dfrac{1}{4}$ to $\dfrac{1}{2}$

Eqns

1. $Q = A_1V_1 = A_2V_2$

2. Apply Bernoulis eqn :

$$\frac{P_1}{\rho g} + \cancel{Z_1} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \cancel{Z_2} + \frac{V_2^2}{2g}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$h_1 - h_2 = \frac{V_2^2 - V_1^2}{2g}$$

$$\Delta h_{12} = h = \left[\frac{Q}{A_2} - \frac{Q}{A_1}\right]^2$$

$$Q = \frac{\sqrt{2gh}}{\sqrt{\frac{1}{A_2^2} - \frac{1}{A_1^2}}}$$

$$\boxed{Q_{th} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2gh}}$$

$$\boxed{Q_{act} = C_d \times Q_{th}}$$ [$C_d = 0.96$ to 0.99]

NOTE:- Differencial manometer connected to venturimeter. The deflection of manometer will not change by changing orientation of pipe bcoz Diff manometer always measures pressure b/w 2 points

Ventruimeter is Best

disadv

Consume space

(15) ~~$A_2 = 3cm^2$~~

~~$A_1 = 5cm^2$~~

~~h = 5cm of water~~

(*) $$\boxed{h_{fluid} = h_m \left[\frac{S_m}{S_{fluid}} - 1\right]}$$

(16) Deflection remains same immaterial of angle of inclinations

h = 25cm

(15) $A_2 = 3cm^2$

$A_1 = 5cm^2$

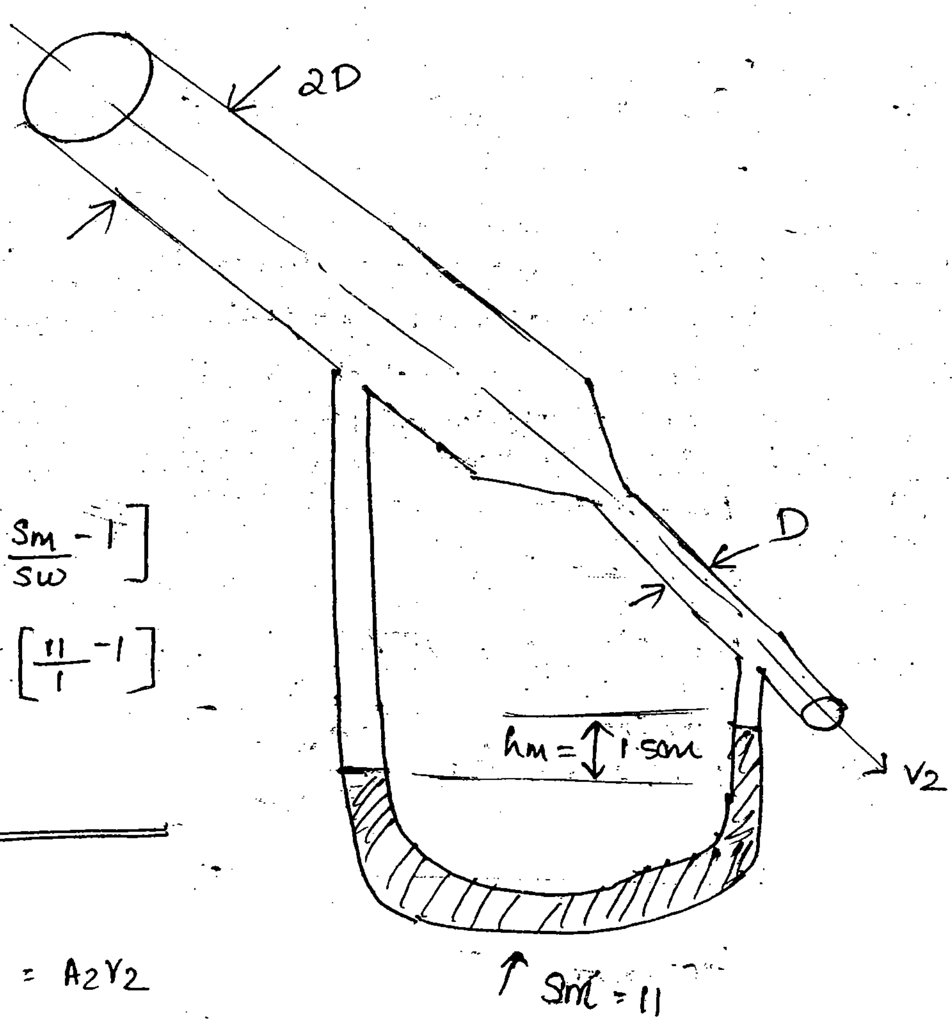
$h = 5cm$ of H_2O

$$Q_{water} = \frac{A_1 A_2}{\sqrt{}} \times \sqrt{2 g h_{water}}$$

$$\frac{5\times 3}{\sqrt{5^2-3^2}} \times \sqrt{2\times 981 \frac{cm}{s^2} \times 5\,cm}$$

$$= \underline{\underline{370\,cm^3/sec}}$$

2) Determine the velocity of water at section ① shown in figure



ANS:-

$$h_w = h_m\left[\frac{S_M}{S_W}-1\right]$$

$$h_w = 1.5\times10^{-2}\left[\frac{11}{1}-1\right]$$

$$= (m)$$

$$Q_{water} = A_1V_1 = A_2V_2$$

$$Q = \frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2gh_{water}}$$

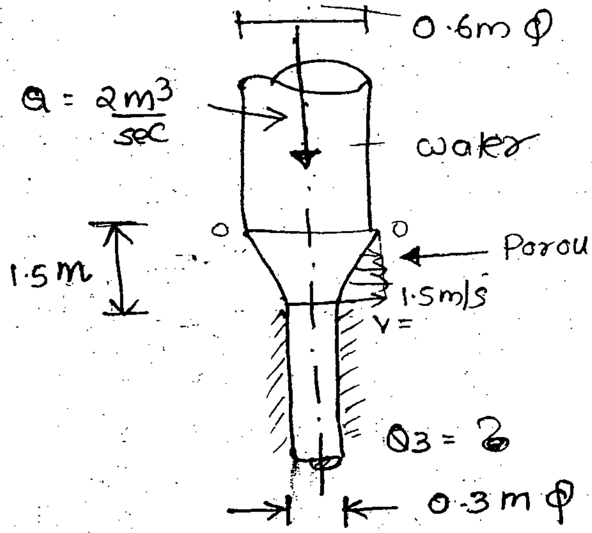
$$\cancel{A_1}V_1 = \frac{\cancel{A_1}A_2\sqrt{2gh_w}}{\sqrt{A_1^2-A_2^2}}$$

$$V_1 = \frac{\frac{\pi}{4}\times D^2}{\sqrt{\frac{\pi}{4}\times(2D)^2-\frac{\pi}{4}\times D^2}} \times \sqrt{2\times 10\frac{m}{sec^2}\times1.5\times10^{-2}\times\left(\frac{11}{1}-1\right)}$$

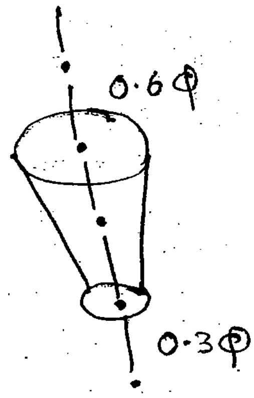
$$\frac{D^2}{\sqrt{16D^4 - D^4}} \times \sqrt{\left(2 \times 10 \times \frac{1.5}{100} \times 10\right)}$$

$$= \underline{\sqrt{0.2} \quad m/s}$$

Q.) Find discharge through a pipe at ③



0.587 1



→ Velocity of fluid through closed pipes

ENERGY EQUATION (For fluid in motion)

OR

BERNOULIS EQUATION

It is Based on law of conservation of energy. ie energy is neither created nor destroyed.

For the Above Assumptions total energy of fluid at any point in a Continous matter remains constant. It is obtained by Integrating the Euler's eqn

$$\int \frac{dp}{\rho} + \int g\,dz + \int v \cdot dv = \int 0$$

For Incompressible flow

$$\rho_1 = \rho_2 = const$$

ENERGY/ UNIT MASS

$$\boxed{\frac{P}{\rho} + gz + \frac{v^2}{2} = constant}$$

$$\frac{N}{m^2} \cdot \frac{m^3}{kg} \qquad \frac{m \times m}{s^2} \qquad \left(\frac{m}{sec}\right)^2$$

$$\frac{\cancel{kg}\,m}{\cancel{m^2}\,s^2} \cdot \frac{m^{\cancel{3}}}{\cancel{kg}} \qquad \frac{m^2}{s^2}$$

$$= \frac{Nm}{kg} \qquad \frac{\cancel{kg}\,m\,m}{\cancel{kg}\,s^2}$$

$$\frac{\text{Energy}}{\text{unit Mass}} \ (N\,m)$$

$$\frac{kg\,\frac{m}{sec^2} \cdot m}{\text{Mass}} = \frac{m^2}{sec^2}$$

$$\boxed{P + \rho g z + \frac{\rho v^2}{2} = C}$$ ENERGY / UNIT VOLUME.

$\left[\frac{N}{m^2}\right]$ $\left[\frac{N}{m^2}\right]$ Dynamic pressure

$$= \frac{\rho v^2}{2} = \frac{kg}{m^3} \times \frac{m^2}{s^2}$$

$$= \frac{kg \; \frac{m}{s^2} \cdot m}{m^3} = \frac{Nm}{m^3}$$

$$= \frac{\text{Joule}}{\text{Volume}}$$

$\div \rho g$

Pressure Head → datum Head

$$\frac{P}{\rho g} + Z + \frac{V^2}{2g} = C$$

m

velocity Head

$$\frac{N}{m^2} \times \frac{m^3}{kg} \times \frac{s^2}{m}$$

$$\frac{\cancel{kg}\,\cancel{m}}{\cancel{m^2}\,\cancel{s^2}} \times \frac{\cancel{m^3}}{\cancel{kg}} \times \frac{\cancel{s^2}}{\cancel{m}}$$

$$\frac{m^2}{\cancel{s^2}} \times \frac{\cancel{s^2}}{m}$$

$$= \underline{\underline{m}}$$

$$\boxed{\frac{P}{\rho g} + Z + \frac{V^2}{2g} = C}$$ ENERGY / UNIT WEIGHT.

$$\frac{\text{Energy}}{\text{weight}} = \frac{Nm}{\cancel{kg}\,N} =$$

Applications of Bernoulis eqn

1. Energy of fluid at a point

2. Identify the direction of fluid flow.

3. Bernoulis eqn ~~app~~lied to Hydraulic Machines. Pumps and Turbines

4. Measurement of discharge [venturimeter, orifice meter nozzlemeter, Bendmeter, Rotameter]

5. vel. of the flow ~~flow~~.

   [pitot tube]

ex 1

MODEL 1 / MODEL 2

A tapered pipe of dia at section 1, 300mm reduces to 100mm at section 2. the elevation [datum] of section 1 is 18m and the section 2 is 12m. The gauge pressure at section 1 is 120kPa and at section 2 is 180kPa. The fluid flow is water and Rate of flow is 210 lit/min. Take $g = 10\,m/s^2$. Assume flow is steady, one dimensional and determine the following

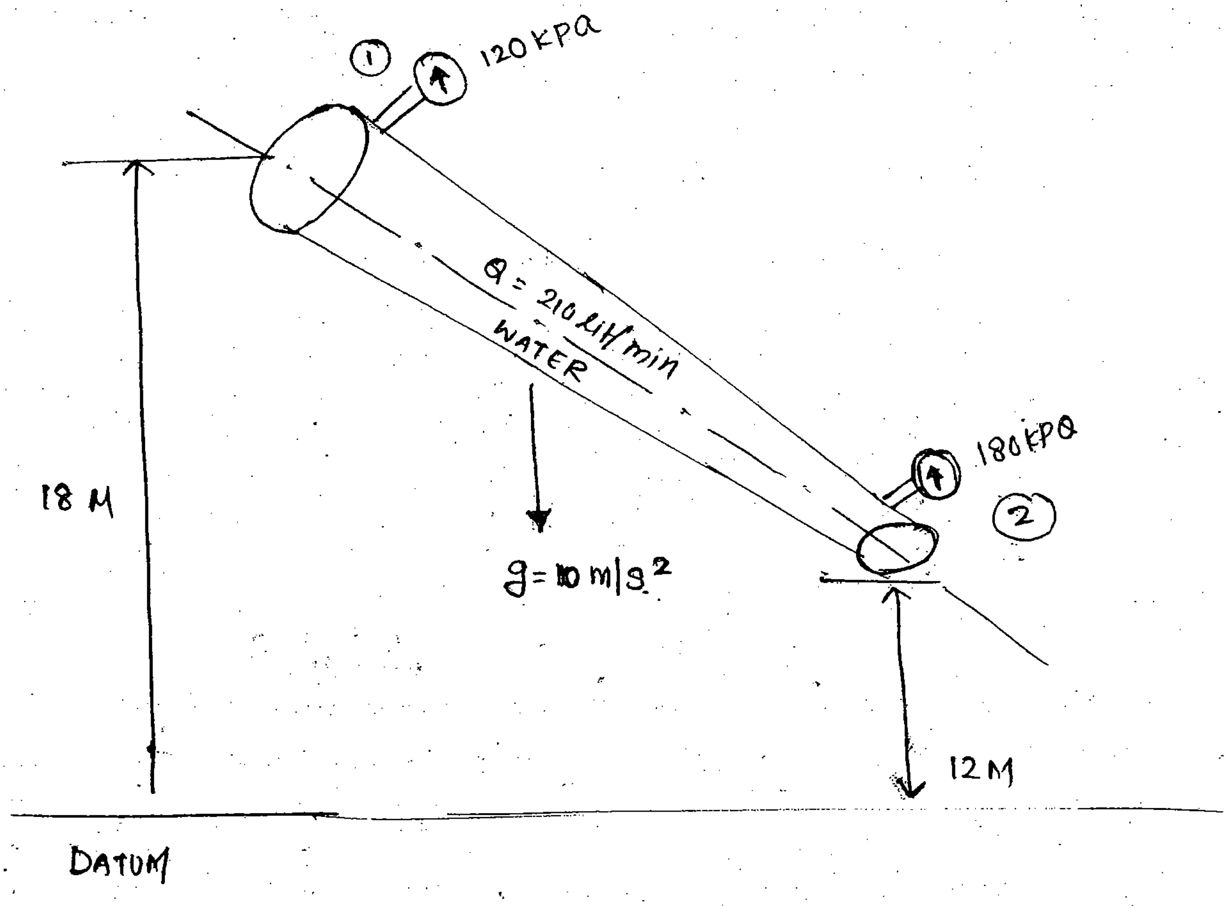
1. Volume flow Rate of the water in $\frac{m^3}{s}$ (discharge)

2. mass flow Rate of water in $\frac{kg}{s}$.

3. vel of water flow at section 1 and at section 2

4. total energy Head of the water at section 1 and at section 2.

⑦ Loss of power during the flow in kW and in H.P



[Fluid Flow due to energy diff]

$Q = \dot{V} = 210 \text{ LPM}$

$= \dfrac{210 \text{ lit}}{60 \text{ sec}} = \dfrac{3.5 \text{ lt}}{\text{sec}}$

$= 3.5 \times 10^{-3} \text{ M}^3/\text{sec}$

$= 0.0035 \text{ cumec}$

$\dot{m} = \rho A V = \rho \times Q$

$= 1000 \times 0.0035$

$= \dfrac{\text{kg}}{\cancel{\text{M}^3}} \times \dfrac{\cancel{\text{M}^3}}{\text{sec}}$

$= 3.5 \dfrac{\text{kg}}{\text{sec}}$

$$\underset{m^2}{(1\times10^{-4})} \times \underset{m/s}{(10\times10^{-2})} = \underset{m2}{(1\times10^{-6})}\ \underset{m/s}{(V_2)}$$

$$V_2 = \underline{\underline{10m/s}}$$

$V_1 = 0.1\ m/s$

~~$V_2 = 40m/s$~~

M.A =

$$\underline{\underline{V_2 = 10m/s}}$$

Apply Bernoulis eqn

GAUGE

GAUGE PRESSURE

$$\frac{P_1}{\rho g} + \cancel{Z_1} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \cancel{Z_2} + \frac{V_2^2}{2g}$$

$\underline{\underline{P_2 = 0}}$

$$\frac{P_1}{\rho g} + \frac{0.1^2}{2g} = 0 + \frac{10^2}{2g}$$

$$\frac{P_1}{\rho \cancel{g}} = \left[\frac{10^2}{2} - \frac{0.1^2}{2}\right] \times \frac{1}{\cancel{g}}$$

$$\frac{P_1}{\rho} = 49.995$$

$$P_1 = 49.995 \times 1000$$

$$= 50{,}000$$

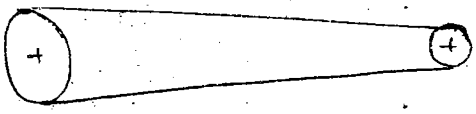
$$\underline{\underline{50\times10^3\ \frac{N}{m^2}}}$$

$$\underline{\underline{50kPa}}$$

$F = P \times Area$

$= 50 \times \cancel{10^3} \times 1\times10^{-4}$

$\underline{5 kN,} = \underline{\underline{5N}}$



5cm

$\frac{P}{\rho g} = 10\,m$

$\frac{V^2}{2g} = 1\,m$

Total Head = $\frac{P}{\rho g} + \frac{V^2}{2g} + 1$

$= 11\,m$

$z_1 = z_2 = 0$

$\boxed{\text{Ans: A}}$

27 Pressure at section 2 is negative

$$\frac{P_1}{\rho g} + \cancel{z_1} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \cancel{z_2} + \frac{V_2^2}{2g}$$

$$10 + 1 = \frac{P_2}{\rho g} + \frac{V^2}{2g} \qquad z_1 = z_2 = 0$$

$$A_1 V_1 = A_2 V_2$$

$$\cancel{\frac{\pi}{4} \times (0.05)^2 \times V_2 = \frac{\pi}{4} \times (}$$

$$\frac{\pi}{4} \times (0.1)^2 \times V_1 = \frac{\pi}{4} \times (0.05)^2 \times V_2$$

$$\left(\frac{0.1}{0.05}\right)^2 \times V_1 = V_2$$

$$4V_1 = V_2$$

$$(4V_1)^2 = (V_2)^2$$

$$16 V_1^2 = V_2^2$$

EQUATION OF CONTINUITY.

$\dot{m} = \text{constant}$

$\rho Q = \text{const}$

$\rho A V = \text{Const}$

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

For Incompressible fluids

$\rho_1 = \rho_2$

$Q = A_1 V_1 = A_2 V_2$

$$\frac{\pi}{4} \times (0.3)^2 V_1 = \frac{\pi}{4} \times (0.1)^2 \times V_2 = 0.0035$$

$V_1 = \underline{\underline{0.0495}}$ M/S $\approx \underline{0.05}$ M/S

$V_2 = \underline{\underline{0.45}}$ M/S

(4) Total energy at Section 1

$E_1 = H_1$ H = Head

$$= \frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} -$$

ALWAYS USE THIS EQN ONLY

$$\frac{120 \times 10^3}{1000 \times 10} + 18 + \frac{0.05^2}{2 \times 10}$$

$E_1 = \underline{\underline{30.000125}}$

(5) Energy at Point 2

$E_2 = H_2$

$$= \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$E_2 = \frac{180 \times 10^3}{} + 12 + 0.45^2$

ie $H_2 > H_1$

Hence flow from $2 \longrightarrow 1$. [Pump think]

⑥ Losses

$H_2 - h_L = H_1$

$30.010125 - h_L = 30.000125$

$h_L = H_2 - H_1$

or

$= E_2 - E_1$

$= \underline{0.01 \text{ m of } \cancel{Hg}\ H_2O}$

$= \frac{0.01}{13.6} \text{ m of Hg}$

$\rho_w \cdot \cancel{g} \cdot h_w = \rho_{Hg} \times \cancel{g} \cdot h_{Hg}$

$h_{Hg} = \frac{10\cancel{00} \times 0.01}{136\cancel{00}}$

$= \underline{\qquad}$

⑦ Loss of Power → Rate of energy

Power = Rate of fluid energy

$= \dot{m} g h_L$

$= 3.5 \times 10 \times 0.01$

$=$

$\underline{0.35 \text{ watt}}$

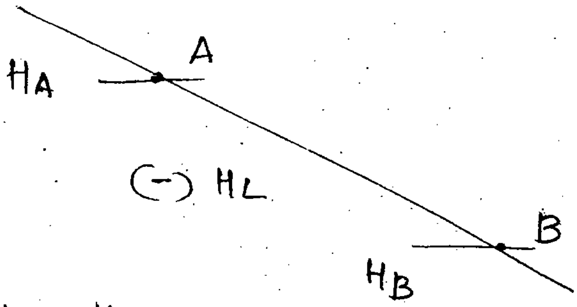
$\boxed{1 HP = 736 \text{ watt}}$ $= \frac{0.35}{\quad} = \underline{\qquad}$ HP

(1.) C

$P = \frac{N}{m^2}$ ~~kg~~ $E = \frac{Nm}{m^3} = \frac{N}{m^2}$

(2.) C

(3.)



If $H_A > H_B$

Flow is $H_A - H_L = H_B$ A — B flow occurs

$$\boxed{H_A = H_B + H_L}$$

If $H_B > H_A$

Flow is from B − A

$H_B - H_L = H_A$

$= \boxed{H_B = H_A + H_L}$

$\boxed{\text{Ans (b)}}$

(4.) GATE 2010

$D = 200mm$

S1

~~10m~~ 10 m

~~$P = 50kPa$~~

$P_1 = 50kPa$

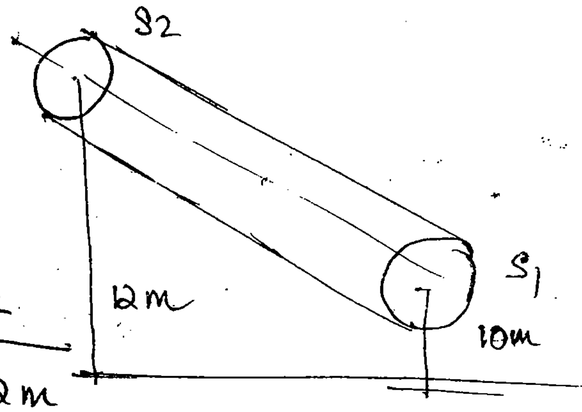
$V = 2m/s$

$a = 9.8 m/s^2$

2

12 m

20 kPa



$$= \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$V_1 = V_2$

$$= \frac{20\times10^3}{9.81\times1000} + 12 + \frac{V_2^2}{2\times9.81}$$

$V_2 = 2\,m/s$

$$= 14.2448$$

$E_1 = H_1$

$$= \frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g}$$

$V_1 = 2\,m/s$

$$= \frac{50\times10^3}{1000\times9.81} + 10 + \frac{V_1^2}{2\times9.81}$$

$$15.300$$

from S_1 to S_2 flow will occur

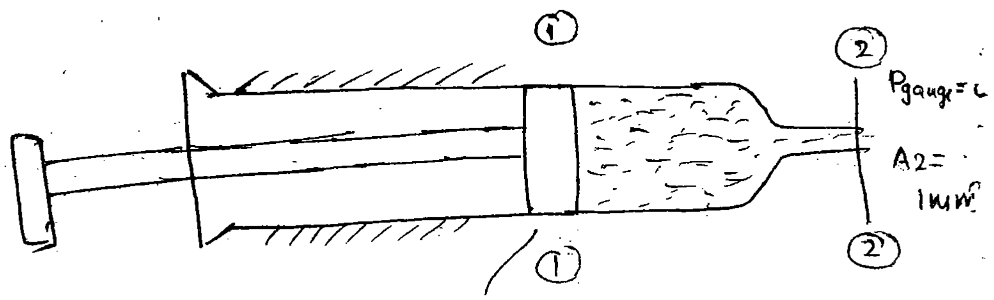
$H_1 - H_L = H_2$

$15.300 - H_L = 14.248$

$H_L = 1.06\,m$

$\boxed{\text{Ans (c)}}$

~~(4)~~ (5)



$A_1 = 1\,cm^2$

$V_1 = 10\,cm/s$

$A_1 V_1 = A_2 V_2$

cm/s

$1\times10 = \cancel{1\times10^{-2}\times V_2}$ $\quad V_2 =$

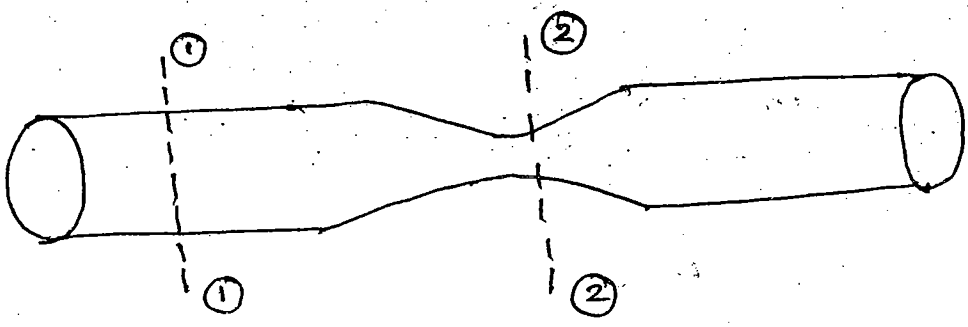
$$16\,\frac{V_1^2}{2g} = \frac{V_2^2}{2g}$$

$$16\times 1 = \frac{V_2^2}{2g}$$

$$10 + 1 = \frac{P_2}{\rho g} + 16$$

$$\frac{P_2}{\rho g} = \underline{-5 \text{ metres of water}}$$

Q.) At section 1 of a Horizontal pipe the fluid pressure head is 32 cm and velocity head is 4 cm. The reduction in Area at section 2 is such that the pressure head dropped down to zero value. What is the Ratio of velocity at section "2" to that at section 1 ?



$$\frac{P_1}{\rho g} = 32\,cm \qquad \frac{P_2}{\rho g} = 0$$

$$\frac{V_1^2}{2g} = 4\,cm \qquad \frac{V_2}{2g} = ?$$

Apply Bernoulis eqn

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$$32 + 0 + 4 = 0 + 0 + \frac{V_2^2}{2g}$$

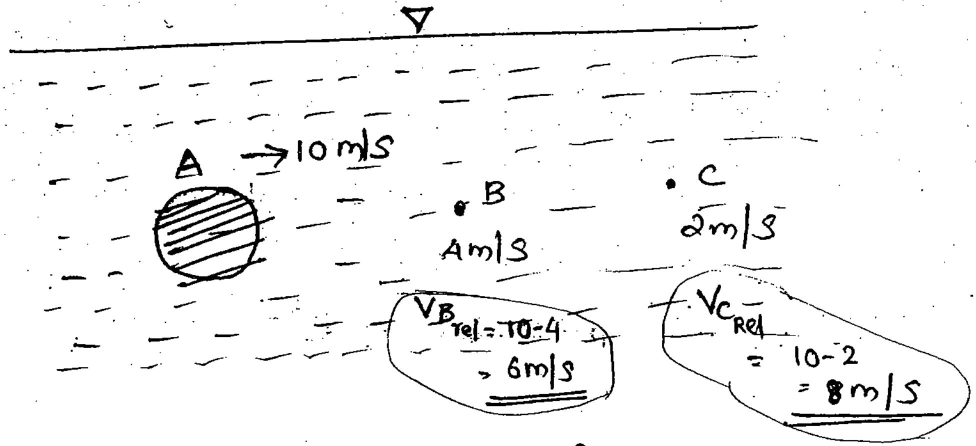
$$\overline{2g} \qquad \overline{V_1^2}$$

$$\frac{\frac{V_2^2}{2g} = 36}{\frac{V_1^2}{2g} = 4} \Rightarrow \longrightarrow \left[\frac{V_2}{V_1}\right]^2 = 9$$

$$\frac{V_2}{V_1} = 3$$

(4) Q.) In a static fluid a Body is thrown with const vel 10 m/s which gives velocity to the water at Point B 4 m/sec ; At point C 2 m/sec. Then what is the value of $P_B - P_C = ?$

a) 12 kPa b) 14 kPa c) 24 kPa d) 28 kPa



$$\cancel{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2}$$

$$\frac{P_B}{\rho g} + \cancel{Z_B} + \frac{V_B^2}{2g} = \frac{V_C^2}{2g} + \frac{P_C}{\rho g} + \cancel{\frac{V_B^2}{2g}}\ \cancel{Z_C}$$

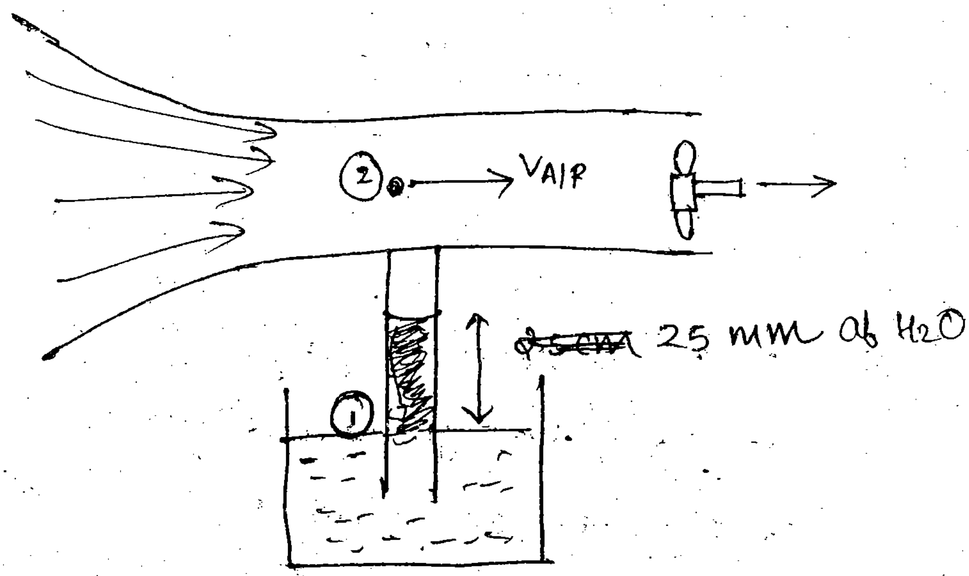
$$\frac{P_B - P_C}{\rho \cancel{g}} = \frac{V_C^2 - V_B^2}{2\cancel{g}}$$

$$P_B - P_C = \frac{\rho}{2} \times \left[V_C^2 - V_B^2\right]$$

$$= \frac{1000}{2}\left[8^2 - 6^2\right] = 500\left[64 - 36\right]$$

$14000 \frac{N}{m^2} = 14 \text{ kPa}$

(9)



$\rho_{AIR} = 1.2 \text{ kg/m}^3$

$$\frac{P}{\rho g} + Z + \frac{V_2^2}{2g} = \text{const}$$

$$P + \rho g Z + \frac{\rho V_2^2}{2} = \text{const}$$

$$\cancel{\frac{P}{\rho} + g Z + \frac{V^2}{2g} = \text{const}}$$

$$\frac{P}{\rho} + gZ + \frac{V_2^2}{2} = \text{const}$$

DYNAMIC HEAD $\quad \frac{\rho V^2}{2} = \frac{1.2 V^2}{2} =$

$$\frac{1.2 V^2}{2} = \rho_w \cdot H_w \cdot g$$

$$\frac{1.2 V^2}{2} = 1000 \times 9.81 \times 25 \times 10^{-3}$$

$$V = \underline{20.21 \text{ m/s}}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$0 + 0 + 0 = D + 25\text{ mm of water} + \frac{V_2^2}{2g}$$

At point
$P_2 = P_{atm}$
$= 0$ gauge pressure

$$-25\text{ mm of water} = \frac{V_{air}^2}{2g}$$

$$\rho_w \cdot g \cdot h_w = \rho_a \cdot g \cdot h_a$$

$$1000 \times 0.025 = 1.2 \times h_a$$

$$h_a = \frac{1000 \times 25 \times 10^{-3}}{1.2} = \underline{\underline{20.833}}$$

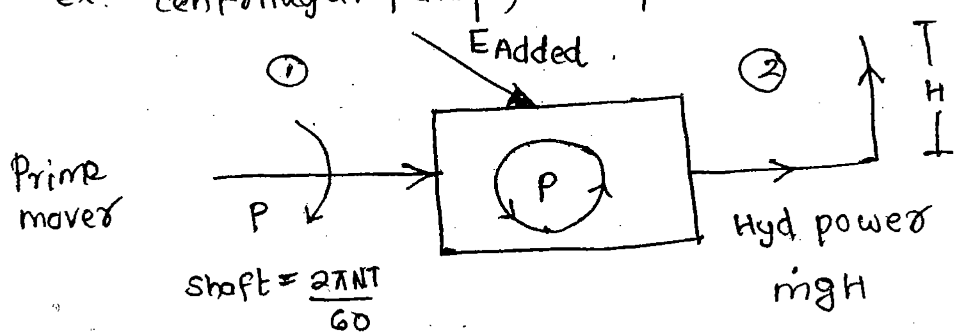
$$20.833 = \frac{V_{air}^2}{2g}$$

$$\underline{\underline{V_{air} = 20.21\text{ m/s}}}$$

Pump

Pump is a device which converts mechanical energy into hydraulic energy.

ex: centrifugal pump, reciprocating pumps.



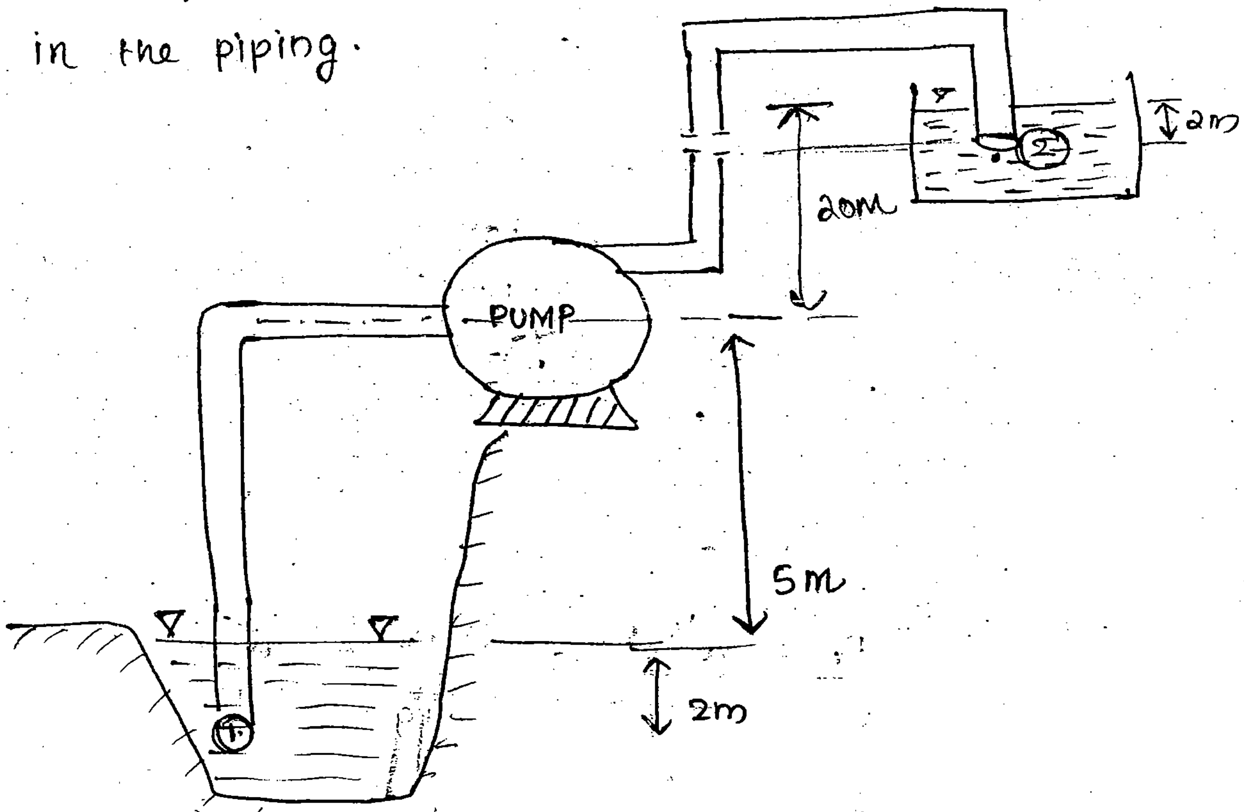
$$\eta = \frac{O/P}{I/P} = \frac{\text{Fluid Power}}{\text{shaft Power}} = \frac{\dot{m}gH}{P_{shaft}} = \frac{\rho \cdot g \cdot Q H}{P_{shaft}}$$

$P_A = 51.5 \times 10^3 \, N/m^2$

$P_A = 51.5 \, kPa$

e) A pump draws water from a reserviour and discharges it to an overhead tank as shown in figure. The Area of the outlet pipe is $20 \, cm^2$ & the average velocity in the outlet pipe is $3 \, m/s$. Neglecting the minor & major loses in the piping.

Ans:-



Q1. Total Head developed by the pump in metres of ~~...~~ water?

(a) 27 m (b) 25 m (c) 24 m (d) 22 m

Q2. If combined efficiency of the pump and motor is 75%. then power reqd to run the pump in kW is ~~1.76 kW~~

a) 1.76 kW b) 1.92 kW c) 2 kW d) 2.16 kW

$V_2 = 3 m/s$

So $A_2 V_2 = 20 \times 10^{-4} \times 3$

$= 60 \times 10^{-4}$

$= \cancel{0.6\ m^3/s}$ $\underline{60 \times 10^{-4}\ m^3/s}$

<u>continuity eqn</u>

$A_1 V_1 = A_2 V_2 = Q$

$A_1 V_1 = 60 \times 10^{-4}\ m^3/s$

$\therefore Q = \underline{60 \times 10^{-4}\ m^3/s}$

$= 6 \times 10^{-3}\ m^3/s$

<u>Mass flow Rate</u>

$= \underline{0.006\ m^3/s}$

$\dot{m} = \rho \cdot Q$

$\dot{m} = 1000 \times 0.006$

$= \underline{6 \frac{kg}{s}}$

Apply Bernoulis eqn

[Generall eqn applied to pump]

$$\left(\frac{P_1}{\rho g}\right)_{gauge} + z_1 + \frac{V_1^2}{2g} + E_{added} = \left(\frac{P_2}{\rho g}\right)_{gauge} + z_2 + \frac{V_2^2}{2g}$$

$P_1 = \rho \cdot g \cdot H_1$

$= \frac{P_1}{\rho g} = 2m$

At optimum
Head = 25m
So it should be
more than that
Since E is added

~~2~~ ~~~~

$$\cancel{2} + 0 + 0 + E_{added} = \cancel{2} + (7+18) + \frac{3^2}{2 \times 10}$$

$$E_{added} = 25 + \frac{3^2}{2 \times 10}$$

$$25 + \frac{9}{\ } = 25.45 m$$

b) ~~η = 100%~~ $\eta = 75\%$

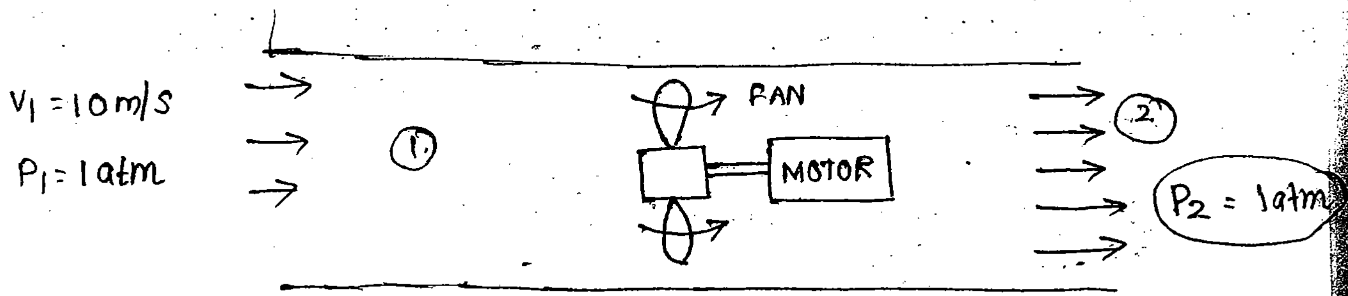
$P = ?$

$$\eta = \frac{\dot{m} g H}{P_{shaft}} = \frac{\rho \times Q \times g \times H}{P_{shaft}}$$

$$P_{shaft} = \frac{6 \times 10 \times \text{[illegible]} \times 27}{0.75}$$

$$= \underline{\underline{2.16 \text{ KW}}}$$

(Q.) Consider one dimensional incompressible flow of water thru a constant c/s Area 1 sqm pipe shown in fig. An electric motor imparts 1 MW power to the fluid via a ~~fan~~ fan. Neglect energy losses and fan doesn't produce any swirl of fluid flow.



1) what is the velocity V_2 in m/s ?

a) 10 m/s b) 17.32 m/s c) insufficent data d) More than 17.32 m/s.

2) what is the pressure P_2 ?

a) 1 atm b) 2 atm 3) 0.5 atm d) 3 atm

ρg $2g$ ρg $2g$

$$\frac{10^2}{2\times10} - 0 + (E_{added}) = \frac{V_2^2}{2g}$$

$$5 + E_{added} = \frac{V_2^2}{2g}$$

$$E_{added} = 1\,MW$$
$$= 1\times10^6\,W$$

$$\eta = \frac{o/p}{i/p} \quad \frac{(watt)}{(watt)}$$

$$100\% = \frac{\dot{m}gH}{1\,MW}$$

$A_1V_1 =$

$Q = A\cdot V$

$10\times1 = 10\,\frac{m^3}{s}$

$$1 = \frac{\rho Q g H}{1\,MW} = \frac{1000\times10\times(10)\times H}{1\times10^6}$$

$$\underline{\underline{H = 10\,m}}$$

$$5+10 = \frac{V^2}{2\times10}$$

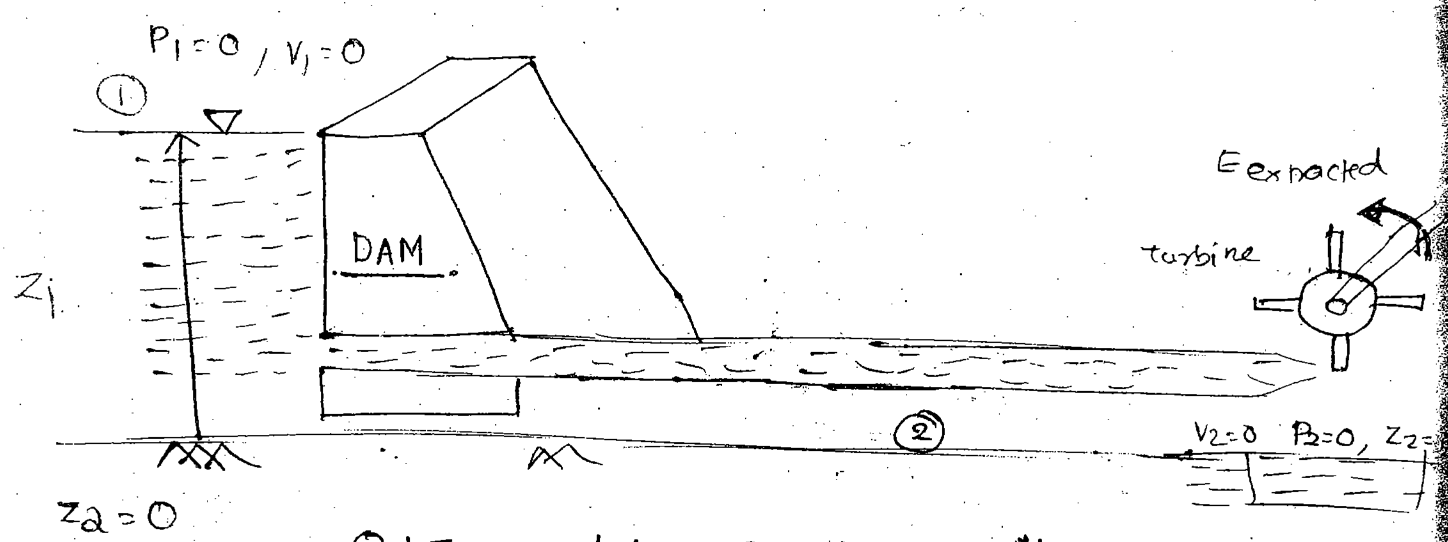
$$15\times20 = V^2$$

$$V = \sqrt{300} = \underline{\underline{17.32\,m}}$$

Q.) A fire fighting Hose with an exit nozzle dia = 50 mm discharges water at the Rate of 3000 lt/min. The height of the nozzle exit from the pump is 4m and water level in the sump is 1m below the pump shown in the fig. Calculate the head developed

Apply Bernoulli's eqn.

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g} + H_{extracted} \text{ or } (E_{extracted})$$



$$0 + z_1 + 0 - h_L = 0 + 0 + 0 + H_{extracted}$$

$$\boxed{z_1 - h_L = H_{extracted}}$$

Q10.

$Q = 1.2\ m^3/s$

$H = 120\ m$

$\eta = 90\%$

$$\eta = \frac{o/p}{i/p} = \frac{P_{shaft}}{\rho \cdot Q \cdot H g}$$

$$P_{shaft} = 0.9 \times 1000 \times 1.2 \times 120 \times (10) \to g$$

$$= 1271 \times 10^3 \text{ watt}$$

$$= 1271\ kW$$

8.

~~H = 50 m~~

$Q = 3\ m^3/s$

$h_L = 5\ m$

$P = 1000\ kW$

SPC

$$\eta_{TURBINE} = \frac{O/P}{I/P} = \frac{1000 \times 10^3 \text{ watt}}{1000 \times 9.81 \times 3 \times H_{extracted}}$$

$$1 = \frac{1 \times 106 \text{ watt}}{1000 \times 9.81 \times 3 \times H_{extracted}}$$

$$H_{extracted} = 33.98\ m$$

$$\underline{z_1 - h_L = H_{extracted}}$$

$$50 - h_L = h_{extracted}$$

$$50 - 5 - h_{residual} = 33.98$$

$$45 - h_{residual} = 33.98$$

$$h_{residual} = 45 - 33.98$$

$$= \underline{\underline{10.95\ m}}$$

Discharge ($Q = \dot{V}$) Measurement.

$$\left.\begin{array}{l} \text{1. venturimeter} \\ \text{2. orificemeter} \\ \text{3. Nozzlemeter} \\ \text{4. Bend meter} \\ \text{5. Rotameter} \end{array}\right\} \text{Discharge Flow Rate meter}$$

VENTURIMETER.

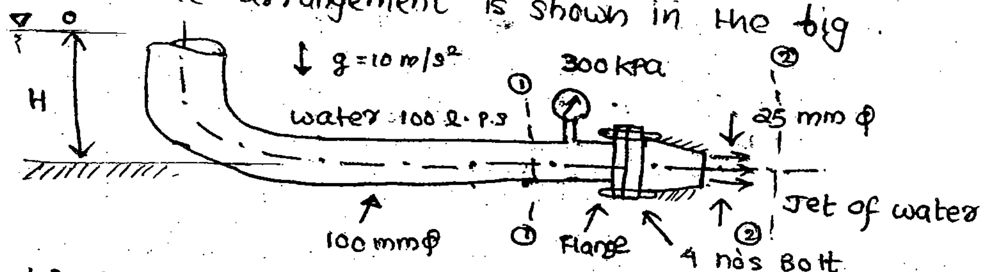
It is a device used to measure volume flow rate ie discharge through closed pipes or conduits; not

Model I :-

1. Force Analysis in pipe flow :-
2. Forces on lifted Bodies :-
3. Force exerted by moving fluid on fixed plates, moving plates
4. Power developed by moving fluid.

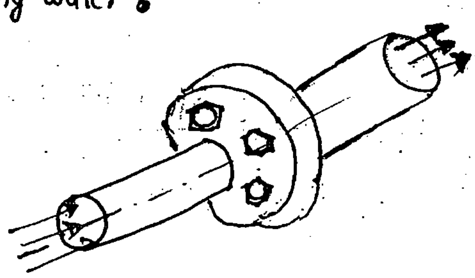
Model I

Q.) A smooth pipe of inner dia 100mm is subjected to water flow at the rate of 100 lit/s. At the end of the pipe a nozzle is connected with the help of a flange and no: of ~~Bores~~ Bolts. The arrangement is shown in the fig.



1. What is the mass flow Rate in pipe system ? (kg/s)
2. avg velocity in pipe line ?
3. velocity of Jet ?
4. What is the possible Head from which pipe is receiving water ?
5. What type of force acting on flange Bolts ?
6. What is the total force on the Flange
7. What is the force per each Bolt ?
8. Determine size of Bolt for the data Given ?
9. Hydraulic losses in the nozzle ?
10. Power lost in the nozzle. ?
11. efficiency of the pipe system ?

$\left\{ \begin{array}{l} \text{Bolt material steel} \\ S_{yt} = 250\ \text{MPa} \\ F.S = 4 \end{array} \right\}$



Ans:

(1) Mass flow Rate $\dot{m} = \rho Q$

$= 1000 \times 100 \times 10^{-3}$ kg/sec $=$ $\underline{100\ \text{kg/sec}}$

(2) By continuity eqn

$A_1 V_1 = A_2 V_2$

$100 \times 10^{-3} = \frac{\pi}{4} \times (0.1)^2 V_1 = \frac{\pi}{4} \times (0.25)^2 \times V_2$

$\therefore V_1 = \underline{12.73\ m/s} \simeq \underline{13\ m/s}$ $\qquad V_2 = \underline{203.7\ m/s} \simeq 204\ m/s$.

(4) Apply Bernoulis eqn at 0 & 1

$$\frac{P_0}{\rho g} + Z_0 + \frac{V_0^2}{2g} - h_{0-1} = \frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g}$$

$$0 + H + 0 - 0 = \frac{300 \times 10^3}{1000 \times 10} + 0 + \frac{13^2}{2 \times 10}$$

$H = \underline{38.45\ m}$

$1000 \times (V_2 - V_1)$

$100(204-13) = 19100\,N$

$= \underline{19.1\,KN}$

(6) Total load on Bolted Flange 19.1 KN

(7) Load Per Bolt $= \frac{19.1\,KN}{4} = \underline{4.78\,KN}$

(8) Diameter of the Bolts.

$$\sigma_{t\,Bolt} = \frac{F_{bolt}}{A_{c/s}}$$

$$\left(\frac{N}{mm^2}\right) \quad \frac{250}{4} = \frac{4.78 \times 10^3 \ (N)}{\frac{\pi}{4} \times d^2 \ (mm^2)} \qquad d = \text{dia of Bolt}$$

$$\therefore d_{Bolt} = \underline{9.86\,mm} \simeq \underline{10\,mm}$$

(9) Hydraulic losses b/w 1 & 2.

$$\frac{P_1}{\rho g} + \cancel{Z_1} + \frac{V_1^2}{2g} - h_L = \frac{P_2}{\rho g} + \cancel{Z_2} + \frac{V_2^2}{2g}$$

$$\frac{300 \times 10^3}{1000 \times 10} + 0 + \frac{13^2}{2 \times 10} - h_L = 0 + 0 + \frac{204^2}{2 \times 10}$$

GM1-GM

$$h_L = -\underline{2042}$$

(10) Power lost

$$P = \dot{m} g h_L$$

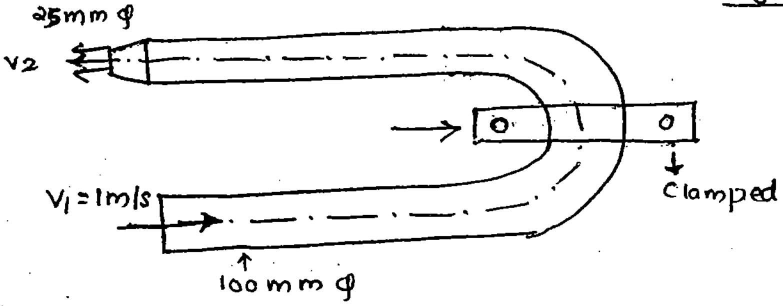
$$= 100 \times 10 \times 2042 \ W$$

1000

(11) $$\boxed{\eta = \frac{\dot{m} g h_L}{\frac{1}{2}\dot{m} V_1^2}}$$

(a) A u Bend pipe of 100 mm kept on the Horizontal plane at dia 100 mm and water flow at vel 1 m/s. At the end of the pipe a nozzle is fitted with exit dia 25 mm. Determine the force necessary to ~~clamp~~ ~~the~~ Hold the pipe in that position

Page 73



$Q = A_1 V_1 = A_2 V_2 = 0.00785$

$\frac{\pi}{4} \times (0.1)^2 \times 1 = \frac{\pi}{4} \times (0.025)^2 \times V_2$

$\underline{V_2 = 16\,m/s}$

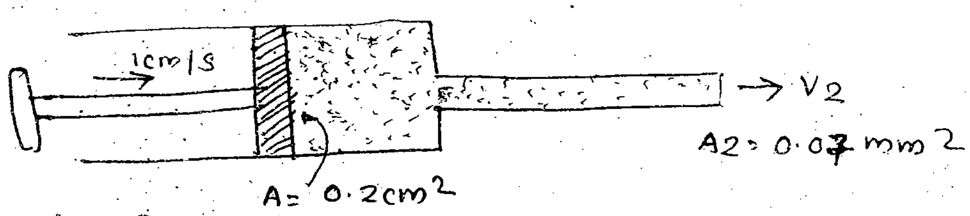
$$\Sigma F = \dot{m}(V_2 - V_1)$$

$$= \rho Q (V_2 - V_1)$$

$$\Sigma F = 1000 \times 0.00785 \,(16-1)$$

$$\underline{\Sigma F = 118\,N}$$

Page 74 (6.)



$\Sigma F = \dot{m}(\Delta v)$

$= \rho Q (V_2 - V_1)$ $\quad Q = A_1 V_1 = (0.2 \times 10^{-4}) \times (1 \times 10^{-2})$

$= 1000 \times 2.7 \times 10^{-7} (\cancel{0.001}\ 2.85 - 0.001)$ $\quad = \underline{\underline{2 \times 10^{-7} \frac{m^3}{sec}}}$

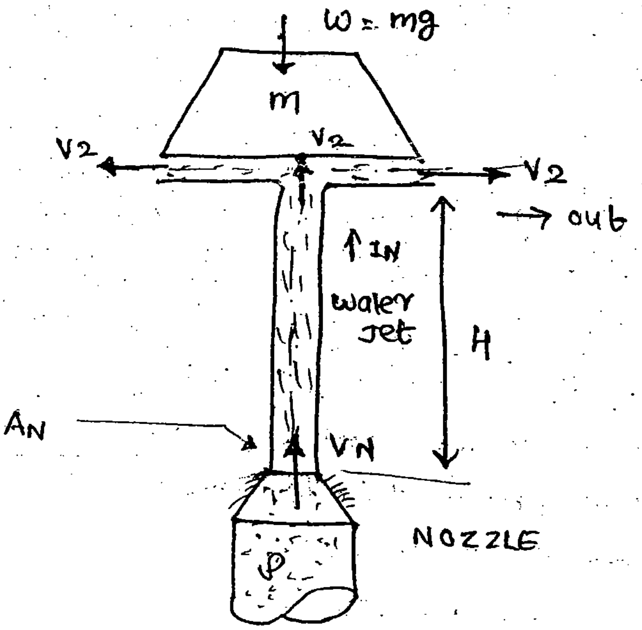
$= \underline{\underline{5.68 \times 10^{-4} N}}$ $\quad A_2 V_2 = 2 \times 10^{-7}$

$= \frac{5.68 \times 10^{-4}}{9.81} (kgf)$ $\quad \frac{\cancel{3}}{4}(0.07 \times 10^{-6}) \times V_2 = 2 \times 10^{-7}$ $\quad m^2 \quad (\frac{m}{s}) \quad (\frac{m^3}{s})$

$= \underline{\underline{5.8 \times 10^{-5} (kgf)}}$ $\quad \underline{\underline{V_2 = 2.85 m/s}}$

Model 2 Force exerted by Fluid on lifting Body

MOVING FLUID



Parameters Available

$A_N, \rho, V_N, m, H.$

Tool

(1) Continuity eqn

$$\boxed{Q = A_N V_N}$$

(2) Bernoulis eqn

Apply Bernoulis eqn at 1 & 2

$$\left(\frac{P_1}{\rho g}\right)_{gauge} + Z_1 + \frac{V_1^2}{2g} - h_L = \left(\frac{P_2}{\rho g}\right)_{gauge} + Z_2 + \frac{V_2^2}{2g}$$

$$0 + 0 + \frac{V_N^2}{2g} - 0 = 0 + H + \frac{V_2^2}{2g}$$

$$\Sigma F_y = \dot{m}(\Delta v_y)$$

$$= \dot{m}(v_{final_y} - v_{initial_y})$$

$$-mg = \dot{m}(0 - v_2)$$

$$-mg = -\rho Q v_2$$

$$mg = \rho Q v_2$$

$$mg = \rho A_N V_N V_2$$

$$\boxed{V_2 = \frac{mg}{\rho \cdot A_N V_N}}$$

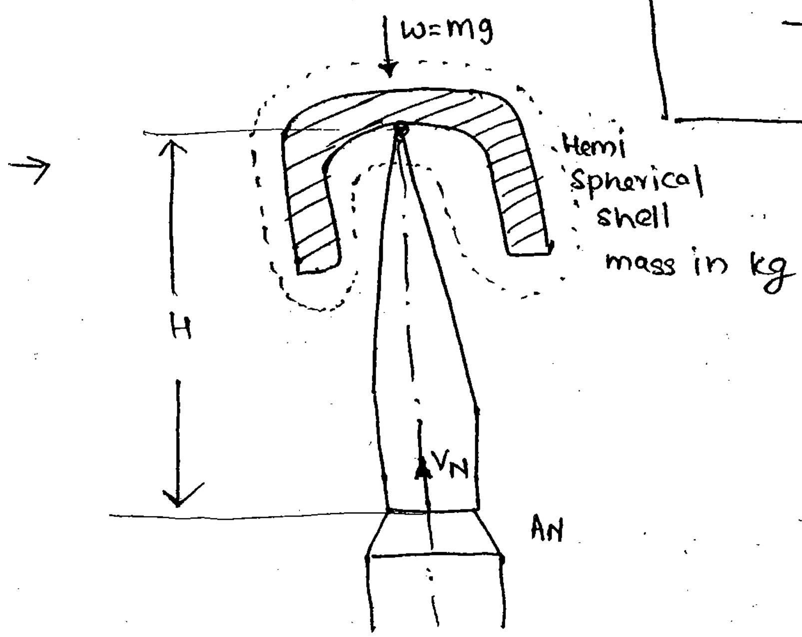
But $V_2 = \sqrt{V_N^2 - 2gH}$

$$\sqrt{V_N^2 - 2gH} = \frac{mg}{\rho \cdot A_N \cdot V_N}$$

Squaring $V_N^2 - 2gH = \left[\frac{mg}{\rho \cdot A_N \cdot V_N}\right]^2$

$$H = \frac{V_N^2 - \left[\frac{mg}{\rho \cdot A_N V_N}\right]^2}{2g}$$

$$\boxed{H = \frac{V_N^2 - \left[\frac{mg}{\rho \cdot A_N V_N}\right]^2}{2g}}$$



Q. A vertical jet of water is able to keep a hemispherical

nozzle of Area A_N with velocity V_N at the Nozzle exit using 108
Bernoulis eqn and momentum theorem develop the expressions
for jet velocities V_2 at the shell point & expression for Height
H at which the shell is Balanced. shell mass is m and
water density is ρ. Assume ideal flow conditions.

1. <u>continuity eqn</u>

$$Q = A_N V_N$$

(2) <u>Bernoulis eqn</u>

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$$0 + 0 + \frac{V_N^2}{2g} = 0 + H + \frac{V_2^2}{2g}$$

$$\boxed{V_2 = \sqrt{V_N^2 - 2gH}}$$

(3) <u>Momentum eqn</u>

$$\Sigma F_y = \dot{m}(\Delta V_y) = \dot{m}(V_{final_y} - V_{initial_y})$$

Since mg is in downward direction

$$-mg = \dot{m}(-V_2 - V_2)$$

$$-mg = -\rho Q 2 V_2$$

$$mg = \rho Q 2 V_2$$

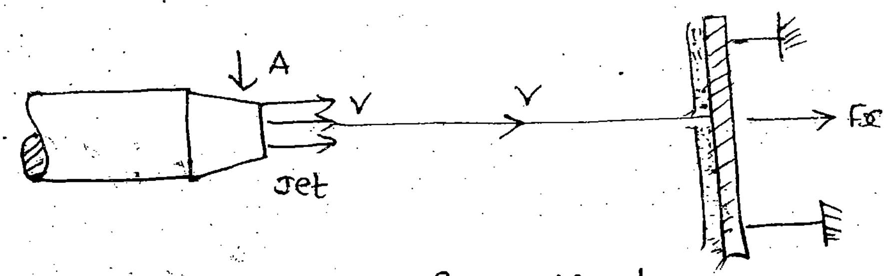
$$mg = \rho \cdot A_N V_N \cdot 2 V_2$$

$$\left[\frac{mg}{\rho 2 A_N \cdot V_N}\right] = V_2$$

$$\frac{mg}{2\rho \cdot A_N \cdot V_N} = \sqrt{V_N^2 - 2gH}$$

$$\left[\frac{mg}{2\rho \cdot A_N \cdot V_N}\right]^2 = V_N^2 - 2gH$$

(1)



$F_x = \rho A V^2$ = Reactive force offered by plate.

$$\frac{kg}{m^3} \times m^2 \times \frac{m}{s^2} = kg\,\frac{m}{s^2} = N$$

$F_x = ma$

$= \dot{m}(\Delta V)_x$

$\dot{m}\left(V_{final_x} - V_{initial_x}\right)$

$= \dot{m}(0 - V)$

$F_x = -\dot{m}V$

-ve sign means energy lost by fluid

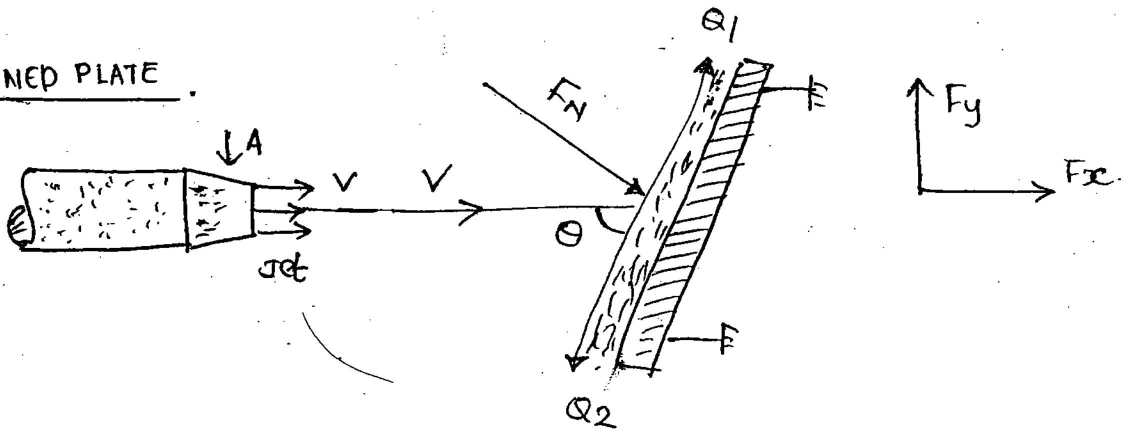
ie ~~free~~ force exerted by jet of fluid on plate (Reaction of plate)

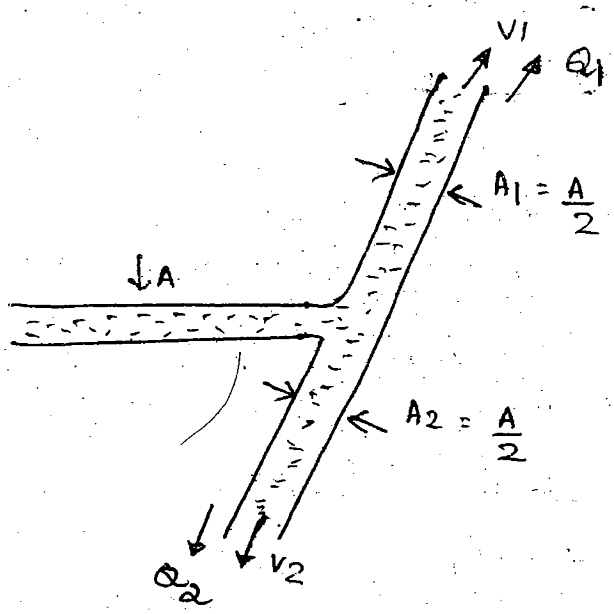
$F_x = \dot{m}V$

$= \rho Q V$

$$\boxed{F_x = \rho A V^2}$$

(2) INCLINED PLATE





$Q = Q_1 + Q_2$ —— By continuity

$\rho A V = \rho A_1 V_1 + \rho A_2 V_2$

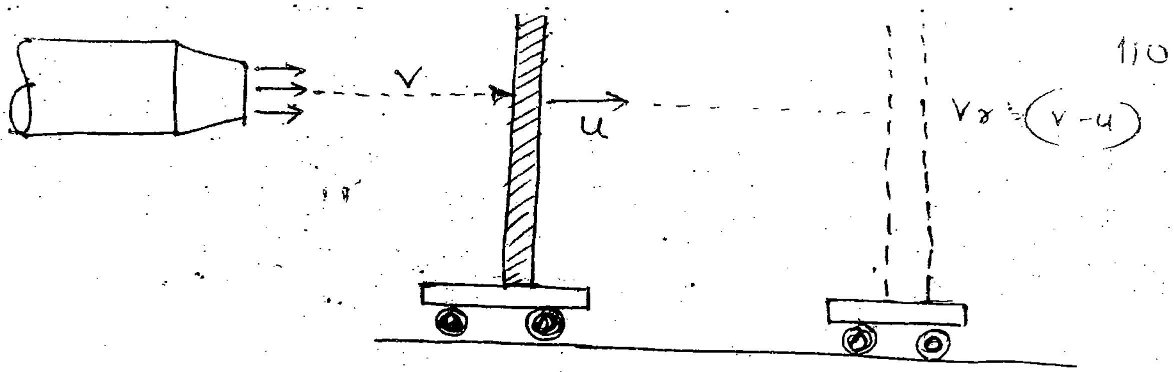
$\cancel{A} V = \frac{\cancel{A}}{2} V_1 + \frac{\cancel{A}}{2} V_2$

$$V = \frac{V_1}{2} + \frac{V_2}{2}$$

$$V = \frac{V_1}{2} + \frac{V_1}{2} \quad \text{or} \quad \frac{V_2}{2} + \frac{V_2}{2}$$

$$V = \frac{2V_1}{2} \quad \text{or} \quad \frac{2V_2}{2}$$

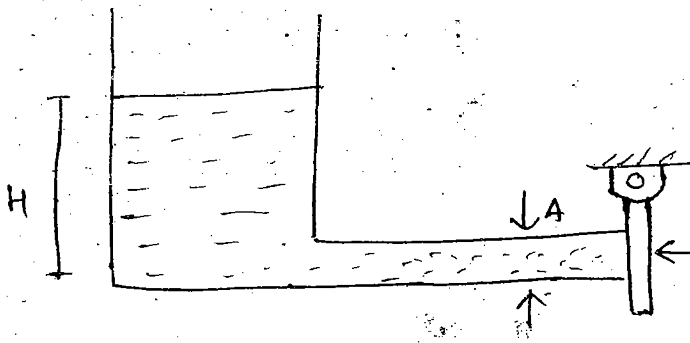
$$\boxed{V = V_1 = V_2}$$



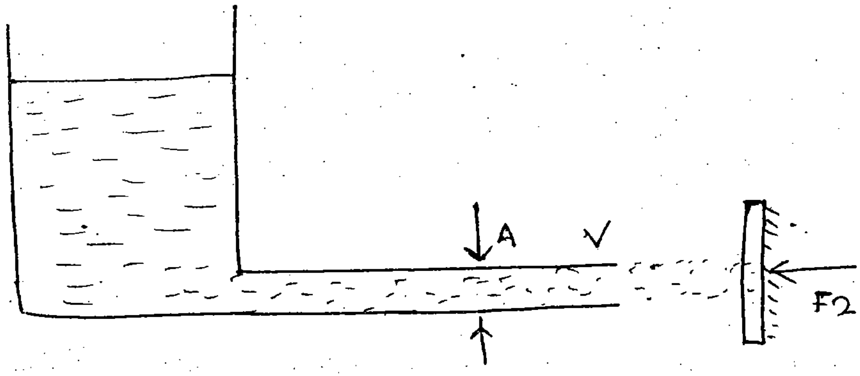
V
u
Vr = (v - u)
110

2009 Q.)

-atic
-luid



$F_1 = P \times A = \rho g H A$



$F_2 = \dot{m} \Delta V$

$= \rho A V (V - 0)$

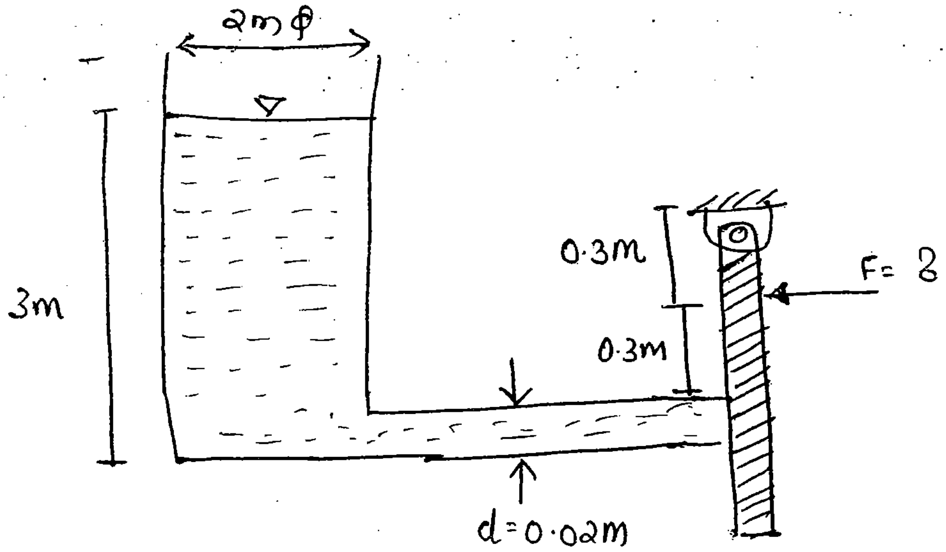
$= \rho A V^2$

$F_2 = \rho A (\sqrt{2gH})^2$

$F_2 = \rho A \times 2gH$

$\frac{F_1}{F_2} = \frac{\rho g H A}{2 \rho g H A} = \frac{1}{2}$

Q.)

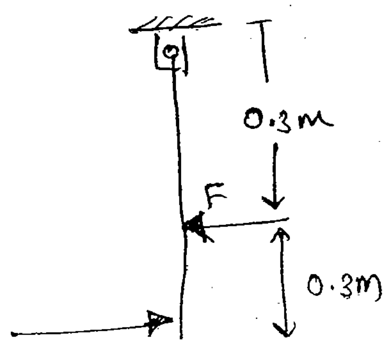


a) 18.5 N

b) 37 N

c) 48.5 N

d) 74 N



F= P.A

$F \times 0.3 = 9.24 \times 0.6$

$F = 18.48\,N$

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10

(Q1.) $F = \rho A(v-u)^2$

$= 1000 \times 0.01\,(20-10)^2$

$= 1000\,N$

(Q2.) Power developed $= F \times V$

$= 1000 \times 10$

$= 10{,}000$ watt

$= 10\,kW$

(Q3) force developed

$F = \rho A\left(v-(-u)\right)^2$

$= \rho A(v+u)^2$

$= 1000 \times 0.01\,(20+10)^2$

$= 9000\,N$

(Q4.) ~~V = V~~ Relative velocity

$V - \left(-\frac{V}{2}\right)$

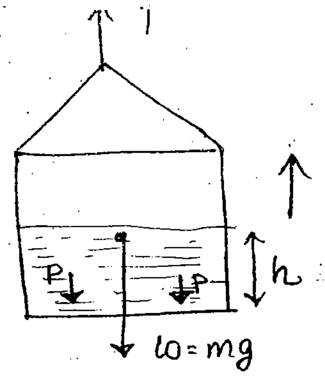
$= \cancel{V+V}\ \ V + \frac{V}{2} = \frac{3V}{2} = 1.5V$

$Q_2 = A \cdot V_R$

$\cancel{= A(u)} = A(v+u)$

$= A\left(v + \frac{V}{2}\right)$

(5)



$$\Sigma F = ma$$

$$T - W = ma$$

$$T - mg = ma$$

$$T = mg + ma$$

$$T = m(g + a)$$

$$\text{Pressure} = \frac{T}{A} = \frac{m(g+a)}{A}$$

$$= \frac{\rho V (g+a)}{A}$$

$$= \frac{\rho \cdot h \cdot \cancel{A} (g+a)}{\cancel{A}}$$

Fluid at Rest

$$P = \rho g h$$

Fluid moving up =

$$\cancel{P = \rho g (g}$$

$$P = \rho h (g + a)$$

Fluid moving down

$$P = \rho h (g - a)$$

$$P = 1200 \times 10 \left(9.81 + 5 \times 9.81\right)$$

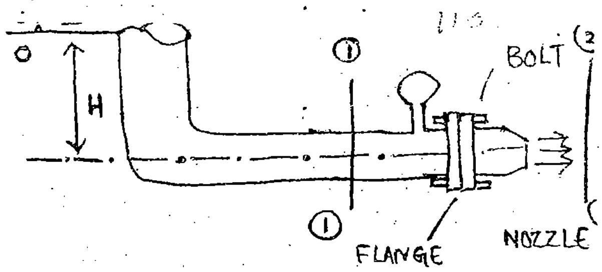
$$= \underline{\underset{\cancel{70632}}{706320}} \ N/m^2$$

$$\frac{706320}{9.81} = \frac{\cancel{70632}}{\cancel{9.81}} \ \cancel{kg/cm^2} \ \frac{Kgf}{M^2}$$

PRAVEEN T.P
PM 15
MECHANICAL
PH NO: 7207237642

FLUID MECHANICS - II

1. – $\boxed{\text{Efficiency of piping system} = \frac{m g h_L}{\frac{1}{2}\dot{m} V_1^2}}$



- Force on Bolts

$$= S_{yt} \text{ of Bolt} \times \text{Area of c/s}$$

$$S_{yt} \times \frac{\pi}{4} \times d^2$$

2. – $\boxed{\text{Force on Bolts} \quad F = \dot{m}\Delta V}$ $\quad \Delta V = V_2 - V_1$

V_2 = Nozzle exit vel

V_1 = vel in piping system

- Force per Bolt = $\frac{F}{\text{No of Bolts}}$

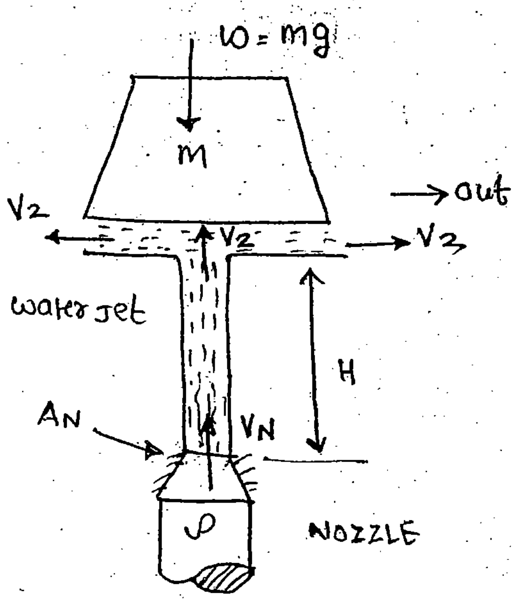
- $\boxed{\text{Power Lost} = \dot{m} g h_L}$

3. Mass Flow Rate.

$$\dot{m} = \rho A V$$
$$= \rho Q$$

MODEL 2

→ Force Analysis in Lifting Bodies ←



Parameters Available:

V_2, A_N, m, H, ρ

Tool:

① continuity eqn:

$$\boxed{Q = A_N V_N}$$

② Bernoulis eqn:

$$\left[\frac{P_1}{\rho g}\right]_{gau} + Z_1 + \frac{V_1^2}{2g} = \left[\frac{P_2}{\rho g}\right]_{gau} + Z_2 + \frac{V_2^2}{2g}$$

$$0 + 0 + \frac{V_N^2}{2g} = 0 + H + \frac{V_2^2}{2g}$$

$$\frac{V_2^2}{2g} = \frac{V_N^2}{2g} - H$$

$$\boxed{V_2 = \sqrt{V_N^2 - 2gH}}$$

③ Momentum eqn:

$$\Sigma F_y = \dot{m}(\Delta V_y) = \dot{m}\left(V_{final_y} - V_{initial_y}\right)$$

$$-mg = \dot{m}(0 - V_2)$$

$$-mg = -\rho Q V_2$$

$$V_2 = \frac{mg}{\rho \cdot Q}$$

$$\boxed{V_2 = \frac{mg}{A_N V_N \cdot \rho}}$$

But $V_2 = \sqrt{V_N^2 - 2gH}$

$$\sqrt{V_N^2 - 2gH} = \frac{mg}{\rho \cdot A_N \cdot V_N}$$

$$V_N^2 - 2gH = \left[\frac{mg}{\rho \cdot A_N \cdot V_N}\right]^2$$

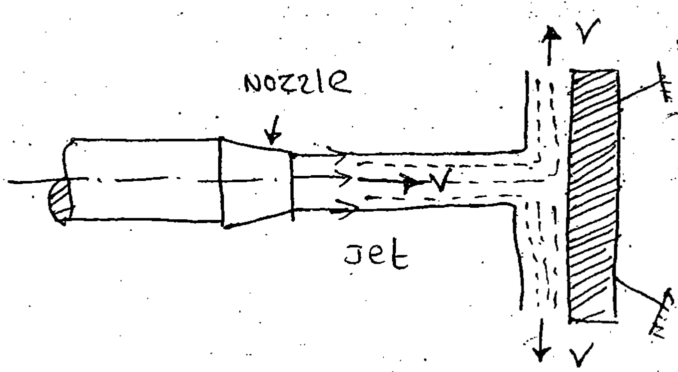
$$\boxed{H = \frac{V_N^2 - \left[\frac{mg}{\rho \cdot A_N \cdot V_N}\right]^2}{2g}}$$

MODEL-3

Force exerted by Moving Fluid on a Fixed plate

① Force exerted by jet on plate

= Rate of change of momentum in direction of force



$= \dot{m}[\Delta V]$

$= \frac{\text{mass}}{\text{Time}}$ [Initial velocity − final velocity]

If force exerted on Jet is to be calculated we will take [final vel − Initial vel]

$\dot{m}[V - 0]$

$\rho AV \cdot V = \rho AV^2$

$$\boxed{F_x = \rho AV^2}$$

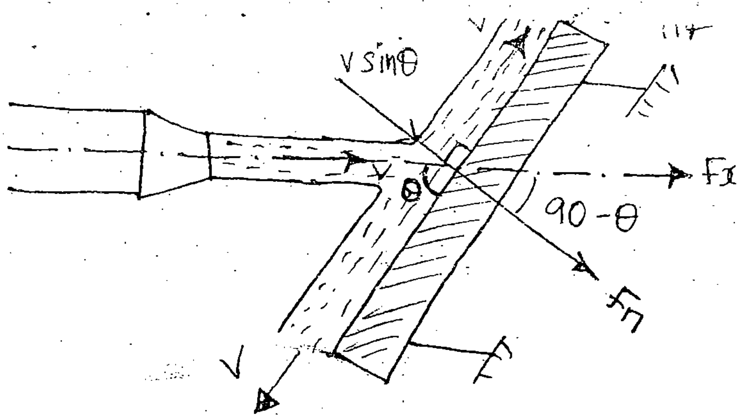
② Inclined plate

$$F_n = \rho AV^2 \sin\theta$$

$$F_n = \frac{\text{Mass}}{\text{Time}} \left[\text{Initial vel} - \text{Final vel} \right]$$

$$F_n = \rho A V \left[V\sin\theta - 0 \right]$$

$$\boxed{F_n = \rho A V^2 \sin\theta}$$



$$F_x = F_n \cos(90-\theta)$$

$$\boxed{F_x = \rho A V^2 \sin^2\theta}$$

$$F_y = F_n \sin(90-\theta)$$

$$= F_n \cos\theta$$

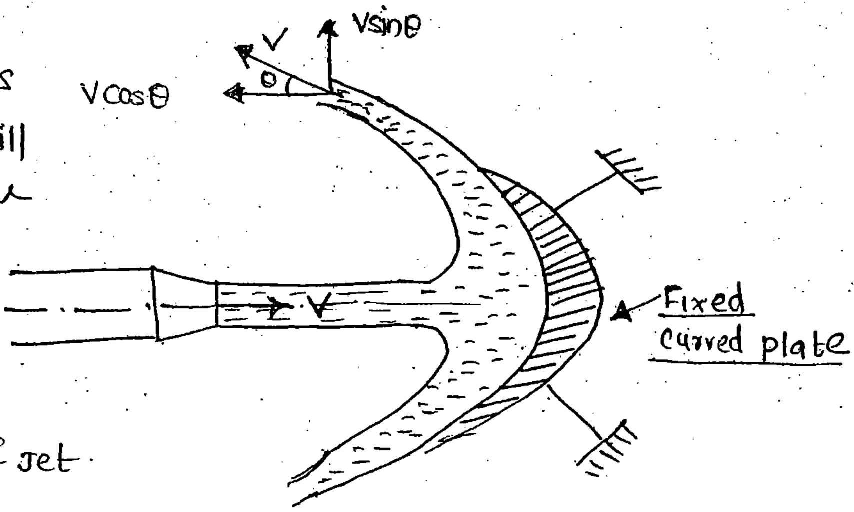
$$\boxed{F_y = \rho A V^2 \sin\theta \cos\theta}$$

Initial vel in the direction of n

Final vel = 0 in dir of n

3. <u>stationary curved plate</u>:

If there is no loss of energy the jet will come out with same vel in the tangential direction.



$V\cos\theta$ = opp dir of jet.

Component in the direction of jet = $-V\cos\theta$

" ⊥r to " of jet = $V\sin\theta$

Initial

V_{1x} = vel in the direction of jet

V_{2x} = final vel in the direction of jet

$$F_x = \frac{\text{Mass}}{\text{sec}} \left[V_{1x} - V_{2x} \right]$$

$$\rho A V \left[V - (-V\cos\theta) \right]$$

$$\boxed{F_x = \rho A V^2 \left[1 + \cos\theta \right]}$$

$$F_x = 2\rho A V^2 \cos^2 \theta/2$$

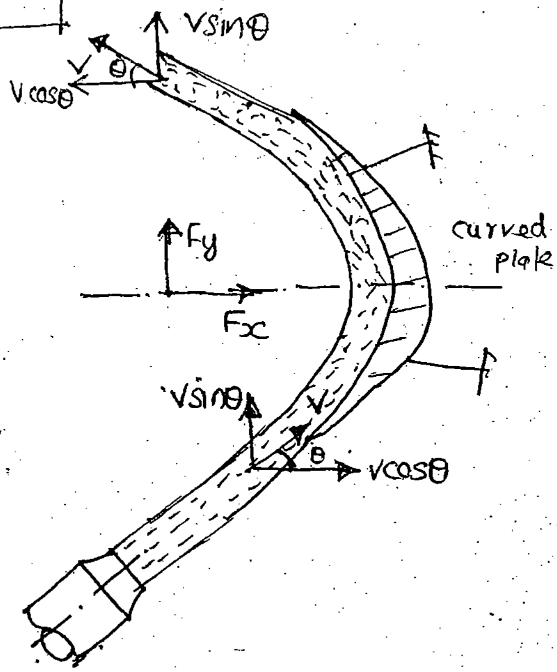
$$F_y = \rho A V \left[V_{1y} - V_{2y} \right]$$

$$= \rho A V \left[0 - V\sin\theta \right]$$

$$\boxed{F_y = -\rho A V^2 \sin\theta}$$

-ve indicates force is acting in downward direction.

$\boxed{\text{Angle of Deflection means } 180 - \theta}$



Jet striking Tangentially

$$F_x = \rho A V \left[V_{1x} - V_{2x} \right]$$

$$= \rho A V \left[V\cos\theta - (-V\cos\theta) \right]$$

$$\boxed{F_x = 2\rho A V^2 \left[\cos\theta \right]}$$

$$F_y = \rho A V \left[V\sin\theta - V\sin\theta \right]$$

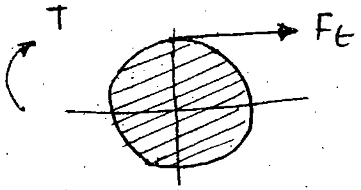
$$F_y = 0$$

Water sprinkler.

It works on Momentum - Momentum equation

Eqn:

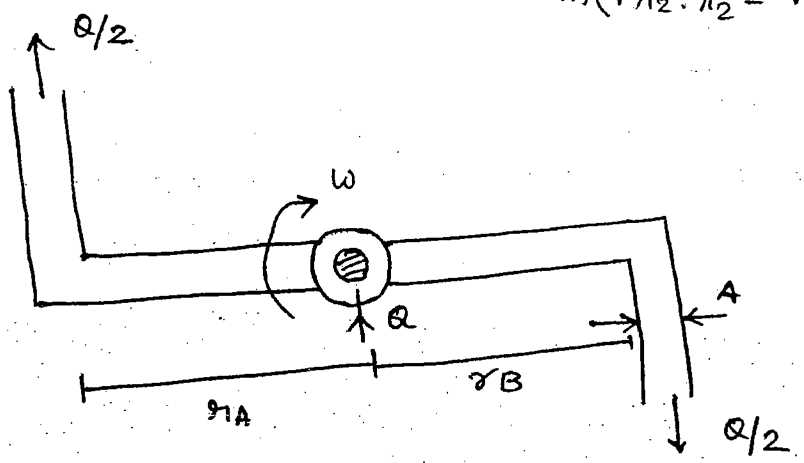
$$\text{Torque} = F \times r$$



$$T = \dot{m}(\Delta V) \cdot r$$

$$= \dot{m}(V_{r2} - V_{r1}) \cdot r$$

$$= \dot{m}(V_{r2} \cdot r_2 - V_{r1} \cdot r_1)$$



$V_t = \omega r_A$

$V_t = \omega \times r_B$

$$\boxed{\frac{Q}{2} = AV}$$

$$V = \frac{\left(\frac{Q}{2}\right)}{A}$$

$$V_{abs} = V_r \pm V_t$$

$$= V_r \pm \omega \times r$$

$$V_{abs} = \frac{Q/2}{\text{Area pipe}} \pm \omega \times r$$

Q11. V_{abs} at A $= V_{rA} - V_{tA}$

$$= \left(\frac{Q}{\ }\right) - \omega \times r$$

$$= \left[\frac{\frac{2.4\times10^{-3}}{2}}{1.2\times10^{-4}}\right] - \left[7.69\times0.2\right]$$

$= 10 - 1.538$

$= \underline{8.462\ m/s}$

Q12. $V_{absolute} = V_{RB} + V_{tB}$

$= 10 + (7.69 \times 0.3)$

$= \underline{12.307\ m/s}$

→ 5. LAMINAR FLOW ←

* Viscous Flow / Layer Flow / Laminar flow through pipes and parallel plates of incompressible fluids

5.1 Introduction

5.2 Reynold No and its Importance

5.3 Analysis of laminar flow (viscous flow through a circular pipe (shear stress distribution, velocity distribution, Pressure Head Loss, Power lost due to viscosity and Ratio of Maximum velocity to Mean velocity in pipe Flow & b/w two parallel fixed plates flow.)

* Laminar flow is a Type of flow which Rarely observed in real life. ex: (1) Blood flow through veins.

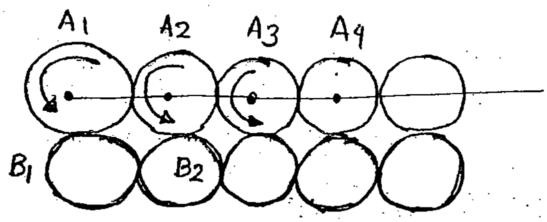
(2) Rain water collected in reservior / Perculation of water (seepage of water)

(3) Lubrication for Hydrostatic Bearings

4. Sap of the plants — Make a drop of water on a dry leaf and notice change in color. due to Laminar flow slowly only there is a color change

The following are the conditions under which flow is said to be a Laminar flow.

1. Each molecule of the fluid moves in its mean path.
2. Fluid flow is a Rotational Flow.
3. Flow is steady.
4. Flow is 1-D.
5. Fluid is viscous in Nature.
6. Reynold's No of Fluid Flow $\leq$ 2000



VISCOUS FLOW

Fluid is incompressible.

5.2 Reynold Number

It is the Ratio of Inertia Force to Viscous force

$$Re = \frac{\text{Inertia Force}}{\text{Viscous Force}} = \frac{ma}{\tau \cdot A} = \frac{m\left(\frac{v-u}{t}\right)}{\mu \frac{V}{y} \cdot A}$$

$$= \frac{\dot{m}\left(\frac{v-0}{t}\right)}{\mu \frac{VA}{y}} = \frac{\rho \cdot Q V}{\mu \frac{VA}{y}} = \frac{\rho A V}{\mu \frac{A}{y}}$$

$$= \frac{\rho V}{\frac{\mu}{y}} = \frac{\rho V y}{\mu}$$

Pipe Flow $y = D$

Characteristic dimension = diameter

$$\boxed{Re_{\text{Pipe}} = \frac{\rho V D}{\mu} = \frac{\rho D}{\vartheta}}$$

$\cancel{Re = \frac{VD}{\nu}}$ $Re = \frac{VD}{\nu} \leq 2000$

Laminar Flow

$Re_{critical} = 2000$

$Re < 1$

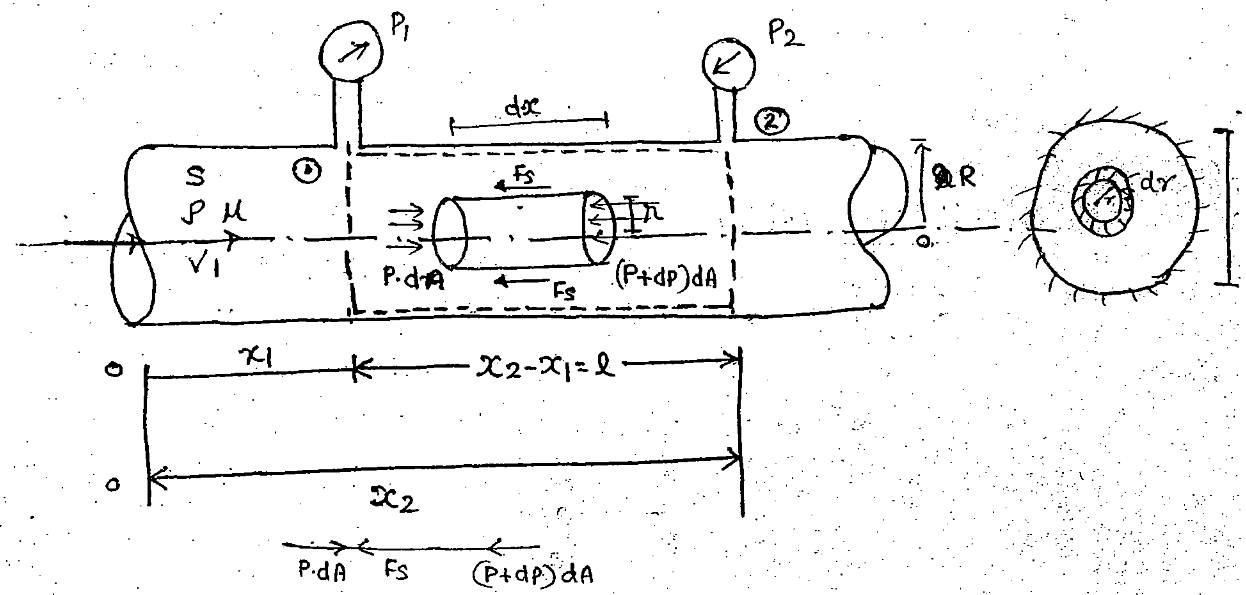
Flow through porous media (soil)

$Re < 500$

Flow through open channels

Viscous Flow Analysis :-

Consider a const dia of pipe 'D' and length 'l' subjected to viscous fluid flow shown in figure.



D = d l	Physical characteristic
S ρ γ μ ν	→ Fluid self properties

$\dot{m}$

Q

V

Fluid is steady. so $a = 0$

$$\Sigma F = ma$$

$$P \cdot dA - F_s - (P+dP)dA = m \times 0$$

$$-F_s - dP\,dA = 0$$

$$-\tau \cdot dA_s - dP\,dA = 0$$

$$\tau\, dA_s = -dP \cdot dA$$

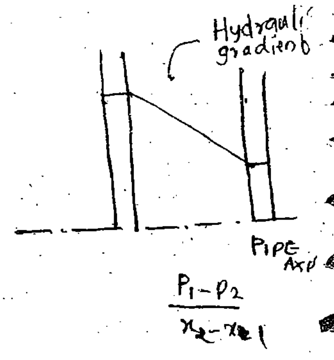
dx = Length of the element

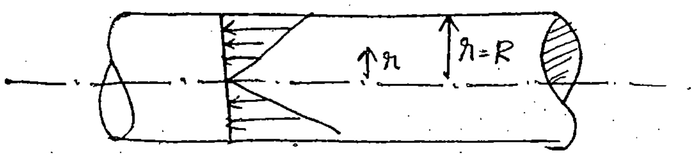
$$\tau \cdot \underbrace{2\pi r \cdot dx}_{\text{Shear}} = -dP \cdot \underbrace{\pi r^2}_{\text{Normal Area}}$$

$$\boxed{\tau = -\frac{dp}{dx} \cdot \frac{r}{2}}$$

Pressure Gradient

LINEAR VARIATION





$$\tau_{wall} = -\frac{dp}{dx} \cdot \frac{R}{2}$$

τ is Max at the WALLS

$$\tau_{centre} = -\frac{dp}{dx} \times 0 = 0$$

$\tau = 0$ at centre since $r = 0$

Velocity distribution

we have

$$\tau_r = -\frac{dp}{dx} \cdot \frac{r}{2}$$

{shear stress at any Radius}

We have Newtons law of viscosity

$$\tau = \mu \frac{du}{dy}$$

where $y = R - r$

y = dist. b/w the 2 layers or thickness of Layer

Diff y

$$dy = 0 - dr$$

$$dy = -dr$$

$$\tau_r = -\mu \frac{du}{dr}$$

$$-\mu \frac{du}{dr} = -\frac{dp}{dx} \cdot \frac{r}{2}$$

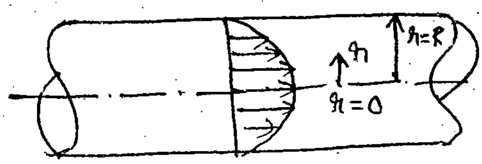
$$\mu \frac{du}{dr} = \frac{dp}{dx} \cdot \frac{r}{2}$$

$$du = \frac{1}{\mu} \frac{dp}{dx} \cdot \frac{r}{2} \cdot dr$$

$$\int du = \frac{1}{2\mu}\frac{dp}{dx}\int_0 r\,dr$$

$$u_r = \frac{1}{2\mu}\frac{dp}{dx}\left(\frac{r^2}{2}\right) + \text{const}$$

$$\boxed{u_r = \frac{1}{4\mu}\left(\frac{dp}{dx}\right) r^2 + C} \quad \text{—— (2)}$$



$$\boxed{u_r \propto r^2}$$

PARABOLIC VARIATION

using boundary conditions

At $r = R$,

$u_r = 0$

$$0 = \frac{1}{4\mu}\left(\frac{dp}{dx}\right)R^2 + C$$

$$C = -\frac{1}{4\mu}\left(\frac{dp}{dx}\right)R^2$$

velocity distribution

$$u_r = \frac{1}{4\mu}\left(\frac{dp}{dx}\right)r^2 - \frac{1}{4\mu}\left(\frac{dp}{dx}\right)R^2$$

$$\boxed{u_r = \frac{1}{4\mu}\left(-\frac{dp}{dx}\right)\left[R^2 - r^2\right]}$$

At $r = 0$

$u_r = u_{max}$ (centre line velocity)

$$u_{max} = \frac{1}{4\mu}\left(-\frac{dp}{dx}\right)\left[R^2 - 0\right]$$

$$\boxed{u_{max} = \frac{1}{4\mu}\left(-\frac{dp}{dx}\right)R^2}$$

III → Pressure drop

we have

$$u_{max} = \frac{1}{4\mu}\left(\frac{-dP}{dx}\right) \cdot R^2$$

$$-\int_{P_1}^{P_2} dP = \frac{4\mu \cdot u_{max}}{R^2} \cdot \int_{x_1=0}^{x_2=l} dx$$

$$-(P_2 - P_1) = \frac{4\mu u_{max}}{R^2}[l]$$

$$P_1 - P_2 = \frac{4\mu u_{max} \cdot l}{R^2}$$

$$\boxed{P_1 - P_2 = \frac{16\mu \cdot u_{max} \cdot l}{D^2}}$$

IV . In pipe Flow.

$$\boxed{u_{max} = 2V}$$

Max vel = 2× Avg velocity

V

Parallel Plate flow

$$u_{max} = 1 \cdot 5\,V$$

Loss of Head due to viscosity

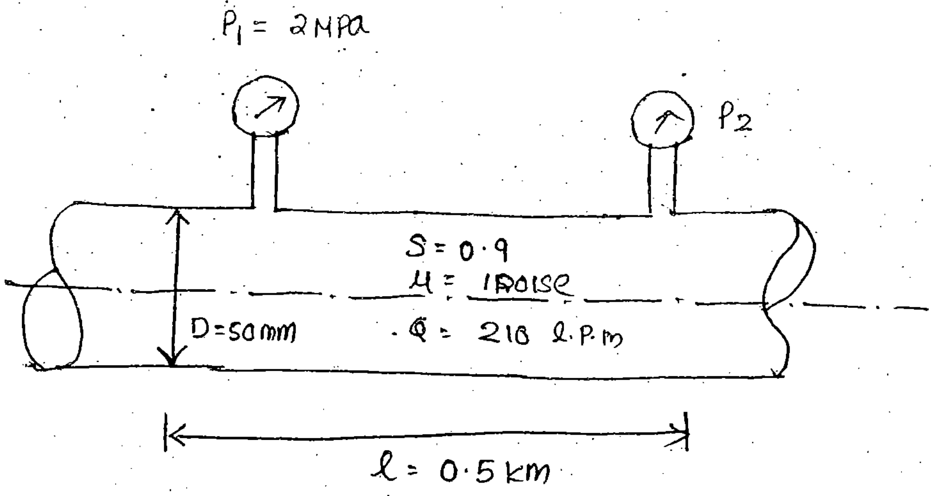
$$h_L = \frac{P_1 - P_2}{\rho g} = \frac{32\mu V l}{\rho \cdot g D^2}$$

Hagen Poissellues Eqn

$$\boxed{\text{Power Lost} = \dot{m} g h_L}$$

Q) A 50 mm internal dia of pipe of length 0.5 km is subjected to an oil flow (S = 0.9), μ = 1 poise at the Rate 210 l.p.m.

1. Mass density of oil in kg/m^3; g/cc or g/ml.
2. Weight density of oil in kN/m^3.
3. dynamic viscosity of oil in S.I unit.
4. Kinematic viscosity of oil in S.I unit and C.G.S unit.
5. Volume flow Rate of oil in the pipe in m^3/s.
6. Mass flow Rate of oil in kg/s.
7. Avg vel of oil flow in m/s.
8. Reynolds no: of oil flow in pipe.
9. state type of the oil flow (whether Laminar or Not)
10. Maximum velocity of the oil flow in pipe in lit/s.
11. Pressure drop due to viscosity in MPa
12. Exit pressure at outlet of pipe in MPa.
13. Pressure Gradient (Hydraulic Gradient) in $\frac{N}{m^3}$ or $\frac{kPa}{m}$
14. Loss of Head in 'm' of oil due to viscosity.
15. Loss of Head in m of water and in metres of mercury.
16. Power lost due to viscosity in kW.
17. Power Reqd to Run the pump to maintain the flow in H.P.
18. Velocity of the oil flow at 8mm from the surface of the pipe.
19. shear stress at the surface of pipe in N/m^2.
20. shear stress at 12mm from the centre line of the pipe.
21. Shear Force on the surface of the pipe.
22. Draw the shear stress and velocity distribution.
23. At what Radial distance Avg vel occurs?



(1) $S_{oil} = \dfrac{\rho_{oil}}{\rho_{water}}$

$\rho_{oil} = 0.9 \times 1000 = \underline{\underline{900\ kg/m^3}}$

(2) $\gamma_{oil} = \rho_{oil} \cdot g$

$= 900 \times 9.81\ N/m^3$

$= \underline{\underline{8.83\ kN/m^3}}$

(3) $\mu = 1\ \text{poise} = \underline{\underline{0.1\ \dfrac{N \cdot s}{m^2}}}$

(4) $\nu = \dfrac{\mu}{\rho} = \dfrac{0.1}{900} = \underline{\underline{1.11 \times 10^{-4}\ m^2/sec}}$

$= 1.11\ cm^2/sec$

$= \underline{\underline{1.11\ \text{stokes}}}$ (c.g.s)

(5) $\dot{V} = Q = 210\ l.p.m$

$= \dfrac{210}{60}\ \dfrac{l}{s}$

$= 3.5\ lt/sec$

$= 3.5 \times 10^{-3}\ m^3/sec$

$= \underline{\underline{0.0035\ m^3/s}}$

$\dot{m} = \rho Q$

$= 900 \times 0.0035 = \underline{3.15 kg/sec}$

(7) $Q = A \cdot V$

$0.0035 = \frac{\pi}{4} \times (0.05)^2 \cdot V$

$\underline{V = 1.78 m/s}$

(8) $Re = \frac{V \cdot D}{\nu} = \frac{1.78 \times 0.05}{1.11 \times 10^{-4}}$

$= \underline{809} < 2000$

(9) Flow is Laminar

(10) $\left(\frac{U_{max}}{V}\right)_{pipe\ flow} = 2$

$U_{max} = 2 \times V = 2 \times 1.78$

$= \underline{3.56 m/s}$

(11) $P_1 - P_2 = \frac{32 \mu V l}{D^2} = \frac{32 \times 0.1 \times 1.78 \times 500}{(0.05)^2}$

$= 1.14 \times 10^6 N/m^2 = \underline{1.14 MPa}$

(12) Exit Pressure

$P_1 - P_2 = 1.14 MPa$

$2 - P_2 = 1.14$

$\therefore P_2 = 2 - 1.14 = \underline{0.86 MPa}$

(13) Pressure Gradient (Hydraulic Gradient)

$-\frac{dP}{dx} = \frac{P_1 - P_2}{x_2 - x_1} = \cancel{1.14 \to 6} \frac{1.14 \times 10^6}{500}$

$2.28 \times 10^3 N/m^3 = \underline{2.28 kPa/m}$

(14) $h_1 - h_2 = \frac{P_1 - P_2}{\rho g} = \frac{1.14 \times 10^6}{900 \times 9.81} = \underline{\underline{129 m}}$

(15) Loss of Hed in metres

$P_1 - P_2 = \rho_{oil} \cdot g \cdot h_{oil} = \rho_{water} \cdot g \, h_w = \rho_{mer} \cdot g \cdot h_{mer}$

$1.14 \times 10^6 = 1000 \times 9.81 \times h_w$

$h_w = \underline{\underline{116.2 \text{ metres}}}$ of water

$1.14 \times 10^6 = 13600 \times 9.81 \times h_{mercury}$

$h_{mercury} = \underline{\underline{8.5 \text{m of water}}}$

(16.) Power lost

$P_{lost} = \dot{m} \, g \cdot h_L$

$= 3.15 \times 9.81 \times 129$

$= 3990$ watt

$= 3.99 \text{ kw} = \underline{\underline{4 \text{ kw}}}$

(17) $1 \text{ HP} = 0.736 \text{ kw}$

Power required to run pump to maintain flow

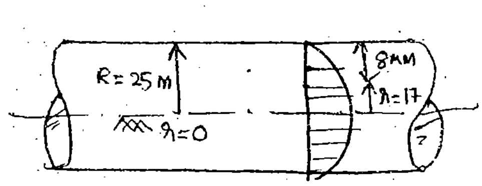
$= \frac{4 \text{ kw}}{0.736} = \underline{\underline{5.43 \text{ HP}}}$

(18) $u_r = \frac{1}{4\mu} \left(\frac{-dp}{dx} \right) \left(R^2 - r^2 \right)$

$u_{max} = \frac{1}{4\mu} \left(-\frac{dp}{dx} \right) R^2$

$u_r = R^2 - r^2 = 1 - (r)^2$

$$\frac{u_r}{3.56} = 1 - \left(\frac{17}{25}\right)^2$$



$\underline{\underline{u_r = 1.91 \, m/s}}$

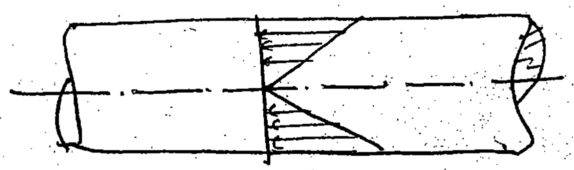
Q19.) $$\tau_r = -\left(\frac{dp}{dx}\right) \cdot \frac{r}{2}$$

$$\tau_{wall} = -\left(\frac{dp}{dx}\right) \cdot \frac{R}{2}$$

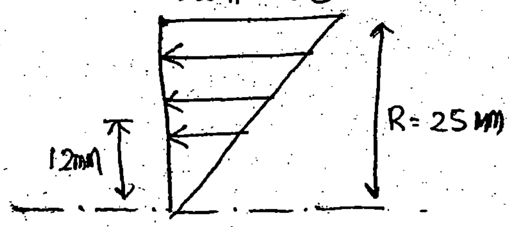
$$= \left(-2.28 \times 10^3\right)\left(\frac{\frac{0.05}{2}}{2}\right) = \underline{\underline{28.5 \, N/m^2}}$$

20.)

$$\tau_r = \frac{-dp}{dx} \cdot \frac{r}{2}$$



$$= \frac{28.5}{25} = \frac{\tau}{12}$$



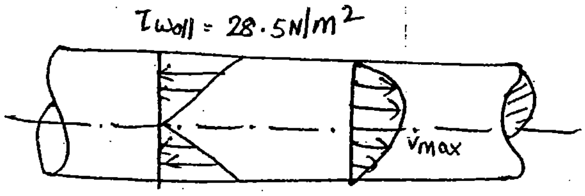
$$\tau = \underline{13.68 \, N/m^2}$$

21.) $$F_{s\,wall} = \tau_{wall} \cdot A$$

$$= 28.5 \times \pi D l = 28.5 \times \pi \times 0.05 \times 500$$

$$= \underline{\underline{2238 \, (N)}} \cdot \underline{\underline{2.24 \, kN}}$$

22.)



23.) $$V_r = \frac{1}{4\mu}\left(\frac{-dp}{dx}\right)\left(R^2 - r^2\right)$$

$$\boxed{r_{AV} = 0.707 R}$$

$$U_{max} = \frac{1}{4\mu}\left(\frac{-dp}{dx}\right)\left(R^2 - r^2\right)$$

$$\frac{1}{8\mu}\left(\frac{-dp}{dx}\right)R^2 = \frac{1}{4\mu}\left(\frac{-dp}{dx}\right)\left(R^2-r^2\right)$$

$$\frac{1}{2}R^2 = R^2 - r^2$$

$$r^2 = R^2 - \frac{1}{2}R^2$$

$$r^2 = \frac{1}{2}R^2$$

$$r = \frac{1}{\sqrt{2}}R$$

Mean vel occurs at a distance of $\boxed{r = 0.707R}$

$$\boxed{\frac{u_r}{u_{max}} = 1 - \left(\frac{r}{R}\right)^2}$$

$$\frac{1}{2} = \left(\frac{r}{R}\right)^2$$

$$r = \frac{R}{\sqrt{2}}$$

Key class work

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4.) B
5) B (3, 4 & 5) correct
6) C
7) C
10) C
11) a
12) C
13) B
9) C
8) d
10.)

1) $Q = A \cdot V$

$= \cancel{\frac{\pi \times d (4)^2}{4} 1.5} = \frac{\pi}{4} \times (d)^2 \times V$

For pipe flow

V_{max} = centre line velocity

Max vel = 1.5 m/s

$V_{max} = 2V$

$V = \frac{V_{max}}{2} = \underline{0.75 \text{ m/s}}$

$Q = \frac{\pi}{4} \times (4 \times 10^{-2})^2 \times 0.75$

$= \underline{\frac{3\pi}{10000} \text{ m}^3/\text{s}}$

2) → FRICTION FACTOR

For Laminar flow

$$\boxed{f = \frac{64}{Re}}$$

$Re = \frac{\rho V D}{\mu}$ ~~or~~

For Laminar

~~to~~ take $Re = 2000$

$f = \frac{64}{2000} = \underline{0.032}$

ISRO

Q8) ~~do~~ $D = 100$ mm

$l = 10$ m

$V = 5$ m/s

$\tau_{wall} = 250 \text{ N/m}^2$

$$\tau_r = \frac{-dp}{dx} \times \frac{r}{2}$$

$$\tau_{wall} = -\left(\frac{dp}{dx}\right) \cdot \frac{R}{2}$$

$dx = 10$

$$250 = -\left[\frac{dp}{dx}\right] \times \frac{50 \times 10^{-3}}{2}$$

$$250 = \frac{-dp}{10} \times \frac{50 \times 10^{-3}}{2}$$

$$\underline{\underline{dp = 1 \times 10^5 \ N/m^2}}$$

9.) $$\frac{dp}{dx} = \frac{N}{m^2 \times m} = \cancel{kgF} \ \frac{kg\,m}{s^2 \times m^3}$$

$$= \frac{ML}{T^2 \times L^3}$$

$$= \underline{\underline{ML^{-2}T^{-2}}}$$ (C)

10.) $\rho = 1000 \ kg/m^3$

$Q = 800 \ mm^3/s$

$L = 2m = 2000 \ mm$

$D = 0.5 \ mm$

$\mu = 3$

$$\frac{dp}{dx} = 2 \ MPa$$

$$\cancel{\tau = \mu \frac{du}{dy}} \qquad dp = \frac{16 \mu \cdot u_{max} \, l}{D^2}$$

$u_{max} = 2 \times V$ for pipe flow

$Q = A \times V$

$$V = \frac{800 \times 4}{\pi \times (0.5)^2} = 4074.36\,mm/s = \underline{\underline{4.07\,m/s}}$$

$$dP = \frac{32 \mu V L}{D^2}$$

$$\cancel{Q = 16 \times \mu \times}$$

$$\frac{2}{10^{-6}} = \frac{32 \times \mu \times 4.07 \times 2}{(0.5 \times 10^{-3})^2}$$

$$\frac{N}{mm^2} \rightarrow \frac{N}{10^{-6} m^2}$$

$$\mu = \frac{\frac{2}{10^{-6}} \times (0.5 \times 10^{-3})^2}{32 \times 4.07 \times 2}$$

$$= \underline{\underline{0.001919 \frac{Ns}{m^2}}}$$

(c)

(11)

$$V = r\omega$$

$$\cancel{V = h\,\omega_y}$$

$$\cancel{\omega_y = \frac{V}{h}} \qquad \cancel{\omega_z = 0} \quad \{\cancel{2\text{-D flow}}\}$$

→ SHEAR VELOCITY ←

$$V_* = \sqrt{\tau_0 / \rho} = V \sqrt{(F/8)}$$

SHEAR STRESS VARIATION

$$\tau = \tau_0 \frac{r}{R}$$

$$\boxed{\tau_0 = \frac{8 \mu V}{D}}$$

→ Pressure drop or Head loss due to viscosity

$$h_L = \frac{32 \mu V L}{\rho g D^2}$$

Hagen Poiselues eqn

→ Darcy's Head loss :-

$$(h_f) = \frac{4 f l v^2}{2 g D}$$

$$\frac{4 f l v^2}{2 g D} = \frac{32 \mu V \cdot L}{\rho g \cdot D^2}$$

$$4F = \frac{32 \cdot \mu V \cdot l}{\rho \cdot g \cdot D^2} \cdot \frac{2 g D}{l v^2}$$

Darcy's friction factor

$$4f = \frac{64 \mu}{\rho V D}$$

$$4F = \frac{64}{\frac{\rho V D}{\mu}}$$

$$\boxed{4f = \frac{64}{Re}}$$

Q.) A 20 cm dia pipe 20 km long laid horizontally carries oil $(s = 0.9)$ $\mu = 0.8$ poise. at the rate of 100 lit/sec. calculate the power reqd to run the pump in H.P to maintain flow against viscosity ?

Ans: $P = \dot{m} g \cdot h_L$ or $\dot{m} g h_f$ $\qquad h_f = \frac{4 f l v^2}{2 g d}$

$$h_L = \frac{32 \mu V L}{\rho \cdot g \cdot D^2}$$

$\cancel{32 \times 0.08 \times}$

$Q = A \times V_{avg}$

$10 \times 10^{-3} = \frac{\pi}{4} \times (0.2)^2 \times V$

$V = \underline{\underline{0.318 \ m/sec}}$

$\mu = \frac{0.8}{10} \frac{Ns}{m^2}$

$S = 0.9$

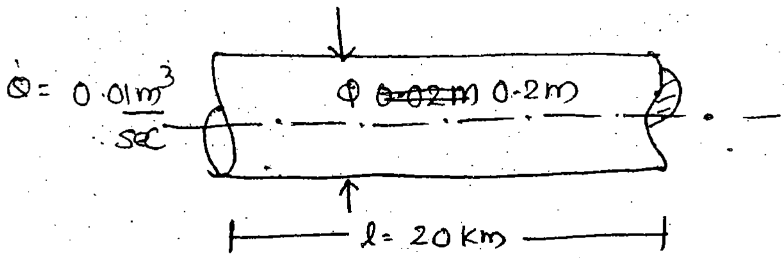
$\rho = 0.9 \times 1000 = 900 \ kg/m^3$

oil

$L = 20 \times 1000 \ m$

$D = 0.2 m$

$Q = 10 \frac{lit}{sec} = 10 \times 10^{-3} \frac{m^3}{sec}$



1 HP = 0.746 kW

$$h_L = \frac{32 \times 0.08 \times 0.318 \times 20000}{900 \times 9.81 \times (0.2)^2}$$

$= \underline{\underline{45.87 \ m}}$

→ <u>Power lost</u>

$P = \dot{m} g \cdot h_L$

$= 9 \times 9.81 \times 45.87 = \underline{\underline{4049 \ W}}$

$= \underline{\underline{4 \ kW}}$

$\frac{4}{0.746} = \underline{\underline{5.5 \ HP}}$

$\dot{m} = \rho \times Q$

$= 900 \times 10 \times 10^{-3}$

$= \underline{9}$

Classwork

(3.) $D = 0.2\,m$

$u_{max} = 1\,m/s$

$u_{r=5} = ?$

$$u_r = \frac{1}{4\mu}\left[\frac{-dp}{dx}\right]\left[R^2 - r^2\right]$$

$$u_{max} = \frac{1}{4\mu}\left[\frac{-dp}{dx}\right] R^2$$

$$\frac{u_r}{u_{max}} = \frac{R^2 - r^2}{R^2} = 1 - \left(\frac{r}{R}\right)^2$$

$$= 1 - \left[\frac{5\times10^{-2}}{0.2}\right]^2$$

$$\frac{u_r}{1} = 1 - \left(\frac{5\times10^{-2}}{0.2}\right)^2$$

$$u_r = \underline{\qquad}$$

(Q.) A 400 m long horizontal pipe is to deliver 900 kg of oil ($s = 0.9$, $\nu = 2$ stokes) per min. if head loss is not to exceed 8m of oil find the dia of pipe ? [Friction factor $f = \frac{64}{Re}$]

Ans: $l = 400\,m$

$$\dot{m} = 900\,\frac{kg}{min} = \frac{900}{60}\,\frac{kg}{s} = \underline{\underline{15\,\frac{kg}{s}}}$$

$S = 0.9$

$\nu = 2\times10^{-4}\,\frac{m^2}{s}$

$\boxed{STOKE = \frac{cm^2}{s}}$

$\rho_{oil} = 900\,\frac{kg}{m^3}$

$$h_f = \frac{4FLV^2}{2gD}$$

$$4F = \frac{64}{Re}$$

$$Re = \frac{\rho VD}{\mu} \text{ or } \frac{VD}{\nu}$$

$$Re = \frac{V \times D}{2 \times 10^{-4}}$$

$$\underline{Re = 0.5 \times 10^4 \times V \times D}$$

$V = ?$

$\dot{m} = \rho \cdot A \cdot V$

$$\boxed{15 = 900 \times \frac{\pi}{4} \times (d^2) \cdot V \qquad V = \frac{60}{900 \times \pi d^2}}$$

$$8 = \frac{64}{0.5 \times 10^4 \times VD} \times \frac{400 \times V^{\cancel{2}}}{2 \times 9.81 \times D}$$

$$8 = \cancel{64} \times \frac{V}{D^2} \quad — ②$$

$\dot{m} = \rho \cdot Q$

$$Q = \frac{15}{900} = 0.016$$

$$0.016 = \frac{\pi}{4} \times D^2 \times V$$

$$VD^2 = \frac{0.016 \times 4}{\pi} \quad — ①$$

$=$

Solve 1 and 2

$\therefore \quad D = \underline{0.162\,m}$

5 Q) oil is being pumped through a straight line pipe. the pipe length, diameter and volumetric flow Rate are all doubled, for the new arrangement of pipe. the pipe friction however remains constant. what is the Ratio of frictional losses in the new arrangement to that in original pipe configuration.

a) 1/4 b) 1/2 c) 2 d) 4.

$Q = A \cdot V$

$\frac{\pi}{4} \times$

$L \rightarrow 2L$

$D \rightarrow 2D$

$Q \rightarrow 2Q$

$$\frac{h_{f_2}}{h_{f_1}} = \frac{\frac{\cancel{4F}LV_1^2}{\cancel{2g}D}}{\frac{\cancel{4F} \times 2L \times V_2^2}{\cancel{2g} 2D}}$$

$$\frac{hf_2}{hf_1} = \frac{\cancel{f} \times \frac{4\cancel{Q}}{D^2} \times \frac{1}{\cancel{D}}}{2\cancel{f} \times \frac{\cancel{8}Q}{\cancel{4}D^2} \times \frac{1}{2\cancel{D}}}$$

$Q = \frac{\pi}{4} \times D^2 \times V$

VOF

$D_2 = 2D$

$2Q = \frac{\pi}{4} \times 4D^2 \times V$

$$\frac{hf_2}{hf_1} = \boxed{\frac{1}{4}}$$

discharge doubled

Q) Two pipes of uniform c/s but different diameters carries water at same volumetric flow Rate. water properties are same in two pipes. The Reynolds no Based on pipe dia. ~~is same in both the pipes~~

a) same in both the pipes b) Larger in Narrow pipe

c) smaller in narrow pipe d) depends on pipe material.

$$Re = \frac{\rho V D}{\mu} \text{ or } \frac{VD}{\vartheta}$$

$$Re_1 = \frac{V D_1}{\vartheta_1} \quad \frac{\left(\frac{Q}{A}\right) \cdot D_1}{\vartheta_1}$$

$$Re_1 = \frac{Q \cdot D_1}{\frac{\pi}{4} \times D_1^2 \times \vartheta_1} = \frac{Q_1}{\frac{\pi}{4} \times D_1 \cdot \vartheta_1}$$

$$Re \propto \frac{1}{D}$$

so Larger pipes have small Re

(b.)

4) A farmer uses a long horizontal pipe line to transport water with 126
1HP pump and it discharges Q. If farmer replaces 1HP motor pump by 5HP water pump for the same pipe line and assuming the friction factor is unchanged then discharge through new Arrangement will be

a) 5Q b) $\sqrt{5Q}$ c) $\sqrt{5}\cdot Q$ d) $\sqrt[3]{5}\cdot Q$

Ans:

$P = \dot{m} g h_f$ $\quad\quad \dot{m} = \rho \cdot Q_1$

$$\frac{P_1}{P_2} = \frac{h_{f1}}{h_{f2}} \cdot \frac{Q_1}{Q_2}$$

$$\frac{1}{5} = \frac{\frac{4fLV_1^2}{\cancel{2gD}}}{\frac{4fLV_2^2}{\cancel{2gD}}} = \frac{V_1^2}{V_2^2}$$

$Q_1 = A \times V_1$

$$\frac{1}{5} = \frac{\left[\frac{Q_1}{A}\right]^2}{\left[\frac{Q_2}{A}\right]^2} \cdot \frac{Q_1}{Q_2}$$

$$\cancel{\frac{1}{5} = \frac{Q_1^2}{Q_2^2}} \quad\quad \frac{1}{5} = \frac{Q_1^3}{Q_2^3}$$

$Q_2^3 = 5Q_1^3$

$\cancel{Q_2^2 = 5 \cdot Q_1^2}$

$\cancel{\boxed{Q_2 = \sqrt{5} \cdot Q}}$ $\quad\quad$ ~~c~~ $\boxed{Q_2 = \sqrt[3]{5} \cdot Q_1}$

d

5. $\frac{h_{f2}}{h_{f1}} = 3$

$$h_f = \frac{4fLV^2}{2gD}$$

$$h_{f1} = \frac{4fLV^2}{2gD_1}$$

New Arrangement

$L_2 = 2L$
$D_2 = 2D$
$Q = 2Q$

$$h_{f2} = 4f \times 2L \times \left[\frac{Q_2}{A_2}\right]^2$$

$$h_{f2} = \frac{4 \times f \times 2L \times (2Q_1)^2}{2 \times g \times 2D \times \left[\frac{\pi}{4} \times\right]^2 \times \cancel{4}D^4 \times 16}$$

$$= \frac{4f \times 2L \times 4Q_1^2}{2 \times g \times 2 \times \left[\frac{\pi}{4}\right]^2 \times \cancel{4}D^5 \times 16}$$

$$\frac{4f \, 2L \times}{2 \times 2D} \left[\frac{2Q}{\frac{\pi}{4} \times}\right]$$

$$\frac{h_{f2}}{h_{f1}} \Rightarrow \frac{\cancel{4f} \times \cancel{2L} \times 4Q_1^2}{16 \times \cancel{2 \times g \times 2 \times \left[\frac{\pi}{4}\right]^2 \times D^5}} \times \frac{\cancel{2 \times g \times 2D \times \left[\frac{\pi}{4}\right]^2 \times D^4}}{\cancel{4 \times f \times L} \times Q^2 \times 16}$$

$$\frac{4Q_1^2}{16Q^2} = \cancel{4Q} = \frac{1}{4}$$

New Arrangement

$$V = \frac{2Q}{\frac{\pi}{4} \times (2D)^2} = \frac{2Q}{\frac{\pi}{4} \times 4D^2}$$

$$h_{f2} = \frac{4 \times f \times 2L \times (2Q)^2}{2 \times g \times 2D \times \left[\frac{\pi}{4}\right]^2 \times (4D^2)^2}$$

$$= \frac{4 \times f \times 2L \times 4Q^2}{2 \times g \times 2D \times \left[\frac{\pi}{4}\right]^2 \times 16D^4}$$

6.1 Introduction

6.2 Types of Losses in the pipe flow (Major losses & Minor Losses)

6.3 Power Transmitted by a pipe to carry the Water (Condition for Maximum power transmission and Max Efficiency)

6.4 Pipe flow Network (Flow in series and Parallel pipe combinations)

A Flow is said to be Turbulent if the following is observed.

1. Flow is fully developed.
2. Flow is in Zig-Zag Manner
3. Fluid molecules never travels in their mean path like Laminar flow
4. Turbulency is involved.
5. Viscosity of flow is increased due to turbulency (Eddy Viscosity)
6. Reynolds No is more than 4000

For every attempt of water flow in a pipe is Bound to be turbulent flow for the following :-

$$Re_{PIPE} = \frac{VD}{\nu}$$

$$Re_{PIPE\ WATER} = \frac{V \cdot D}{\nu_{WATER}} = \frac{V \cdot D}{1 \times 10^{-6}}$$

$$= VD \times 10^{6}$$

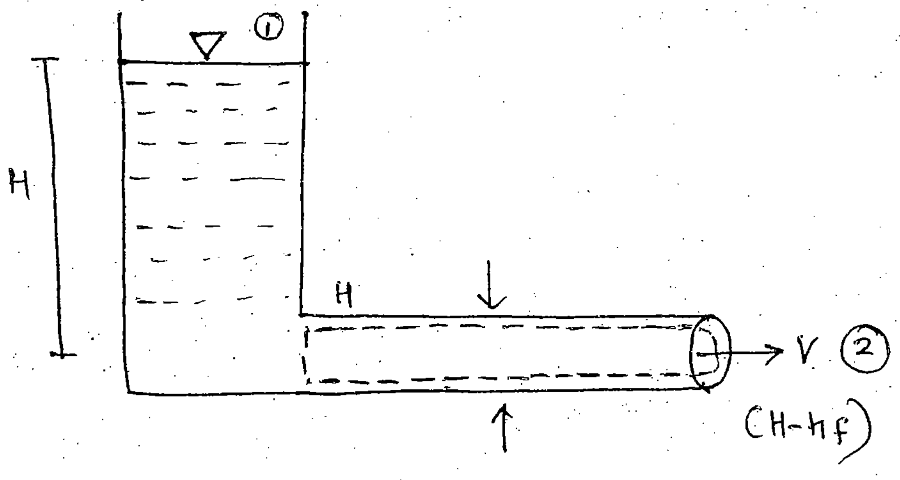
0.1 m/s   0.1 m

$$= \underline{\underline{10000}}$$

From the above it is observed that water Flow through pipe line is always a Turbulent flow. Hence a detailed

Analysis for water flow is Required.

→ 6.2 <u>Power transmitted by a pipe line by carrying fluid</u> :-



$$P_{o/p} = \dot{m} g (H - h_f)$$

$$\rho Q g (H - h_f)$$

$$\rho g \cdot A \cdot V (H - h_f)$$

$$\rho g A (H \cdot V - h_f V)$$

$$\rho g A \left(H \cdot V - \frac{4 f L V^2}{2 g D} \cdot V\right) = \rho g A \left(H \cdot V - \frac{4 f L V^3}{2 g D}\right)$$

→ To Maximise power Transmission through pipes

$$\frac{d P_{o/p}}{dv} = \rho g A \left[H \cdot 1 - \frac{4 f L 3 V^2}{2 g D}\right]$$

$$= \rho g A \left[H - 3 h_f\right]$$

For Maximum Power transmission

$$\frac{d P_{o/p}}{dv} = 0 \quad \Rightarrow$$

$$H - 3h_f = 0$$

$$3h_f = H$$

$$\boxed{h_f = \frac{1}{3} H}$$

$$\eta_{PIPE} = \frac{P_o/p}{P_I/p} = \frac{\dot{m} g (H - h_f)}{\dot{m} g H}$$

$$\eta = \frac{H - h_f}{H}$$

$$\eta_{PIPE} = 1 - \frac{h_f}{H}$$

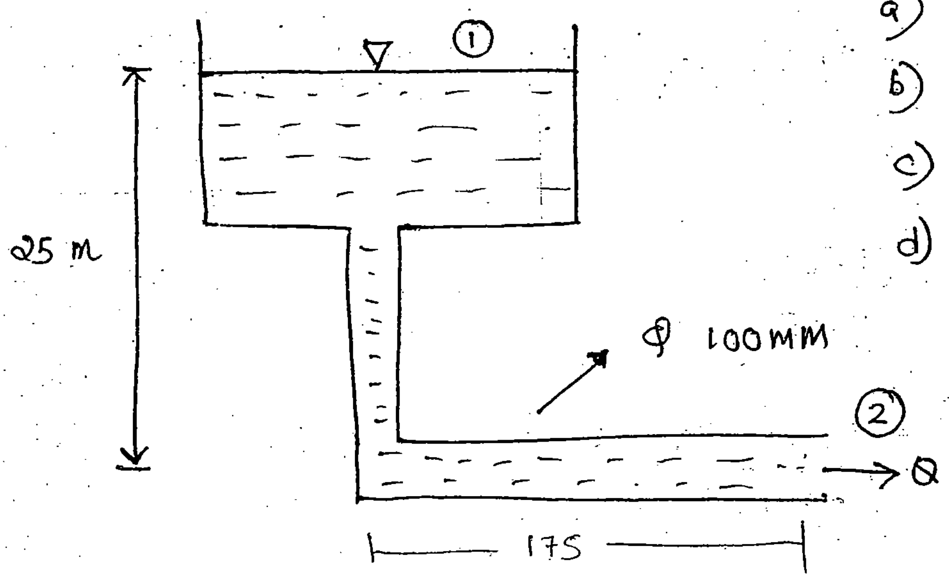
$$= 1 - \frac{\frac{1}{3}\cancel{H}}{\cancel{H}}$$

$$\frac{3-1}{3} = \frac{2}{3} = 0.66$$

$$\boxed{\eta_{PIPE} = 66\%}$$

CARNOT PIPE

Q.) A PIPE ~~protection~~ Production system supplies water to a bend pipe shown in the figure. The pipe minor losses neglected and consider friction factor 0.03. Estimate maximum discharge Q at the end of the pipe.



a) Q = 31.7 lts

b) Q = 24 lps

c) 15.9 lps

d) Q = 12 lps

Ans: $4f = 0.03$ $\qquad Q_2 = A_2 V_2$

Apply Bernoulis eqn

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 - h_f = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$0 + 0 + 25 - h_f = 0 + \frac{V_2^2}{2g} + 0$$

$$25 - \frac{4 f l V_2^2}{2gD} = \frac{V_2^2}{2g}$$

$$25 - \frac{0.03 \times 175 \times V_2^2}{2 \times 9.81 \times 0.1} = \frac{V_2^2}{2 \times 9.81}$$

$$\underline{V_2 = 3.028 \text{ m/s}}$$

$$Q_2 = \frac{\pi}{4} \times (0.1)^2 \times 3.028 = \underline{0.024 \text{ m}^3/\text{sec}}$$

Q. Water is to be supplied to a college campus through one Single main pipe.

1. Distance b/w Reserviour and college campus is 3km
2. Loss of Head due to friction 18m
3. Coeff of friction for the pipe flow = 0.007
4. consumption of water per student per day = 180 lit
5. Daily pump operation is 8hrs at ~~50%~~ 50% capacity.

No of students = 4000

The dia of the main pipe will be a

a) 445mm b) 345 mm c) 245 mm d) 145 mm ✓

Ans: F = 0.007

$$h_f = \frac{4FLV^2}{2gD} \qquad D = ? \qquad h_f = 18$$

$$18 = \frac{4 \times 0.007 \times 3000 \times V^2}{2 \times 9.81 \times D}$$

$$18 = \frac{4 \times 0.007 \times 3000 \times Q^2}{2 \times 9.81 \times \left[\frac{\pi}{4}\right]^2 D^5}$$

18 =

$$V = \left[\frac{Q}{A}\right]^2$$

$$\frac{Q}{\frac{\pi}{4} \times D^2}$$

$$V^2 = \frac{Q^2}{\left(\frac{\pi}{4}\right)^2 D^2}$$

$$Q = \frac{180 \text{ lit}}{\text{student in a day}}$$

$Q = \frac{180 \text{ lit}}{\text{Student - 8hrs}} \times 50\%$

$$\frac{180 \text{ lit}}{\cancel{\text{student}}\ 8 \times 3600 \text{ S}} \times 0.5 \times 4000 \cancel{\text{students}}$$

$$= \frac{180 \times 10^{-3} (m^3) \times 0.5 \times 4000}{8 \times 3600 \text{ S}}$$

$$= 0.0125 \frac{m^3}{sec}$$

$$18 = \frac{4 \times 0.007 \times 3000 \times \cancel{0.000125}\ (0.0125)^2}{2 \times 9.81 \times \left[\frac{\pi}{4}\right]^2 \times D^5}$$

$D^5 = $

$D = 0.142$ m say 145 mm.

Q.) Water is Being pumped at the Rate of 1200 lit/mts to an overhead Tank through 150mm dia pipe of length 300m. (out of which 15m above the pump centre line). Darcy's friction factor is 0.03. what is the

2 MARKS

ⓐ ~~pr~~ pressure developed by pump.

a) 1.85 bar b) 2.85 bar c) 3.65 bar d) None of these

ⓑ what is the Power Reqd to Run the pump in kw ?

Ans:

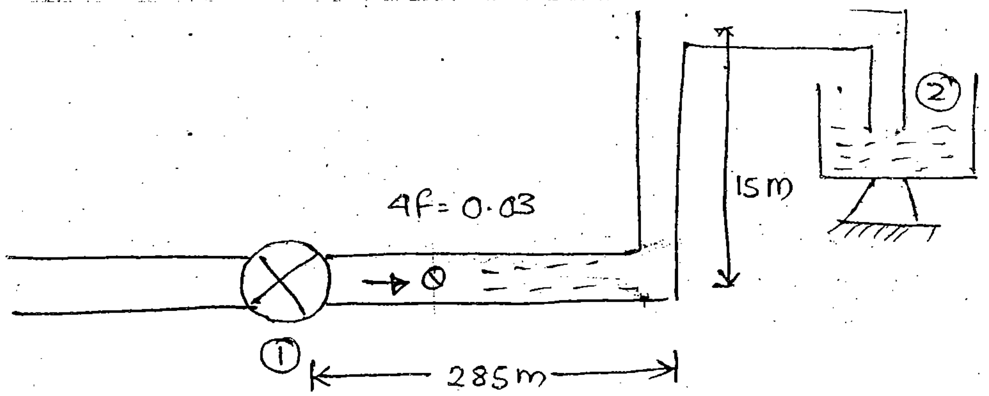
$Q = 1200 \frac{lit}{mt}$

$= \frac{1200 \times 10^{-3}}{60} \frac{m^3}{s}$

Darcy friction factor

$4f = 0.03$

$D = 150$ mm



Ans $Q = A \cdot V$

$$0.02 = \frac{\pi}{4} \times (0.15)^2 \cdot V$$

$$\underline{V = 1.137 \text{ m/sec}}$$

$$\boxed{\begin{aligned} Q &= 1200 \text{ lpm} \\ &= \frac{1200 \times 10^{-3}}{60} \\ Q &= 0.02 \text{ m}^3/\text{sec} \end{aligned}}$$

Apply B.E (1) and (2)

$$\frac{P_1}{\rho g} + z_1 + \cancel{\frac{V_1^2}{2g}} - h_L = \frac{P_2}{\rho g} + z_2 + \cancel{\frac{V_2^2}{2g}}$$

$$\frac{P_1}{\rho g} + 0 + 0 - \frac{4fLV^2}{2gD} = 0 + 15$$

$$\frac{P_1}{1000 \times 9.81} - \frac{0.03 \times 300 \times (1.137)^2}{2 \times 9.81 \times 0.15}$$

$$\underline{P_1 = 1.85 \times 10^5 \text{ N/m}^2}$$

$$\underline{P_1 = 1.85 \text{ bar}}$$

$$h_f = \frac{4FLV^2}{2gD} = \frac{0.03 \times 300 \times (1.137)^2}{2 \times 9.81 \times 0.15} = \underline{3.9}$$

Power Required to Run the pump

$$= \dot{m} g (H + h_f)$$

$$= \rho g Q (15 + 3.9)$$

$$= \quad \text{watt}$$

Class Work

(1) $\nu = 0.4\ cm^2/sec$

$\nu = 0.4 \times 10^{-4}\ m^2/sec$

$D = 8cm = 0.08m$

for pipe Flow $V_{max} = 2V_{avg}$

$$V_{avg} = \frac{V_{max}}{2}$$

$$Re = \frac{\rho V D}{\mu} \text{ or } \frac{VD}{\nu}$$

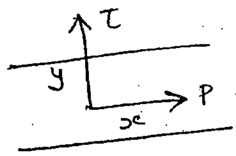
$2000 =$

* ~~Fu~~ → **LAMINAR FLOW CONTD.** ← | SOME XTRA POINTS |

* Navier stoke eqn:-

$$\left(\frac{\partial p}{\partial x}\right) = \left(\frac{\partial \tau}{\partial y}\right)$$

It means Pressure Gradient in the direction of flow is equal to shear Gradient in the direction normal to the flow.



Velocity at a section

$$V = +\frac{1}{4\mu}\left[\frac{-dP}{dx}\right]\left[R^2 - r^2\right]$$

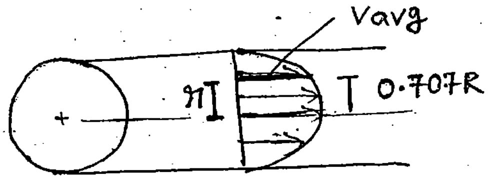
$$V_{max} = \frac{1}{4\mu}\left[\frac{-dP}{dx}\right]R^2$$

$$\boxed{\frac{V}{V_{max}} = \left[1 - \left(\frac{r}{R}\right)^2\right]}$$

V = local velocity at a point whose Radius is 'r'.

Vmax = centre line velocity which is maximum.

Since vel distribution is parabolic $V \propto r^2$



velocity is Average : (Vmean) at a distance $= \frac{R}{\sqrt{2}} = 0.707R$ from the centre of

$$V_{avg} = U_{max}\left[1 - \left(\frac{R}{\sqrt{2}R}\right)^2\right]$$

$$= V_{max}\left[1 - \frac{1}{2}\right]$$

$$\boxed{V_{avg} = \frac{V_{max}}{2}}$$

Shear stress distribution :—

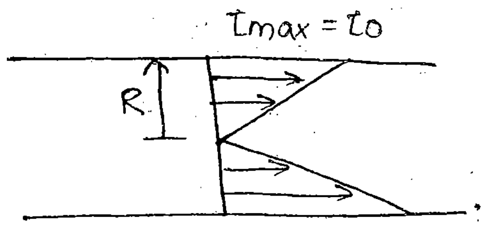
Assertion :— vel is Maximum at centre, zero at the at Boundary

Reason: ...

the Boundary

$$\boxed{\text{Velocity} \propto \frac{1}{\text{Shear stress}}}$$

$$\boxed{\tau_0 = \tau_{max} = -\frac{dp}{dx}\left(\frac{R}{2}\right)}$$



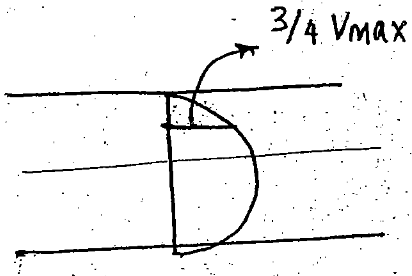
$\frac{\partial p}{\partial x}$ means change in pressure in a pipe of length dx or simply pressure Gradient

$$V = \frac{1}{4\mu}\left[-\frac{\partial p}{\partial x}\right]\left[R^2 - r^2\right]$$

If $r = 0$, $V = V_{max}$

so $$\boxed{V_{max} = \frac{1}{4\mu}\left[-\frac{\partial p}{\partial x}\right] R^2}$$

→ Velocity at Quarter Point

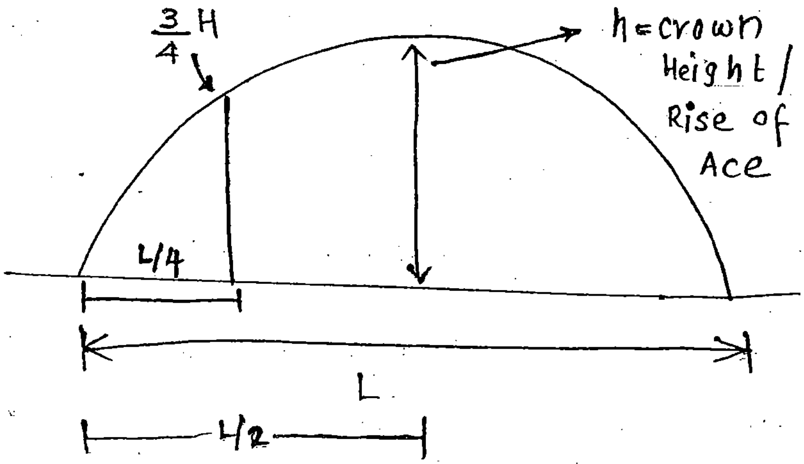


$$r = \frac{R}{2}$$

$$V = \frac{1}{4\mu}\left[-\frac{\partial p}{\partial x}\right]\left[R^2 - \frac{R^2}{4}\right]$$

$$= \frac{1}{4\mu}\left[-\frac{\partial p}{\partial x}\right]\left[\frac{3R^2}{4}\right]$$

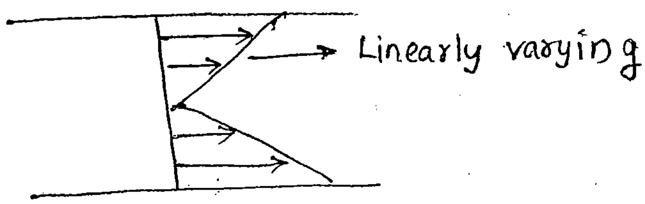
$$\boxed{V = V_{max} \times \frac{3}{4}}$$



→ VARIATION OF SHEAR STRESS

$$\boxed{\frac{\tau}{r} = \frac{\tau_{max}}{R}}$$

$$\tau \propto r$$



Shear stress varies linearly with distance from centre of pipe.

$$\boxed{\tau_0 = \tau_{max} = \underbrace{\frac{8\mu V}{D}}_{\text{Laminar}} = \underbrace{\frac{\rho f V^2}{8}}_{\text{Turbulent flow}}}$$

* SHEAR VELOCITY (V_*)

$$\boxed{V_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{8\mu V}{D\rho}} = \sqrt{\frac{8\nu V}{D}}}$$ LAMINAR FLOW.

$$\boxed{V_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{\rho f V^2}{8\rho}} = V\sqrt{\frac{f}{8}}}$$ TURBULENT FLOW

Significance

Shear velocity is a fictious parameter. shear vel is called so as it uses shear stress in ~~to~~ its ~~direction~~ derivation & it has units same as that of velocity [Mathematical significance]

→ **Physical significance :-**

Shear velocity is used in classfying Turbulent flow into 3 types.

1. Hydrodynamically Smooth
2. " Transition
3. " Rough

- Shear velocity is used in Analysis of Boundary Layer concepts.

For eg: Laminar Sub Layer.

1cc → 16

→ **Relation b/w Friction factor & Reynold No :**

$$h_f = \underbrace{\frac{4 f L V^2}{2 g D}}_{\text{TURBULENT}} = \underbrace{\frac{32 \mu V L}{\rho g D^2}}_{\text{LAMINAR}}$$

Equate Head loss due to friction of laminar and Turbulent flow

$$h_f = \frac{4 f l v^2}{2gD} = \frac{32 \mu V L}{\rho g D^2}$$

$$4 F \times \frac{v}{2} = \frac{32 \mu}{\rho D}$$

$$\frac{\rho V D}{\mu} = \frac{64}{4F}$$

$$\boxed{4F = \frac{64}{Re}}$$

4f = Darcy friction factor

Coeff of friction = f

$$F = \frac{64}{4 Re}$$

$$\boxed{F = \frac{16}{Re}}$$

** In Laminar flow friction factor depends on Reynold NO only. It is independant of Surface ~~str~~ Roughness

* In Laminar Flow $F \propto \frac{1}{Re}$

The Max possible Re is 2000. it is called critical Re for Laminar flow

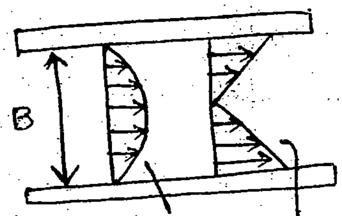
$$\therefore f = \frac{64}{Re} = \frac{64}{2000} = 0.032$$

→ for Flow through circular pipes — Max permissible friction factor = 0.032

→ LAMINAR FLOW B/W PLATES ←

Both plate Fixed

eg: cracks in the wall and Fluid moving (thick)

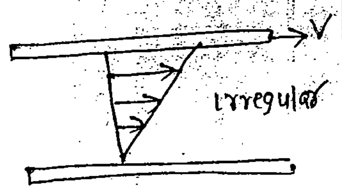


$V_{avg} = \frac{2}{3} V_{max}$

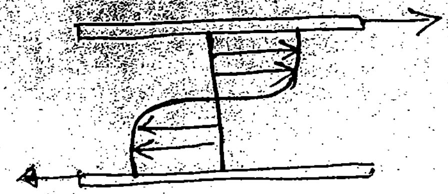
$h_f = \frac{12 \mu V L}{\rho g B^2}$ → Gap b/w

one plate Fixed and other Moving

Couette Flow



Both plate Moving



* Printing Press

$$\boxed{\tau_0 = \tau_{max} = \frac{6\mu V}{D}}$$

Measurement of viscosity

(1) $$\boxed{\tau = -\mu \frac{du}{dy}}$$

Coaxial cylinder viscometer uses the concept of Newtons law of viscosity for measuring viscosity

(2) Hagen Poiselles eqn :-

$$h_f = \frac{32 \mu V L}{\rho g D^2}$$

** Say Bolt viscometer [capillary type viscometer] uses Hagen Poiselles eqn

(3) Stokes eqn :-

$$\boxed{V = \frac{D^2}{18\mu}(\gamma_s - \gamma_f)}$$

V = terminal velocity

γ_s = sp wt of settling particl.

γ_f = sp " " fluid

"Falling sphere viscometer" uses stokes eqn for measuring viscosity

CREEPING MOTION

If Re is < 1 such flow is called Creeping motion.

Q10.) $\frac{dQ}{Q} = 25\%$

$\frac{d(4f)}{4f} = ?$

$$h_f = \frac{4fLv^2}{2gD} = \frac{4f \cdot l \left(\frac{Q}{A}\right)^2}{2 \times g \times D}$$

$$= \frac{4f\,l\,Q^2}{2 \times g \left[\frac{\pi}{4}\right]^2 \cdot D^2}$$

$4fQ^2 = C$

$f' \cdot Q^2 = \text{constant}$

differenciate

$$f' \cdot 2Q \cdot dQ + Q^2 \cdot d(f') = 0$$

$$\frac{df'}{f'} = -\frac{2Q \times dQ}{Q^2}$$

$$\frac{df'}{f'} = -2\left(\frac{dQ}{Q}\right)$$

$$\frac{df}{f} = -2 \times 25\% = \underline{-50\%}$$

Q14

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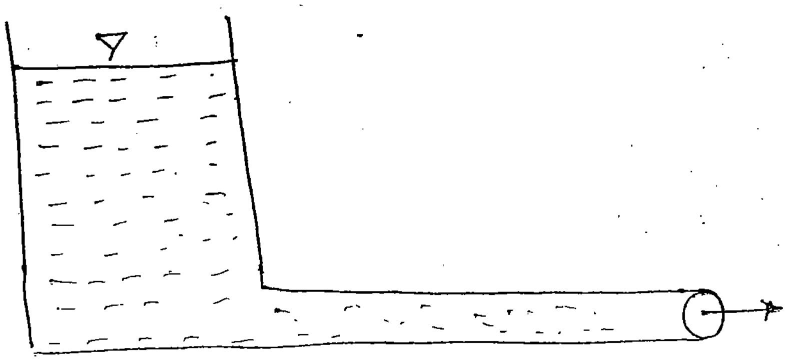


$P_{o/p} = \rho Q (H - h_f)$

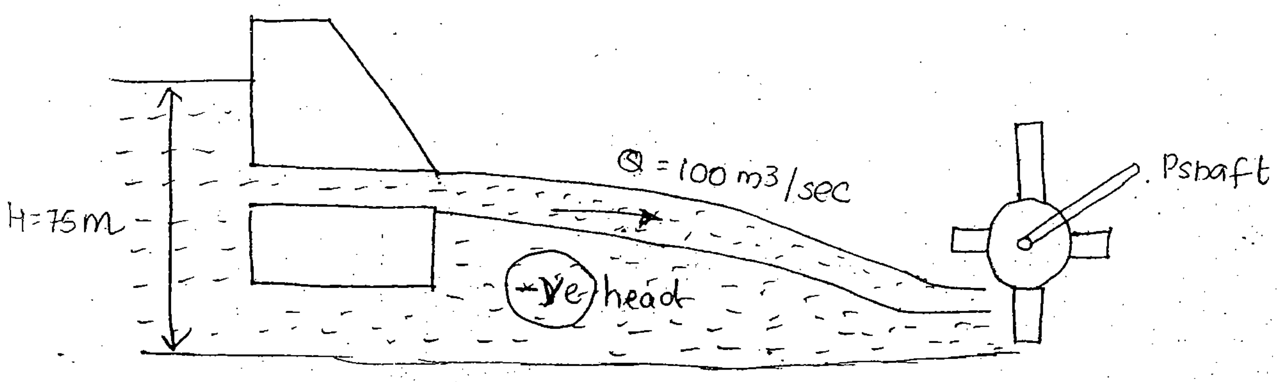
Maximum power ~~developed~~ transmitted by pipe

Condition is $h_f = \frac{H}{2}$

$$P_{o/p} = 1000 \times 9.81 \times 1 \times \left[99 - \frac{99}{3}\right]$$

$$= \underline{\underline{660 \times 10^3 W}} = \underline{\underline{660 kW}}$$

Q15



$$\eta_{Turbine} = \frac{P_{shaft}}{P_{water}} = \frac{P_{shaft}}{\rho g Q (H - h_f)}$$

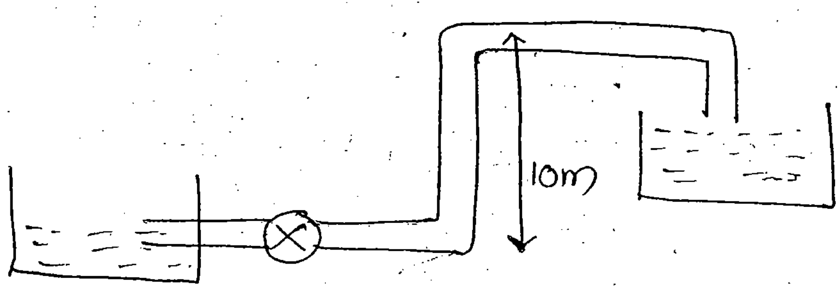
$$100\% = 1 = \frac{(P_{shaft})_{theoritical}}{1000 \times 10 \times 100 (H - 0)}$$

$$(P_{shaft})_{theo} = 1 \times 1000 \times 10 \times 1000 (75 - 0)$$

$$= 75000 \times 10^3 W$$

$$= \underline{\underline{75000 kW}}$$

Q16



$$\eta_{pump} = \frac{P_{water}}{P_{shaft}} = \frac{\rho Q g (H + h_f)}{P_{shaft}}$$

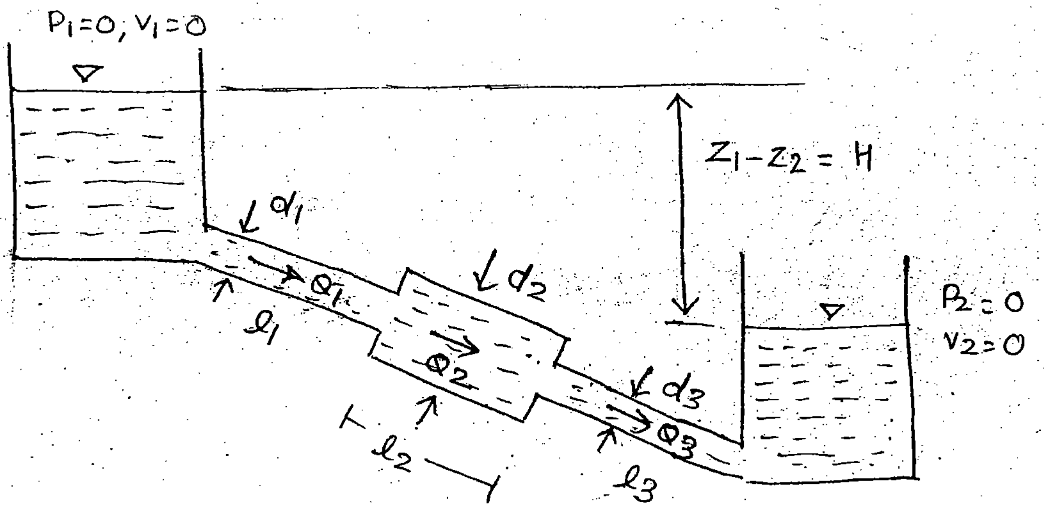
$$100\% = 1 = 1000 \times 9.81 \times 0.1 \times (10+5)$$

$P_{shaft} = 1 \times 1000 \times 9.81 \times 0.1 \times .15 = 14.715 \, kW$

PIPE NETWORK (Neglect Minor Loses)

- Pipe in series
- Pipes in Parallel
- Series and parallel

When No of pipes are connected end to end, discharge in each pipe is same and loss of Head for all the pipe is the summation of the individual ~~price~~ pipe head loses.
Purpose is to carry the Fluid from one plate to another plate without changing discharge.



PIPES IN SERIES

(1) $Q = Q_1 = Q_2 = Q_3$

Same discharge

STEP I

By Continuity eqn,

$$Q_e = A_1 V_1 = Q_2 = A_2 V_2 = A_3 V_3$$

$$d_e^2 . V_e = d_1^2 V_1 = d_2^2 V_2 = d_3^2 V_3$$

STEP-II

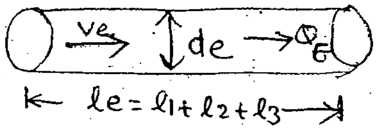
By Bernoulis equation

$$-h_f + \frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$$

$$z_1 - h_f = z_2$$

$$z_1 - z_2 = h_f$$

$$\boxed{H = h_f}$$



equivalent pipe

STEP - III

$$H = h_f$$

$$= h_{f1} + h_{f2} + h_{f3}$$

$$H = h_{fe} = \frac{4f_1 l_1 v_1^2}{2g d_1} + \frac{4f_2 l_2 v_2^2}{2g d_2} + \frac{4f_3 l_2 v_3^2}{2g d_3}$$

$$\frac{4f_e l_e v_e^2}{2g d_e} = \frac{4f_1 l_1 v_1^2}{2g d_1} + \frac{4f_2 l_2 v_2^2}{2g d_2} + \frac{4f_3 l_3 v_3^2}{2g d_3}$$

Assume friction factor is same

$$4f_e = 4f_1 = 4f_2 = 4f_3$$

$$\frac{l_e v_e^2}{d_e} = \frac{l_1 v_1^2}{d_1} + \frac{l_2 v_2^2}{d_2} + \frac{l_3 v_3^2}{d_3}$$

$$v_1 = \frac{d_e^2 . v_e}{d_1^2}$$

$$\frac{d_e^4 . v_e^2}{d_1^4}$$

eliminate velocities v_1, v_2 and v_3 in terms of v_e

$$\boxed{\frac{l_e}{d_e^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5}}$$

Equivalent Pipe (d_e)

Q same

H is same

$d_1 = 100m$ $l_1 = l_2 = l$

$d_2 = 200m$

$$\frac{l_e}{d_e^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} \qquad l_e = l_1 + l_2 = 2l$$

$$\frac{2l}{d_e^5} = \frac{l}{(10)^5} + \frac{l}{(20)^5}$$

$$\frac{2\cancel{l}}{d_e^5} = \cancel{l}\left[\frac{1}{10^5} + \frac{1}{20^5}\right]$$

$$\therefore d_e^5 = \frac{2 \times (10)^5 \times (20)^5}{(20)^5 + (10)^5} = \_\_\_\_\_\_$$

$$d_e = \underline{\underline{11.4}} \text{ cm}$$

(Q2.) $l_A = l_B$

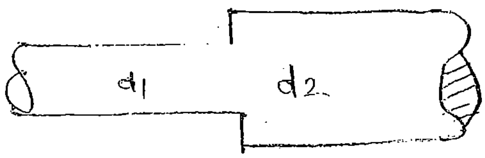
$d_A =$ 20% more than d_B

$d_A = 1.2\, d_B$

$$h_f = \frac{4flv^2}{2gd} = \frac{4fl\left(\frac{Q}{A}\right)^2}{2 \times g \times d}$$

$$h_f = \frac{4fl \cdot Q^2}{2g\left[\frac{\pi}{4}\right]^2 \times d^5}$$

$$\therefore H = h_f = \frac{4fl}{12.1}\frac{Q^2}{d^5}$$



$4f_1 = 4f_2$

$l_1 = l_2$

$d_2 = 1.2 d_1$

~~$h_f d^5$~~

$$h_f \cdot d^5 = \text{const}$$

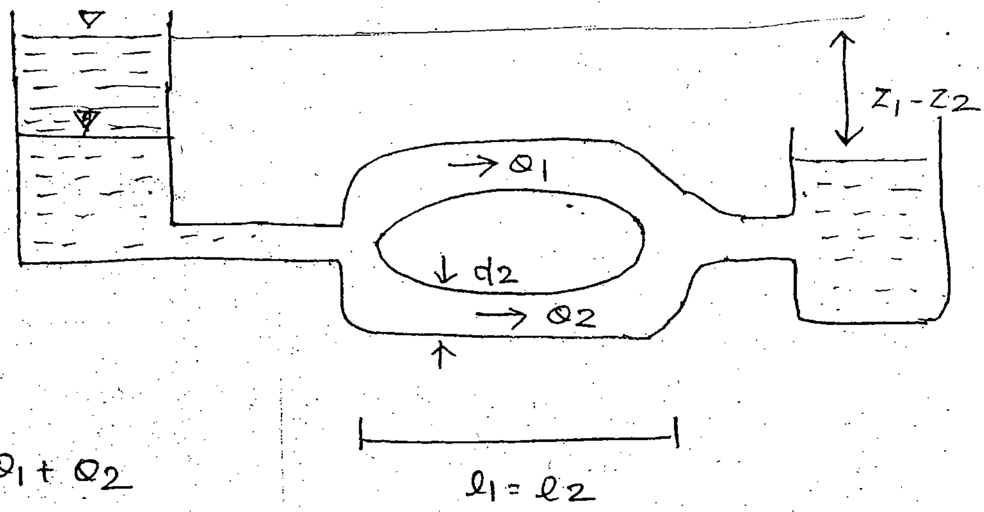
$$\frac{h_{fA}}{h_{fB}} = \left(\frac{d_B}{d_A}\right)^5 = \left(\frac{d_B}{1.2\, d_B}\right)^5$$

$$\frac{h_{fA}}{h_{fB}} = \left(\frac{1}{1.2}\right)^5$$

$$= 0.402$$

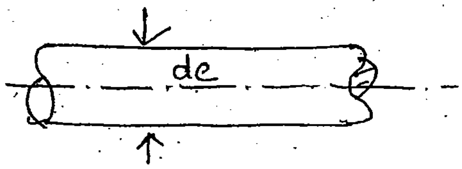
PIPES IN PARALLEL

When No of pipes connected in such a way that the discharge is distributing and head loss is same across each pipe



1.) $Q = Q_1 + Q_2$

$Q_e = A_1V_1 + A_2V_2$



$l_e = l_1 = l_2$

$h_{f_1} = h_{f_2} = h_{f_2}$

$$\frac{4F_e l_e V_e^2}{2gd_e} = \frac{4f_1 l_1 v_1^2}{2gd_1} = \frac{4f_2 l_2 V_2^2}{2gd_2} \ldots$$

$$\frac{l_e v_e^2}{d_e} = \frac{l_1 v_1^2}{d_1} = \frac{l_2 v_2^2}{d_2} : \ldots \quad \text{— (2)}$$

$Q_e = A_1V_1 + A_2V_2$

$d_e^2 \cdot v_e = d_1^2 \cdot v_1 + d_2^2 \cdot v_2 \quad \text{— (1)}$

eliminate v_1, v_2 in terms of v_e

$$V_2 = \frac{d_e^2 V_e}{d_2^2} \qquad V_1 = \frac{d_e^2 V_e}{d_1^2}$$

$$d_e^{5/2} = d_1^{5/2} + d_2^{5/2}$$ conditions friction factor same, length are equal

If $d_1 = d_2 = d$

$$(d_e)^{5/2} = 2(d)^{5/2}$$

$$d_e = (2)^{2/5} \cdot d$$

$$d_e = (2)^{0.4} \cdot d$$

For n pipes of equal dia $\boxed{d_e = (n)^{0.4}\, d}$

Eg: suppose

$x = 25\%$ more discharge

$$Q_e = 1.25 \times Q$$

$$\boxed{d_e = \left(\frac{n}{1.25}\right)^{0.4} \times d}$$

Q3.) $d_A = d_B$

$l_A = l_B$

$4f_A = 4 \times 4f_B$

$\frac{Q_A}{Q_B} \; \cancel{\frac{Q_A}{Q_B}} = ?$

4:00
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for parallel pipes

$$h_{f_A} = h_{f_B}$$

$$\frac{4f_A \cdot \cancel{l_A}\, Q_A^2}{12.1\, D_A^2} = \frac{4f_B \cdot \cancel{l_B} \cdot Q_B^2}{12.1 \cdot D_B^2}$$

$$\left(\frac{Q_A}{\ }\right)^2 = \frac{1}{4}$$

Energy losses in pipes

Major loses (h_f)

This is due to friction.
It can be calculated by

1. Darcy weisbach formula
2. chezy's Formula

Minor loses (h_L)

This is due to

1. Sudden expansion
2. sudden contraction
3. Bend in pipe
4. pipe fittings
5. obstruction in pipe.

Darcy Weisbach Formula

$$\boxed{h_f = \frac{4flv^2}{2gD}}$$

chezy's Formula

$$\boxed{V = C\sqrt{mi}}$$

$$C = \text{chezy's const} = \sqrt{\frac{\rho g}{4f}}$$

i = loss of head per unit length

$$i = \frac{h_f}{L}$$

m = Hydraulic mean depth

$$= \frac{\text{Area}}{\text{Perimeter (wetted)}} = \frac{\frac{\pi}{4} \times D^2}{\pi D} = \frac{D}{4} \quad \text{for pipes}$$

✡ Flow Through pipes:—

It is also called Pressure flow. Because In pipes the pressure is other than atmospheric. it may be positive or negative

$$\text{Reynold No:} \ \frac{\text{Inertia force}}{\text{Viscous force}}$$

Inertia force $= m \times a$

$= \rho \times V \left(\frac{v-u}{t}\right)$

$\rho L^3 \frac{V}{t} = \rho L^2 \times \frac{L}{t} \times V$

$$\boxed{\text{Inertia force} = \rho L^2 V^2}$$

Viscous force $= \tau \times A$

$= \mu \frac{du}{dy} \times A$

$= \mu \times \frac{V}{L} \times L^2$

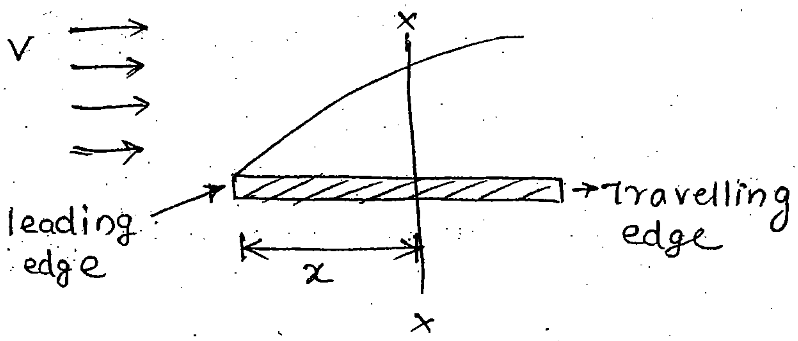
$$\boxed{\text{Viscous Force} = \mu V L}$$

L = characteristic dimension

for pipes it is diameter

$= x$ for flow over the plates

Reynold No $= \frac{\rho L^2 V^2}{\mu V L} = \frac{\rho V D}{\mu}$



L = characteristic dimension

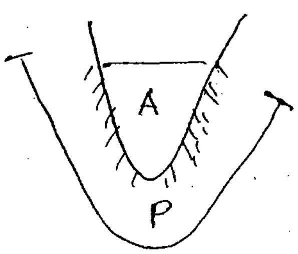
L = 4 Rm

$Re_{(x-x)} = \frac{\rho V x}{\mu} = \frac{V x}{\nu}$ $\left\{ \text{Reynolds No from leading edge} \right\}$

& $\boxed{L = 4 R_m}$ in case of non-circular ~~plates~~ pipes

R_m = hydraulic mean Radius

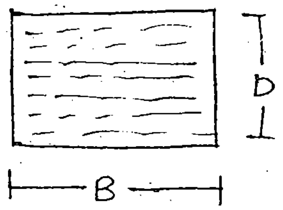
$R_m = \frac{\text{Wetted Area}}{\text{wetted perimeter}} = \frac{A}{P}$



the Boundary length in contact with liquid is wetted perimeter

for pipes $R_m = \frac{\pi/4\, D^2}{\pi D} = \frac{D}{4}$

Q.) A Rectangular duct of width B and height 'D' is running full of water. cal. the Reynolds no: of flow in terms of ρ, V, μ.



Ans: Hydraulic mean Radius $R_m = \frac{A}{P} \Rightarrow \frac{\text{wetted Area}}{\text{wetted Perimeter}}$

$$= \frac{BD}{2(B+D)}$$

$$Re = \frac{\rho VL}{\mu} = \frac{\rho V(4R_m)}{\mu}$$

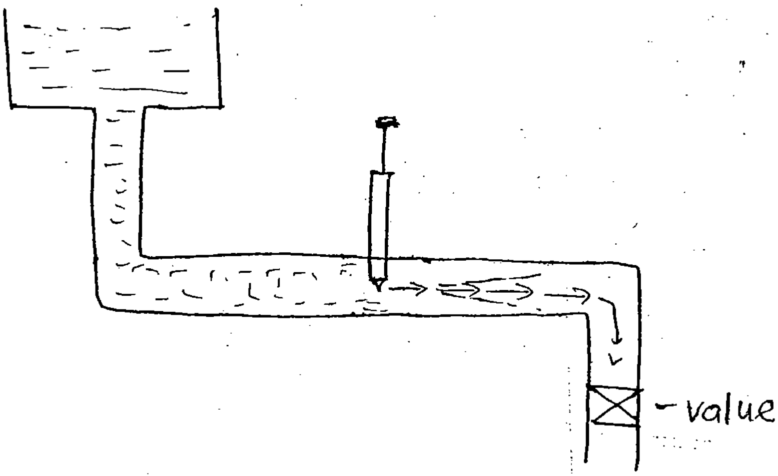
$$= \frac{\rho V 4}{\mu} \cdot \frac{BD}{2(B+D)} = \frac{2\rho_w V \cdot BD}{\mu(B+D)}$$

$$\frac{2000\, VBD}{\mu(B+D)}$$

Reynolds Experiment:-

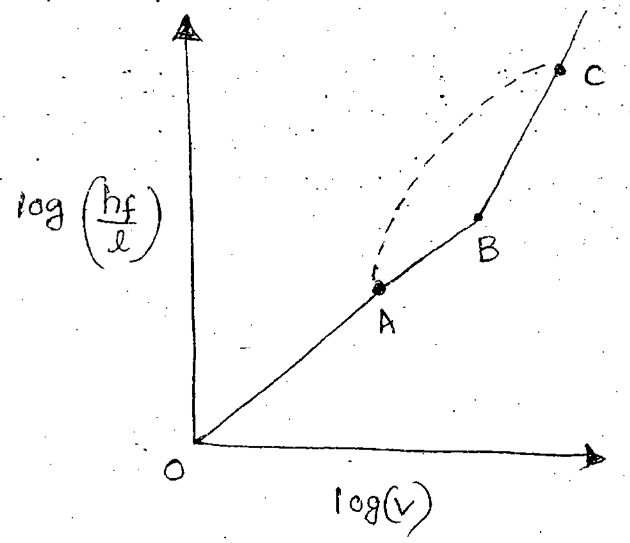
Aniline is used. It has same specific weight as that of water

Layered Motion = laminar
Intermixing = turbulent (zig zag)
Transitional = slightly dispered



Reynold chart:-

log-log plot ✡✡



A = Lower critical point

B = upper critical point

C = Limit of Transition

OA = Laminar flow

AB = Transition

BC = Turbulent

* In Engg purpose Lower critical point is of greater importance

Used for defining Laminar flow

For flow through circular pipe

if $Re \leq 2000$ laminar

$2000 < Re < 4000$ Transitional

$Re \geq 4000$ Turbulent

From Reynold chart

$$\boxed{h_f \propto V^n}$$

$n = 1$ for Laminar flow $h_f \propto V$

$n = 2$ for Turbulent (Rough pipes)

$n = 1.75$ for smooth pipes Turbulent

General Formula:-

$h_f \propto V^n$

$n = 1.75$ to 2 For Turbulent flow

* Minor losses
* They are those loses whoose Contribution is less than 5% of total loses.
* ~~elongation,~~ ductile materials have more than 15% elongation. Brittle materials have less than 5% elongation. If percentage ~~is m~~ of elongation is in b/w 5-15% it is called material of intermediate ductility

N.B :-

Major ~~loss~~ elongation of M.S = 35%

→ Major loss [frictional loss]

$$h_f = \frac{32 \mu V L}{\rho g D^2}$$ [LAMINAR FLOW THROUGH O/r PIPES]

* In Laminar flow viscous friction (μ) is the Reason for Friction loss.

$$h_f = \frac{128 \mu Q L}{\gamma \pi D^4}$$ $\left\{ V = \frac{4Q}{\pi D^2} \right\}$

$Q = AV$

$\boxed{V = \frac{Q}{A}}$

i) $h_f \propto V^n$, $n=1$ [Laminar]

ii) $h_f \propto \mu$ [hf proportional to viscosity of liquid]

∴ as Temp ↑ μ ↓ ∴ Hence frictional losses ↓.

iii) Frictional losses of laminar flow is independant of nature of surface of inner pipe Boundary.

iv) For a given discharge

$$\boxed{h_f \propto \frac{1}{D^4}}$$

As dia/area of pipe ↑ Frictional loss ↓

v) $h_f \propto L$ is when length ↑ Frictional loss ↑

vi) $\frac{h_f}{L}$ = hydraulic gradient

$$\frac{h_f}{L} = \frac{32 \mu V}{\rho g D^2}$$

$$\frac{\Delta P}{\gamma} = \frac{32 \mu V L}{\rho g D^2}$$

$$\Delta P = \frac{32 \mu V L}{D^2} = \frac{128 \mu Q L}{\pi D^4} = P_2 - P_1$$

$\frac{p}{M^2} \times \frac{M^2}{kg}$

$\frac{kgm}{m s^2} \times \frac{m^2}{kg}$

$\frac{m^2}{s^2}$

V_{n} L.S.L

$$\frac{32\mu VL}{\gamma D^2}$$

$P = \dot{m} g h_f$

$P = \rho Q g h_f$

$$P = \gamma Q\left[h_f\right] = \gamma Q \times \frac{128 \mu Q L}{\gamma \pi D^4}$$

$$= \frac{128 \mu Q^2 L}{\pi D^4} = Q \Delta P$$

$$Q(P_1 - P_2)$$

→ Head loss due to friction in Turbulent flow

$$h_f = \frac{4fLV^2}{2gD}$$ Darcy weisbach eqn

- $F \propto V^n$
- independant of Pressure
- depend on Nature of surfaces in contact.

i) $h_f \propto V^n$ where $n = 2$

actually n varies from 1.75 to 2

$n = 1.75$ smooth pipe

$n = 2$ Rough pipe

ii) h_f depends on Nature of surfaces (f) ie roughness protrusion

With Age the inside boundary of pipe gets collected with dust particles, debris etc - so Roughness ↑. so friction factor increases.

iii) h_f is independant of Temperature because it is independant of viscosity.

iv) $h_f \propto l$.

v) $h_f \propto \frac{1}{D^5}$ As Dia ↑ h_f ↓

vi) Hydraulic Gradient $\frac{H_f}{L} \Rightarrow \frac{4fV^2}{2gD}$

$$\Rightarrow \frac{4fQ^2}{2 \times g \times \frac{\pi^2}{4} D^5} \Rightarrow \frac{8fQ^2}{\pi^2 D^5 g}$$

UNIT-3

FLUID KINEMATICS

2 MARKS

3.1 Introduction
3.2 Classification of fluid flows
3.3 Continuity eqn:
3.4 Fluid particle Motion in space coordinate (cartesian coordinates)
3.5 Equation of fluid motion (stream line eqn, velocity and acceleration of fluid)
3.6 Fluid ~~Flow~~ field functions [stream fn and vel potential fn]
3.7 Angular speed of components of fluid flow in space (vorticity, discharge b/w 2 points, shear strain Rate of fluid flow)

Introduction:-

Fluid kinematics is a branch of fluid science which deals with motion of fluid in space which considering forces causing motion. Hence fluid kinematics is restricted to study of displacement, velocity, acceleration, Mass flow Rates and discharge in given space.

Types of fluid flows:-

1. Steady and unsteady flow
2. Uniform and Non uniform flow
3. laminar and Turbulent flow
4. Compressible and Incompressible fluid flow
5. 1D, 2-D, 3-D fluid flows
6. Rotational and Irrotational flow

→ **Steady Flow:-**

It is the one in which the fluid characteristics (flow or self properties) remains const w.r.t time

ie $\frac{\partial V}{\partial t} = 0$

$$\frac{\partial P}{\partial t} = 0$$

$$\frac{\partial \sigma}{\partial t} = 0$$

$$\frac{\partial \mu}{\partial t} = 0$$

(b) **unsteady Flow:-**

The properties change w.r.t. time

$$\frac{\partial v}{\partial t} \neq 0, \ \frac{\partial P}{\partial t} \neq 0 \ \ldots$$

2a) **uniform flow:-**

Fluid properties w.r.t space coordinates remains const

$$\frac{\partial v}{\partial s} = 0$$

$\partial s \rightarrow$ space coordinates

$$s = f(x, y, z)$$

eg: Flow through a const dia pipe.

2b.) **Non uniform flow:-**

Flow properties change w.r.t. space coordinates

$$\frac{\partial v}{\partial s} \neq 0$$

eg: Flow through variable dia pipe.

3a) **Compressible Flow:-**

Flow in which density of fluid change at every location of flow.

eg: Air & other gas flow.

3b) **Incompressible Flow:-**

Flow in which the density 'ρ' of fluid remains const at all locations. eg: oil, water, mercury etc.

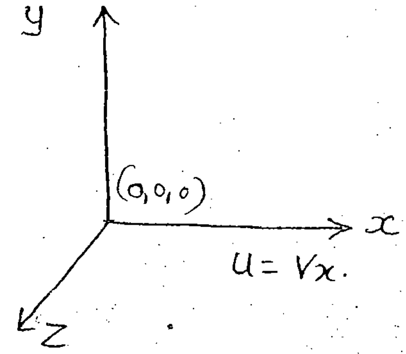
4.9) 1-D Flow:-

In this flow the velocity of fluid flow in one direction is considered. all the other directions are ignored.

$$\text{velocity} = f(x, y, z, t)$$

$$\vec{V} = (\vec{u})\hat{i} + (\vec{v})\hat{j} + (\vec{w})\hat{k}$$

$$= (\vec{V_x})\hat{i} + (\vec{V_y})\hat{j} + (\vec{V_z})\hat{k}$$



For 1D flow:-

$V = f(x, 0, 0, 0) \rightarrow$ 1D steady flow

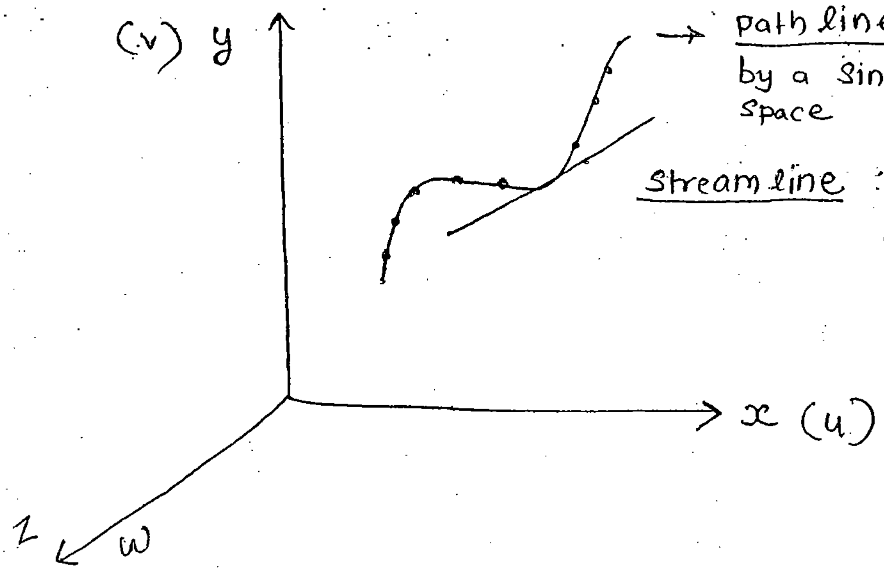
$V = f(x, 0, 0, t) \rightarrow$ 1D unsteady flow

For 2D flow:-

$V = f(x, y, 0, 0)$ - 2D steady flow

$V = f(x, y, 0, t)$ - 2D unsteady flow

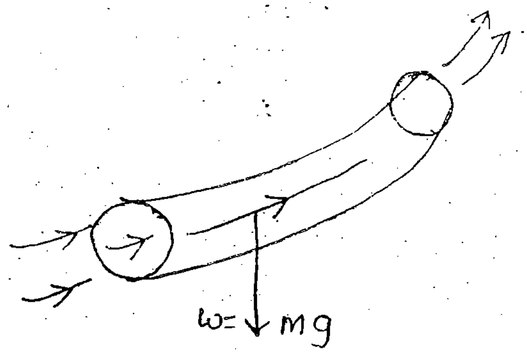
3.3 Fluid particle in Space



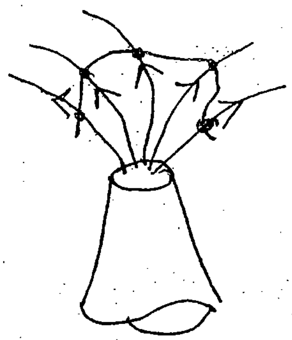
→ path line :- A line traced by a single fluid particle in space

stream line : A line tangent at a point in flow

Stream Tube : combination of stream lines across which no flow takes place.



→ **Streakline:-** It is a line which traces the different fluid particles at given time



Along x direction : $u = \frac{dx}{dt}$ —— ①

Along y direction : $v = \frac{dy}{dt}$ —— ②

$$\frac{u}{v} = \frac{dx}{dy}$$

$$\frac{u}{dx} = \frac{v}{dy}$$

for 3-D

$$\frac{u}{dx} = \frac{v}{dy} = \frac{w}{dz}$$

→ **Acceleration:**

$$\vec{a} = (a_x)\hat{i} + (a_y)\hat{j} + (a_z)\hat{k}$$

$$\boxed{a_x = u \cdot \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}}$$ → Local Acceleration

$$a_y = u \cdot \frac{\partial v}{dx} + v \cdot \frac{\partial v}{\partial y} + \omega \cdot \frac{\partial v}{dz} + \frac{dv}{dt}$$

$$a_z = u \cdot \frac{\partial \omega}{\partial x} + v \cdot \frac{\partial \omega}{\partial y} + \omega \cdot \frac{\partial \omega}{\partial z} + \frac{\partial \omega}{\partial t}$$

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Q NO1

$$u = -x$$
$$v = \cancel{2x}\ 2y \qquad (1,1)$$

we have $u = u_x = \frac{dx}{dt} = -x$

$$v = v_y = \frac{dy}{dt} = 2y$$

$$\frac{u}{v} = \frac{dx}{dy}$$

$$\frac{u}{dx} = \frac{v}{dy}$$

$$\frac{-x}{dx} = \frac{2y}{dy}$$

$$+\frac{dx}{x} = -\frac{dy}{2y}$$

$$+\log_e x = -\frac{1}{2} \log y + \text{constant}$$

$$\log_e x + \frac{1}{2} \log_e y = \text{const}$$

$$\log_e x + \log_e \sqrt{y} = \log_e C$$

$$\log x\sqrt{y} = \log_e C$$

$$x\sqrt{y} = C$$

at $(1,1)$: $C = 1$

Q No: 2 $\vec{V} = u\hat{i} + v\hat{j}$

$$|V| = \sqrt{u^2 + v^2}$$

$$V = 4\sqrt{x^2+y^2}$$

$$V = 4\sqrt{16+9} = \underline{\underline{4 \times 5 = 20}}$$

$$a = \frac{dv}{dt} = \frac{d}{dt} 4\sqrt{x^2+y^2}$$

$$V = \sqrt{(4x)^2 + (-4y)^2}$$

$$V = \sqrt{u^2 + v^2}$$

$u = 4x$

$v = -4y$

continuity eqn :→ Conservation of Mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\vec{a} = (\vec{a_x})\hat{i} + (\vec{a_y})\hat{j}$$

$$a_x = u \cdot \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial u}{\partial t}$$

$$= 4x \cdot 4 + (-4y) \cdot (0) + 0 = \underline{\underline{16x}}$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{\partial u}{\partial t}$$

$$= 4x \times 0 + (-4y) \times (-4) + 0 = \underline{\underline{16y}}$$

$$\vec{a} = (\vec{a_x})\hat{i} + (\vec{a_y})\hat{j}$$

$$= 16x\,\hat{i} + 16y\,\hat{j}$$

At $(x, y) = (4, 3)$

$= (16\times 4)\hat{i} + (16\times 3)\hat{j}$

$= 64\hat{i} + 48\hat{j}$

$|a| = \sqrt{64^2 + 48^2}$

$\bar{V} = (u)\hat{i} + (v)\hat{j}$

$\vec{V}_{(x,y)} = (4x)\hat{i} + (4y)\hat{j}$

$= 16\hat{i} + 12\hat{j}$

$|v| = \sqrt{16^2 + 12^2}$

$=$ m/sec

(3) $V = 2x\hat{i} + y\hat{j}$

$u = 2x$

$v = y$

$a_x = u \cdot \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$

$= 2x \times 2 + 0 = 4x$

$a_y = u \frac{dv}{\partial x} + v \frac{\partial v}{\partial y}$

$= 0 + y = y$

$a = (a_x)\hat{i} + (a_y)\hat{j}$

$= 4x\hat{i} + y\hat{j}$

At $(1,1)$: $4\hat{i} + \hat{j}$

$|a| = \sqrt{16+1} = \sqrt{17}$

(28.) $\vec{V} = 2\hat{i} + 3\hat{j}$

$\cancel{u = 2\hat{i}}$ $u = 2$ $\qquad \frac{u}{V} = \frac{dx}{dy}$

$\cancel{v = 3\hat{j}}$ $V = 3$

$$\frac{2}{3} = \frac{dx}{dy}$$

$$3dx - 2dy = 0$$

~~(29.)~~

(26.) $V = 2y\hat{i} + 3x\hat{j}$

$u = 2y$

$v = 3x$

$$a_x = u \cdot \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

$$= \cancel{2y \times 0 + 3x \times 0} = 0$$

$$2y \times 0 + 3x \times 2$$

$$= 6x$$

At $(1,1) =$ $6 \, m/s^2$

(27.) $u = 1.5x$

$V = 0$ $\qquad$ ~~$V = 1.5x$~~

At $(x, y) = (1, 0)$ $\qquad V_y = V = 0$

$$u = \frac{dx}{dt} \qquad \frac{u}{v} = \frac{dx}{dy}$$

$$v = \frac{dy}{dt}$$

$$\frac{u}{dx} = \frac{v}{dy}$$

$$\frac{dx}{u} = \frac{dy}{v}$$

Inorder to Have fluid flow condition is

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$$

$$\frac{\partial}{\partial x}(1.5x) = -\frac{\partial}{\partial y}(v)$$

~~1.5 =~~

$$1.5 = \frac{-\partial v}{\partial y}$$

$$1.5\, dy = -\partial v$$

$\int$ ing $\quad dv = -1.5\, dy$

$$v = -1.5y + \text{constant}$$

$$v = -1.5y + 0$$

$$\underline{v = -1.5y}$$

Boundary conditions

$$(x, y) = (1, 0)$$

$$0 = -1.5 \times 0 + C$$

$$\underline{C = 0}$$

(14) $u = 6xy - 2x^2$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$$

$$6y - 4x = -\frac{\partial v}{\partial y}$$

$$dv = -(6y - 4x)\, dy$$

$\int$ ing $\quad v = -\left[\frac{6y^2}{2} - 4xy\right]$

$$\underline{v = 4xy - 3y^2}$$

(12) $v \propto \frac{1}{r^2}$

$$v = \frac{c}{r^2}$$

$$a = \frac{\partial v}{\partial t} = \frac{\partial v}{\partial r} \cdot \frac{\partial r}{\partial t}$$

$$= \frac{\partial v}{\partial r} \cdot v$$

$$a = V \cdot \frac{\partial V}{\partial r}$$

$$a = \frac{c}{r^2} \cdot \frac{\partial}{\partial r}\left[\frac{c}{r^2}\right]$$

$$= \frac{c}{r^2} \cdot c \frac{\partial}{\partial r}\left[r^{-2}\right] = \frac{c^2}{r^2} - 2r^{-3}$$

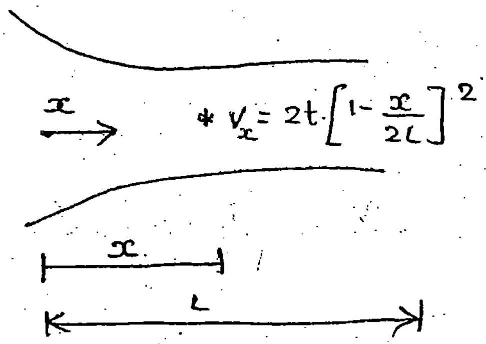
$$= \frac{-2c^2}{r^2 r^3} = -\frac{2c^2}{r^5}$$

$$a = \frac{-2c^2}{r^5}$$

$$\boxed{a \propto \frac{1}{r^5}}$$

KE

8. $u = 2t\left[1 - \frac{x}{2L}\right]^2$



Total Acceleration

= convective Accln + Local Accln.

$$a = a_x + a_t$$

$$= \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}$$

Conv Accln — Local Accln

$$\text{Local Accln} = \frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left[2t\left(1 - \frac{x}{2L}\right)^2\right]$$

$$= 2 \times 1 \left[1 - \frac{x}{2L}\right]^2$$

$$= 2\left[1 - \frac{x}{2L}\right]^2$$

$$= 2 \times \left[1 - \frac{0.5}{2 \times 0.8}\right]^2 = \underline{\underline{0.94}}$$

9.) Convective accln

$$a_x = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t}$$

$$= \frac{\partial u}{\partial x} \cdot u$$

$$= u \cdot \frac{\partial}{\partial x}(u)$$

$$2t\left[1-\frac{x}{2L}\right]^2 \times \frac{\partial}{\partial x}\left[2t\left(1-\frac{x}{2L}\right)^2\right]$$

$$= 2t\left[1-\frac{x}{2L}\right]^2 \times 2t \times \left(2\left(1-\frac{x}{2L}\right) \times \left(\frac{-1}{2L}\right)\right]$$

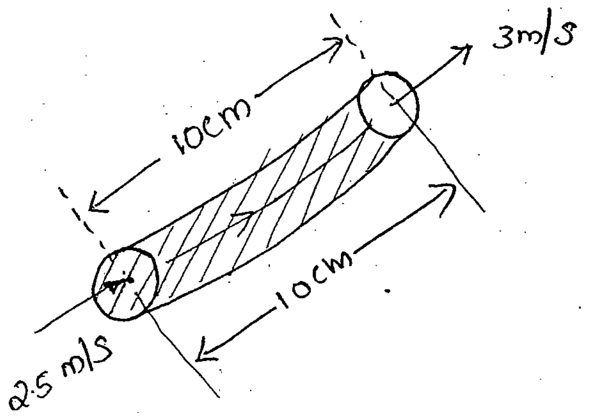
$$- 2t\left[1-\frac{x}{2L}\right]^2 \times \frac{4t}{2L} \times \left[1-\frac{x}{2L}\right]$$

$$a_x = \frac{-4t^2}{L}\left[1-\frac{x}{2L}\right]^3$$

At $\left.\begin{array}{l} t = 3\,sec \\ L = 0.8\,m \\ x = 0.5\,m \end{array}\right\}$

$$a = \frac{-4(3)^2}{0.8}\left[1-\frac{0.5}{2\times 0.8}\right]^2 = \underline{14.6\ m/sec^2}$$

(17)



$$a_{convection} = \frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t}$$

$$\boxed{a_{conv} \leq u \cdot \frac{\partial u}{\partial x}}$$

$$u \times \left(\frac{u_2 - u_1}{x_2 - x_1}\right)$$

$$\left(\frac{2.5+3}{2}\right)\left(\frac{3-2.5}{0.1}\right)$$

$$2.75 \times 5$$

$$= 13.75 \ m/sec^2$$

$$\frac{\partial a}{\partial t} = \frac{\partial u}{\partial x} \times \frac{\partial x}{\partial t}$$

u

Q3. d.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\lambda y^3 - 2\cancel{x}y + 2\cancel{x}y - \left(\frac{3}{4}\right)4y^3 = 0$$

$$\lambda \cancel{y^3} = \frac{3}{4} \times \cancel{y^3} \times 4$$

$$\lambda = 3$$

LAW OF CONSERVATION OF MASS:-

CONTINUITY EQUATION

In ~~fluid~~ flow ~~Rto~~ Field, it is a constrained particle for which the velocity components differenciation w.r.t their direction (u, v, w) must be equal to zero. ie ~~mass~~ total mass flow Rate remains constant at every point in the continous fluid flow path.

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

for 3-D steady flow incompressible fluid flow

$$\boxed{\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{d\rho}{\partial t} = 0}$$

3-D Compressible flow

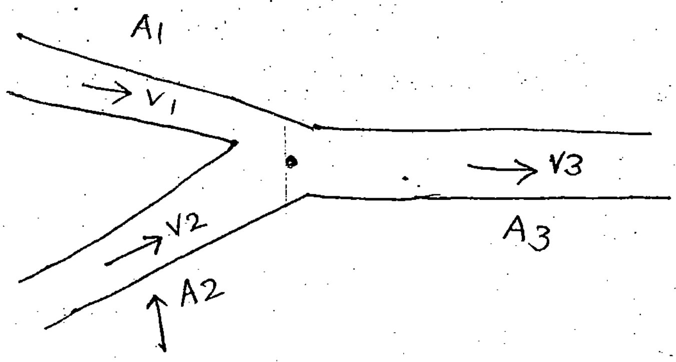
$\dot{m} = const$

$\rho Q = const$

$\rho AV = const$

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ (compressible fluid)

$Q = A_1V_1 = A_2V_2 = \dots$ [Incompressible flow]



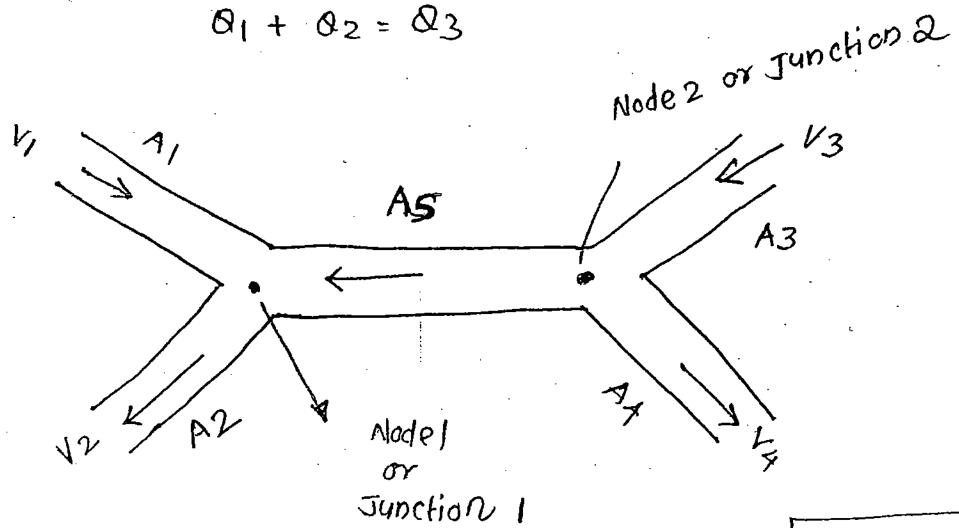
$\underline{\Sigma m = 0}$

$\dot{m}_1 + \dot{m}_2 = m_3$

for incompressible fluid } $\cancel{\rho_1} A_1 V_1 + \cancel{\rho_2} A_2 V_2 = \cancel{\rho_3} A_3 V_3$

$A_1V_1 + A_2V_2 = A_3V_3$

$Q_1 + Q_2 = Q_3$



Entering +ve
Leaving -ve

$Q_3 = Q_4 + Q_5$

$Q_1 + Q_5 = Q_2$

$A_1V_1 + A_5V_5 = A_2V_2$

$A_3V_3 = A_4V_4 + A_5V_5$

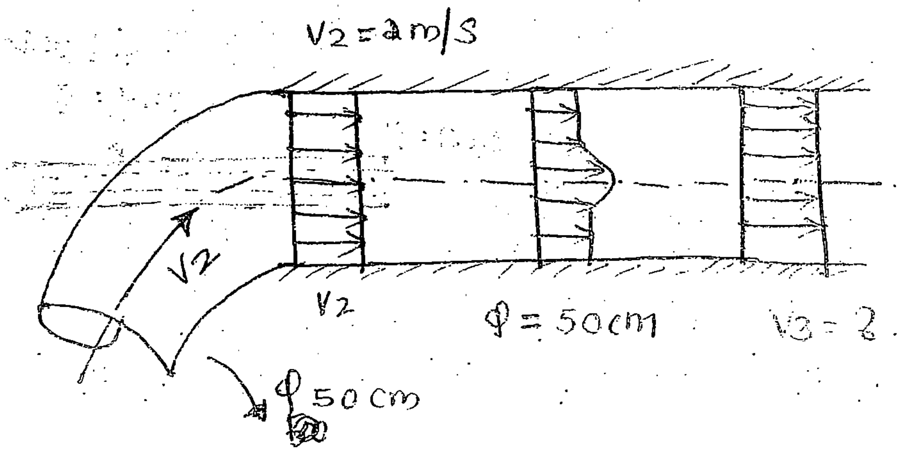
Pipe No:	Area	Velocity (m/se)
1	1	10
2	2	V_2
3	2.5	8
4	A_4	5
5	3	3

$A_4 = ?$

$V_2 = 9.5\ m/s$

$A = 2.2\ m^2$

Mixing of Fluids

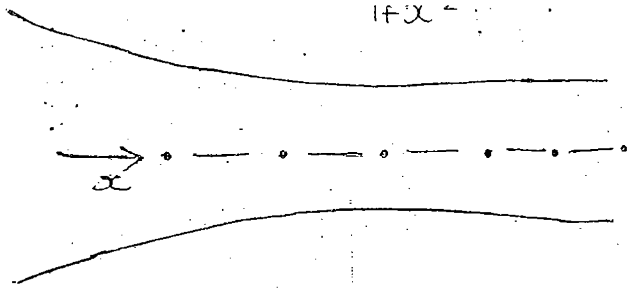


$$A_1V_1 + A_2V_2 = A_3V_3$$

$$\frac{\pi}{4} \times (0.1)^2 \times 5 + \frac{\pi}{4} \times \left(0.5^2 - 0.1^2\right) \times 2 = \frac{\pi}{4} \times (0.5)^2 \times V_3$$

$$V_3 = \underline{2.12\ m/s}$$

Q) Figure shows a nozzle through which water flowing at the Rate of $300\ m^3/min$ steadily. the ~~Area~~ velocity of the Nozzle is given as $\frac{1}{1+x^2}$



Determine the Acceleration of the fluid

a) $-50(1+x^2)$ ⓑ $-50(x+x^3)$

$$a_x = \frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t}$$

$$= \frac{\partial u}{\partial x} \cdot u$$

$$\therefore a_x = u \cdot \frac{\partial u}{\partial x}$$

we have

$$Q = AV$$

$$300 \frac{m^3}{min} = \frac{1}{1+x^2} \cdot u$$

$$\frac{300}{60} = \frac{1}{1+x^2} \cdot u$$

$$u = 5(1+x^2)$$

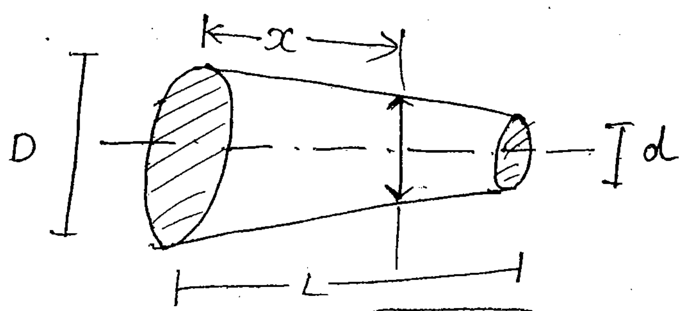
$$a_x = 5(1+x^2) \times \left[\frac{\partial}{\partial x}(5(1+x^2))\right]$$

$$= 5(1+x^2) \times [5(2x)]$$

$$= 50x(1+x^2)$$

$$a_x = 50(x+x^3)$$

NOTE:-



$$\boxed{d_x = D - \left[\frac{D-d}{L}\right]x}$$

$$\boxed{A_x = A_1 - \left(\frac{A_1 - A_2}{L}\right)x}$$

P42

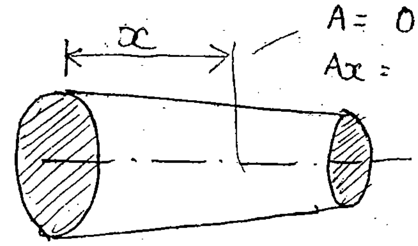
(6) $L = 2m$

$A = (0.4 - 0.1x)$

$Q = 0.48\ m^3/s$

$\frac{dQ}{dt} = \frac{0.12\ m^3/sec}{sec}$

$a_{x=0}$



$A = 0.4 - 0.1x$

$A_x = A_1 - \left[\frac{A_1 - A_2}{L}\right]x$

So $A_1 = 0.4$

$\frac{A_1 - A_2}{L} = 0.1$

$\frac{0.4 - A_2}{2} = 0.1$

$\therefore \underline{\underline{A_2 = 0.2\ m}}$

Local Acceleration

$$\dot{a} = \frac{\partial u}{\partial t}$$

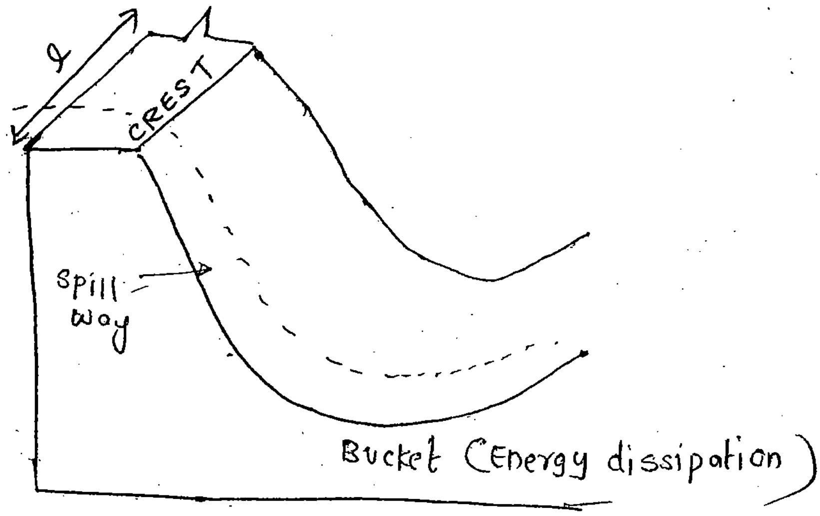
$$= \frac{\partial}{\partial t}\left(\frac{Q}{A}\right)$$

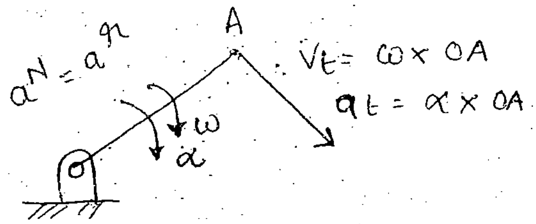
$$a = \frac{1}{A}\frac{\partial Q}{\partial t}$$

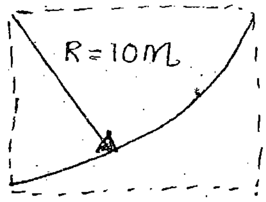
$$a_{x=0} = \frac{1}{0.4} \times 0.12 \qquad \frac{m^3}{s} \times \frac{1}{s}$$

$$= \underline{\underline{0.3\ m/s^2}}$$

(4) spill way: A concrete structure constructed Across the dam:-







$$a^N = a^r = a^{\text{centrepetal}}$$

$$= \frac{V_{AO}^2}{OA}$$

$$\boxed{a^N = a^r = \frac{v^2}{R}}$$

Soln:-

$$Q = AV$$

$$\frac{5m^3}{sec}/\text{unit length} = (L \times B) \times V$$

unit length. so 1m taken.

$$5 = (1 \times 0.5) \times V$$

$$\therefore 5 = 0.5V$$

$$V = \frac{5}{0.5} = \underline{10 m/s}$$

$$a^N = \frac{v^2}{R} = \frac{10^2}{10} = \underline{10 \frac{m}{s^2}}$$

STREAK LINE:-

A streak line at a fixed time t is the line connecting locus of all fluid particles, which have or will pass through a fixed location $\vec{y}$ at time t'.

ENGG SERVICES

Fluid kinematics is the Branch of science which deals with motion of particles without considering forces causing motion

→ Methods of describing fluid Motion

1. Lagrangian Method
2. Eulerian Method

In Lagrangian method a single fluid Particle is followed ~~and~~ during its motion and its vel, accln, ρ etc are described. But in Eulerian method, the vel, accln and ρ are described at a point in Fluid Flow field.

→ Steady Flow:-

Fluid chara like vel, Pressure, ρ etc do not change with time

ie $\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} = 0$ $\quad \left(\frac{\partial P}{\partial t}\right)_{x_0, y_0, z_0} = 0$

$\left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$

→ Unsteady flow:-

Fluid chara change with respect to time

Mathematically

$\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \quad \left(\frac{\partial P}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$ ..etc.

→ Uniform Flow:-

It is type of flow in which velocity at any ~~poin~~ given time does not change with respect to space (ie length of direction of flow)

$\boxed{\left(\frac{\partial v}{\partial s}\right)_{t = const} = 0}$

∂v = change in velocity

∂s = Length of flow in the direction s

→ Non uniform Flow:-

Type of flow in which velocity at any given time changes w.r.t space

$\boxed{\left(\frac{\partial v}{\partial s}\right)_{t = const} \neq 0}$

→ LAMINAR FLOW:

Type of flow in which fluid particles move along well defined paths or stream line and all the stream lines are straight and parallel

Here the fluid particles move in laminas or they are gliding smoothly over the adjacent layers. This type of flow is also called streamline flow or viscous flow

In terms of Reynold NO:

$$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu} \quad \boxed{\text{PIPE FLOW}}$$

If $Re < 2000$, flow is called Laminar

→ TURBULENT FLOW ←

It is the type of flow in which fluid particles move in zig-zag pattern. Here due to zig-zag movement of particles eddies formation takes place which result in high energy loss

If $Re > 4000$ [Turbulent]

[If $2000 < Re < 4000$] Flow is Laminar or Turbulent

→ Compressible flow ←

$$\rho \neq const$$

→ Incompressible Flow ←

$$\rho = const$$

→ Rotational Flow ←

Type of flow in which fluid particles move along streamlines and also rotate w.r.t its own axis

→ Irrotational Flow: ←

Type of flow in which fluid particles while flowing along streamlines do not rotate about their axis

→ 1-D Flow ←

Type of flow in which flow parameter such as velocity is a function of time and one space coordinate only. say 'x'

The variation of velocities in other directions are assumed to be negligible

For steady 1-D Flow

$u = F(x) \quad v = 0 \; , \; \omega = 0$

u, v, ω are velocity components in x, y and z directions

→ 2D Flow ←

$u = f_1(x, y)$ $\quad \omega = 0$

$v = f_2(x, y)$

→ 3D Flow ←

Velocity is a function of three mutually ⊥r space coordinates and time

$u = f_1(x, y, z)$

$v = f_2(x, y, z)$

$\omega = f_3(x, y, z)$

Rate of Flow or Discharge :-

Quantity of fluid flowing per second through a section of pipe or channel

For Liquids $\quad Q$ is in $\frac{m^3}{s}$ or $\frac{\text{litres}}{s}$

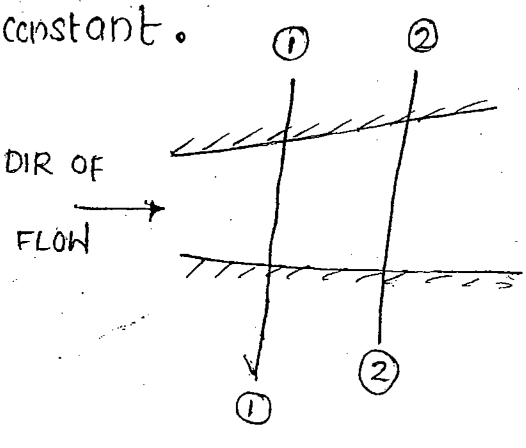
For Gases $\quad Q$ is $\frac{kgf}{s}$ or $\frac{\text{Newton}}{s}$

$$\boxed{Q = A \times V}$$

→ CONTINUITY EQN : ←

Based on Law of Conservation of Mass.

For a fluid flowing through a pipe at ~~are~~ all cross section the qty of fluid per second is constant.



Rate of flow at section 1

$= \rho_1 A_1 V_1$

Rate of flow at section 2

$= \rho_2 A_2 V_2$

Rate of flow at Section 1 = Rate of flow at Section 2

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$\rho AV = \text{const}$$

$$\boxed{\rho Q = \text{const}}$$

For incompressible fluids

$$Q = \text{const}$$

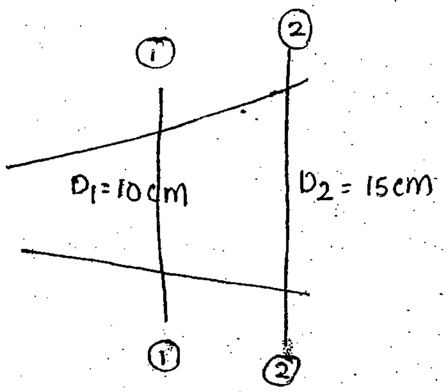
$$\boxed{AV = \text{const}} \qquad A_1V_1 = A_2V_2$$

~~FM~~

FM TEXT
Page 163

5.1) $D_1 = 0.1\,m$ $\quad V_1 = 5\,m/s$

$D_2 = 0.15\,m$



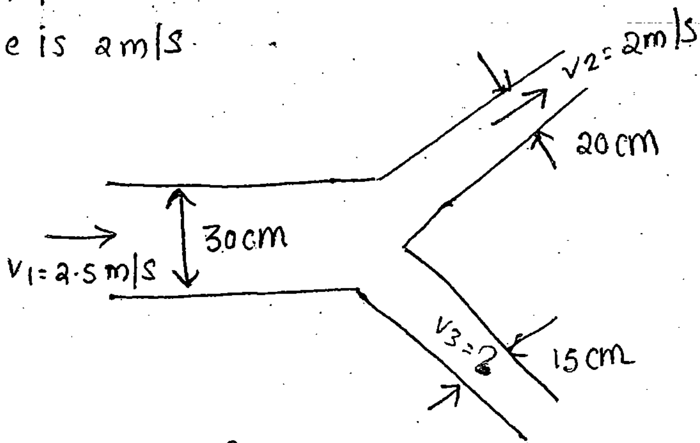
$$A_1V_1 = A_2V_2$$

$$\frac{\pi}{4} \times D_1^2 \cdot V_1 = \frac{\pi}{4} \times D_2^2 \cdot V_2$$

$$\cancel{\frac{\pi}{4}} \times (0.1)^2 \times 5 = \cancel{\frac{\pi}{4}} \times (0.15)^2 \times V_2$$

$$V_2 = \frac{(0.1)^2 \times 5}{(0.15)^2} = \underline{2.22\,m/s}$$

2) A 30 cm dia pipe conveying water, Branches into two pipes of dia 20 cm and 15 cm. If Average vel in the 30 cm pipe is 2.5 m/s. find the discharge in pipe. Also find vel in 15 cm pipe if average vel in 20 cm pipe is 2 m/s.



Sol:-

$$Q_1 = A_1V_1$$

$$= \frac{\pi}{4} \times (0.30)^2 \cdot 2.5$$

$$= \underline{0.1767\,m^3/s}$$

$$Q_1 = Q_2 + Q_3$$

$$0.1767 = 0.062 + Q_3$$

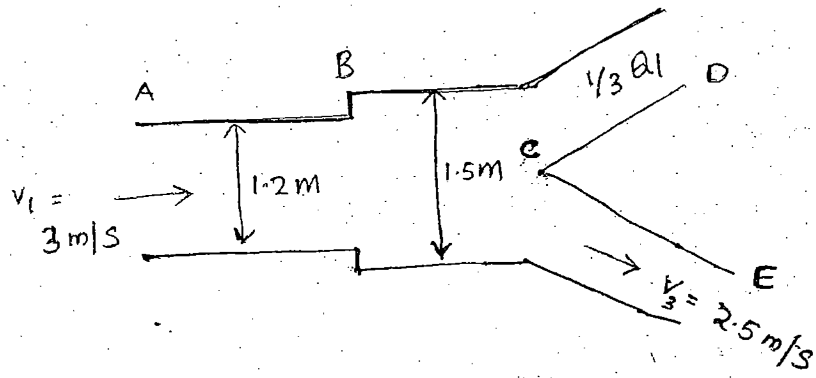
$$\underline{Q_3 = 0.11\,m^3/s}$$

$$Q_2 = \frac{\pi}{4} \times (0.2)^2 \times 2$$

$$= 0.062\,m^3/s$$

$$V_3 = \frac{4 \times 0.11}{\pi \times (0.15)^2} = \underline{6.22\ m/s}$$

Q.) Water flows through a pipe AB 1.2 dia at 3m/s and ~~then~~ passes through pipe BC 1.5 m dia. At C, the pipe branches. Branch CD is 0.8m in dia and carries one third flow of AB. The flow vel in branch CE is 2.5 m/s. Find vel rate of flow in AB, vel in BC, vel in CD and dia of CE ~~3~~



$$Q_{CD} = \frac{1}{3} Q_1$$

$$Q_1 = \frac{\pi}{4} \times (1.2)^2 \times 3$$

$$\underline{Q_1 = \cancel{1.08\ m^3/s}\ 3.39\ m^3/s}$$

$$Q_1 = Q_2\ \cancel{Q_1 = Q_2} = Q_3 + Q_4$$

$$Q_1 = Q_2$$

$$Q_2 = \frac{\pi}{4} \times (1.5)^2 \times V_2$$

By continuity eqn

$$A_1 V_1 = A_2 V_2$$

$$3.39\ \cancel{1.08} = \frac{\pi}{4} \times (1.5)^2 \times V_2$$

$$\underline{V_2 = \cancel{0.61\ m/s}\ 1.92\ m/s}$$

$$\underset{(CD)}{Q_4} = \frac{1}{3} Q_1$$

$$Q_1 = Q_3 + Q_4$$

$$3.39 = Q_3 + \frac{3.39}{3}$$

$$\underline{Q_3 = 2.26\ m^3/s}$$

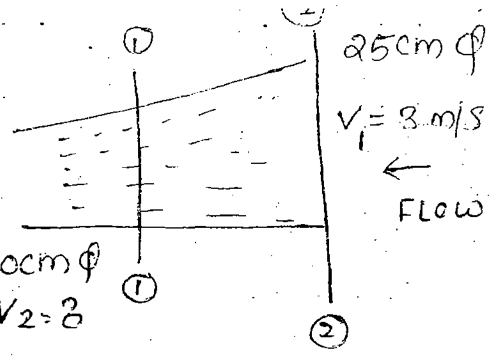
$$Q_3 = \frac{\pi}{4} \times d_3^2 \times V_3$$

$$2.26 = \frac{\pi}{4} \times d_3^2 \times 2.5$$

$$\underline{d_3 = \cancel{0.60\ m}\ 1.07\ m}$$

Q.) A 25 cm dia pipe carries oil of sp gr 0.9 at a vel of 3 m/s. At another section the dia is 20cm. Find the vel at this section and also mass rate of flow.

Soln: $S = 0.9$



$$A_1V_1 = A_2V_2$$

$$\cancel{\frac{\pi}{4}}\times[0.25]^2\cdot 3 = \cancel{\frac{\pi}{4}}\times[0.20]^2\times V_2$$

$$V_2 = \left[\frac{0.25}{0.20}\right]^2 \cdot 3$$

$$\underline{V_2 = 4.6875\ m/s}$$

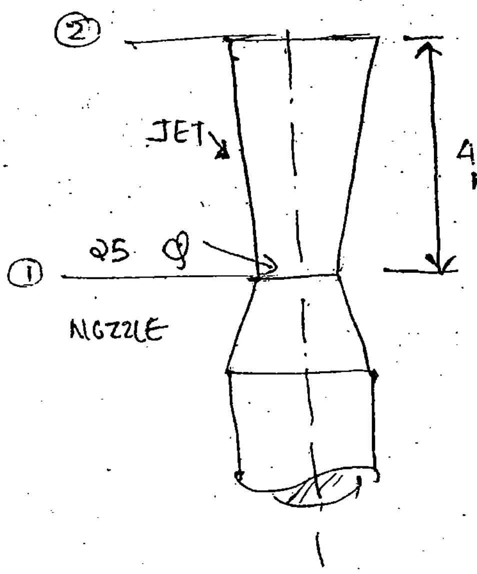
Mass flow Rate $(\dot{m}) = \rho_1 A_1 V_1$

$$= (0.9\times1000)\times\left[\frac{\pi}{4}\times(0.25)^2\right]\cdot 3$$

$$= \underline{132.53\ kg/s}$$

Q.) A jet of water from a 25 mm dia nozzle is directed vertically upwards. Assuming that jet Remains Circular and Neglecting any loss of energy, that will be the dia at point 4.5 m above the nozzle. If the vel with which jet leaves nozzle is 12 m/s.

Apply Bernoulli's eqn



$$\cancel{\frac{P_1}{\rho g}} + z_1 + \frac{V_1^2}{2g} = \cancel{\frac{P_2}{\rho g}} + z_2 + \frac{V_2^2}{2g}$$

$$\frac{0}{\rho g} + 0 + \frac{12^2}{2\times9.81} = \frac{0}{\rho g} + 4.5 + \frac{V_2^2}{2\times9.81}$$

$$\frac{12^2}{2\times9.81} = 4.5 + \frac{V_2^2}{2\times9.81}$$

$$2.83 = \frac{V_2^2}{2\times9.81}$$

$$\underline{V_2 = 7.46}$$

$$A_1V_1 = A_2V_2$$

$$\cancel{\frac{\pi}{4}}\times[0.025]^2\times 12 = \cancel{\frac{\pi}{4}}\times[d_2^2]\times 7.46$$

$$\underline{d_2 = 31.70\ mm}$$

$$\frac{\partial}{\partial x}\left[\rho u\right] + \frac{\partial}{\partial y}\left[\rho v\right] + \frac{\partial}{\partial z}\left[\rho w\right] + \frac{\partial \rho}{\partial t} = 0$$

continuity eqn in cartesian coordinates — BASIC EQN

this eqn is applicable to

(i) steady and unsteady flow

(ii) uniform and Nonuniform flow

(iii) Compressible and incompressible flow

Case 1: Steady Flow

$$\boxed{\frac{\partial \rho}{\partial t} = 0}$$

so continuity eqn becomes

$$\frac{\partial}{\partial x}\left[\rho u\right] + \frac{\partial}{\partial y}\left[\rho v\right] + \frac{\partial}{\partial z}\left[\rho w\right] = 0$$

Case 2: Incompressible flow:

$\rho = C$

so $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

For 2-D flow $\frac{\partial w}{\partial z} = 0$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

→ CONTINUITY EQN IN POLAR COORDINATES: ←

$$\boxed{\frac{\partial}{\partial r}(r \cdot u_r) + \frac{\partial}{\partial \theta}(u_\theta) = 0}$$

2D steady flow incompressible.

$$\frac{\partial}{\partial r}(r \cdot u_r) = u_r + r \cdot \frac{\partial u_r}{\partial r}$$

General eqn:-

$$\boxed{\frac{\partial \rho}{\partial t} + \rho \frac{\partial}{\partial r}(r \cdot u_r) + \rho \cdot \frac{\partial}{\partial \theta}(u_\theta) = 0}$$

→ VELOCITY AND ACCELERATION ←

u, v and ω are vel components in x, y and z directions

V = Resultant velocity at a point in fluid flow.

$u = f_1(x, y, z, t)$

$v = f_2(x, y, z, t)$ $\qquad V = ui + vj + \omega k$

$\omega = f_3(x, y, z, t)$ $\qquad = \sqrt{u^2 + v^2 + \omega^2}$

Resultant velocity

$$\boxed{\begin{aligned} V &= u\hat{i} + v\hat{j} + \omega\hat{k} \\ V &= \sqrt{u^2 + v^2 + \omega^2} \end{aligned}}$$

a_x, a_y and a_z are total accln in x, y & z directions.

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x} \times \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial u}{\partial t}$$

$$\boxed{a_x = u \cdot \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial z}} + \frac{\partial u}{\partial t}$$

$$a_y = \frac{dV}{dt} = \frac{\partial v}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial v}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial v}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial u}{\partial t}$$

$$\underbrace{\boxed{a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \omega \frac{\partial v}{\partial z}}}_{\text{Convective Accln}} + \frac{\partial u}{\partial t} \rightarrow \text{Local Accln}$$

$$a_z = \frac{d\omega}{dt} = \frac{\partial \omega}{\partial x} \times \frac{dx}{dt} + \frac{\partial \omega}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial \omega}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial u}{\partial t}$$

$$\boxed{a_z = u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} + \omega \frac{\partial \omega}{\partial z}} + \frac{\partial u}{\partial t}$$

For steady flow $\boxed{\frac{\partial V}{\partial t} = 0}$ $\qquad$ V is Resultant velocity

$\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial t}$, $\frac{\partial w}{\partial t} = 0$

→ Acceleration vector (A) ←

$$A = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$A = \sqrt{a_x + a_y^2 + a_z^2}$$

→ Local Acceleration ←

It is defined as Rate of increase of velocity ~~ab~~ with respect to time at ~~any~~ a point in a flow field

ie $\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial t}$ or $\frac{\partial w}{\partial t}$

→ Convective Acceleration ←

Terms other than $\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial t}$, $\frac{\partial w}{\partial t}$ in the a_x, a_y, a_z are called Convective Acceleration.

(5.6) The velocity vector in a fluid flow is given by

$\vec{V} = 4x^3\hat{i} + (-10x^2y)\hat{j} + 2t\hat{k}$. find Accln and velocity at (2,1,3) at time t = 1.

ANS:- $u = 4x^3$; $V = -10x^2y$; $z = 2t$

$$\vec{V}_{(2,1,3)} = 4 \times 2^3\,\hat{i} - 10 \times 2^2\,\hat{j} + 2 \times 3\,\hat{k}$$

$$\vec{V}_{(2,1,3)} = 32\hat{i} - 40\hat{j} + 6\hat{k}$$

$$|\vec{V}| = \sqrt{(32)^2 + (-40)^2 + 6^2}$$

$$= \sqrt{2660}$$

$$|\vec{V}| = 51.57 \text{ m/s}$$

$\vec{A} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$= 4x^3 \left[12x^2\right] + (-10x^2y)(0) + 2t \times 0 + 0$$

$\omega = 2t$
$u = 4x^3$
$v = -10x^2y$

$a_x = 48x^5$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \omega \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$\cancel{-10x^2y}\; 4x^3 \left[-20x\right] + -10x^2y\left(-10x^2\right) + 2t \times 0$$

$$a_y = -80x^4 + 100x^4y$$

$$a_z = u \frac{d\omega}{dx} + v \frac{d\omega}{dy} + \omega \frac{d\omega}{dz} + \frac{d\omega}{dt}$$

$$= 4x^3(0) + (-10x^2y) \times 0 + 2t \times 0 + 2$$

$\underline{a_z = 2}$

$$\vec{A} = 48x^5 \hat{i} + (-80x^4 + 100x^4y)\hat{j} + 2\hat{k}$$

$$\text{At}(2,1,3) = 48 \times (2)^5 \hat{i} + \left[(-80) \times 2^4 + 100 \times 2^4 * 1\right]\hat{j} + 2\hat{k}$$

$$1536\,\hat{i} + \left[\overset{320}{\cancel{12880}}\right]\hat{j} + 2\hat{k}$$

$$|\vec{A}| = \sqrt{(1536)^2 + \underset{(320)^2}{\cancel{(12880)^2}} + 2^2} = \underline{1568.98\ m/s^2}$$

Resultant Acceleration $= 1568.98\ m/s^2$

(2.) Determine the third component of velocity such that it satisfies continuity eqn:-

(i) $u = x^2 + y^2 + z^2$

$v = xy^2 - yz^2 + xy$

(ii) $v = 2y^2$, $\omega = 2xyz$

Soln – we have continuity eqn $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$

$$(2x+0+0)+(2xy\overset{-z^2+x}{\wedge})+\frac{\partial w}{\partial z}=0$$

$$\frac{\partial w}{\partial z} \cancel{\pm} \cancel{\oplus}2x+2xy-z^2+x=0$$

$$\partial w=(-2x+2xy-z^2+x)\,dz$$

$$\int dw=\int(-2x+2xy-z^2+x)\,dz$$

$$w=-2xz-2xyz-\frac{z^3}{3}+xz+C$$

C is not a fn of ~~x~~ z. It can be $f(x,y)$

Soln :—

$$(2x+0+0)+(2xy-z^2+x)+\frac{\partial w}{\partial z}=0$$

$$\frac{\partial w}{\partial z}+2x+2xy-z^2+x=0$$

$$\frac{\partial w}{\partial z}=-2x-2xy+z^2-x$$

$$\partial w=(-3x-2xy+z^2)\,\partial z$$

$$\int\partial w=\int(-3x-2xy+z^2)\,\partial z$$

$$w=\left[-3xz-2xyz+\frac{z^3}{3}\right]+\text{constant}$$

$\searrow F(x,y)$

(ii) $v=2y^2,\ w=2xyz$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$\frac{\partial v}{\partial y}=4y$$

$$\frac{\partial w}{\partial z}=2xy$$

$$\frac{\partial u}{\partial x}+4y+2xy=0$$

$$\frac{\partial u}{\partial x}=-4y-2xy$$

$$\int\partial u=\int(-4y-2xy)\,\partial x$$

$$u=-4xy-\frac{2x^2y}{2}+\text{const}$$

$$u=-4xy-x^2y+\text{const}$$

Q.) A flow field is given by

$$\vec{V} = x^2y\,\hat{i} + y^2z\,\hat{j} - (2xyz + yz^2)\,\hat{k}$$

P.T it is a case of steady incompressible Flow ?

Soln:-

$u = x^2y$

$v = y^2z$

$w = -(2xyz + yz^2)$

$\frac{\partial u}{\partial x} = 2xy$

$\frac{\partial v}{\partial y} = 2yz$

$\frac{\partial w}{\partial z} = -(2xy + 2yz)$

Continuity equation :-

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$2xy + 2yz - 2xy - 2yz = \underline{0}$$

Q2.) From the above Given vel field vector $\vec{V}$, calculate velocity and acceleration at Point (2, 1, 3)

$$\vec{V}_{(2,1,3)} = (2^2 \cdot 1)\hat{i} + (1^2 \cdot 3)\hat{j} - (2\times2\times1\times3 + 1\times3^2)\hat{k}$$

$$\vec{V}_{(2,1,3)} = 4\hat{i} + 3\hat{j} - 21\hat{k}$$

$$|\vec{V}| = \sqrt{16 + 9 + 441}$$

$$|\vec{V}| = \sqrt{466}$$

$$|\vec{V}| = \underline{21.58\ m/s}$$

Acceleration at (2, 1, 3)

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$= x^2y\,(2xy) + y^2z\,(x^2) + -(2xyz + yz^2)\times 0 + 0$$

$$= \underline{2x^3y^2 + x^2y^2z}$$

$$a_y = u\frac{dv}{dx} + v\frac{dv}{dy} + w\frac{dv}{dz} + \frac{dv}{\partial t}$$

$$x^2y\times 0 + y^2z\,(2yz) - (2xyz + yz^2)(y^2) + 0$$

$$= 2y^3z^2 - 2xy^3z - y^3z^2$$

$$\vec{A} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$= \underline{\underline{28\hat{i} - 3\hat{j} + 105\hat{k}}}$$

5 Q31.) $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$

3D un steady compressible fluid flow

$\rho = \rho_0 e^{-2t}$

$u = 5x + 6y + 7z$

$v = 6x + 5y + 9z$

$w = 3x + 2y + \lambda z$

$\frac{\partial u}{\partial x} = 5$

$\frac{\partial v}{\partial y} = 5$

$\frac{\partial w}{\partial z} = \lambda$

$\frac{\partial \rho}{\partial t} = \rho_0 e^{-2t} \times (-2)$

$-\rho_0 e^{-2t} \times 2 + \rho[5] + \rho[5] + \rho[\lambda] = 0$

$\rho[5+5+\lambda] = \rho_0 e^{-2t} \times 2$

$\rho[10+\lambda] = \rho_0 e^{-2t} \times 2$

$\cancel{\rho_0} \times \cancel{e^{-2t}} [10+\lambda] = \cancel{\rho_0} \cancel{e^{-2t}} \times 2$

$10 + \lambda = 2$

$\lambda = 2 - 10$

$\boxed{\lambda = -8}$ (c)

a) A velocity field is given by

$\vec{V} = (0.3x)\hat{i} + (-0.3y)\hat{j}$

or

$\vec{V} = (kx)\hat{i} + (-ky)\hat{j}$

(1) - Is the following steady or ~~flow~~ unsteady ?

(2) Is it 2D or 3D

(5) Accln vector

$u = 0.3x$

$v = -0.3y$

$$\frac{\partial u}{\partial x} = 0.3$$

$$\frac{\partial v}{\partial y} = -0.3$$

Continuity eqn:-

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$= +0.3 + (-0.3) = 0$$

$\therefore$ It satisfies continuity eqn.

~~So steady~~ yes

(6) Total Accln at $(-2,-2)$

(7) obtain the stream line Equation

Soln:- flow is steady because time doesnot appear in velocity components

2. flow is 2D because $u, v \neq 0$ and $w = 0$

3. $u = 0.3x$ $\qquad \frac{\partial u}{\partial x} = 0.3$

$v = -0.3y$ $\qquad \frac{\partial v}{\partial y} = -0.3$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$0.3 - 0.3 = 0$$

so it satisfies continuity eqn
Hence it obeys Law of conservation of energy.

4. $u(1,1) = 0.3$

$v(1,1) = -0.3$

$$\vec{V} = 0.3\hat{i} + (-0.3)\hat{j}$$

$$|\vec{V}| = \sqrt{0.3^2 + (-0.3)^2}$$

$$|\vec{V}| = 0.3\sqrt{2} \text{ m/s}$$

5. $$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial v}{\partial t}$$

$$a_x = 0.3x(0.3) + (0)(-0.3y) + 0$$

$$a_y = 0.3x(0) + (-0.3y)(-0.3) + 0$$

$$\vec{A} = a_x\hat{i} + a_y\hat{j}$$

$$\vec{A} = (0.09x)\hat{i} + (0.09y)\hat{j}$$

$$|\vec{A}| = \sqrt{(0.09)^2 + (0.09)^2} = 0.18 \times \sqrt{2} \text{ m/s}^2$$

Given $u = 0.3x \Rightarrow \frac{dx}{dt} = 0.3x$ — (1)

$v = -0.3y \Rightarrow \frac{dy}{dt} = -0.3y$ — (2)

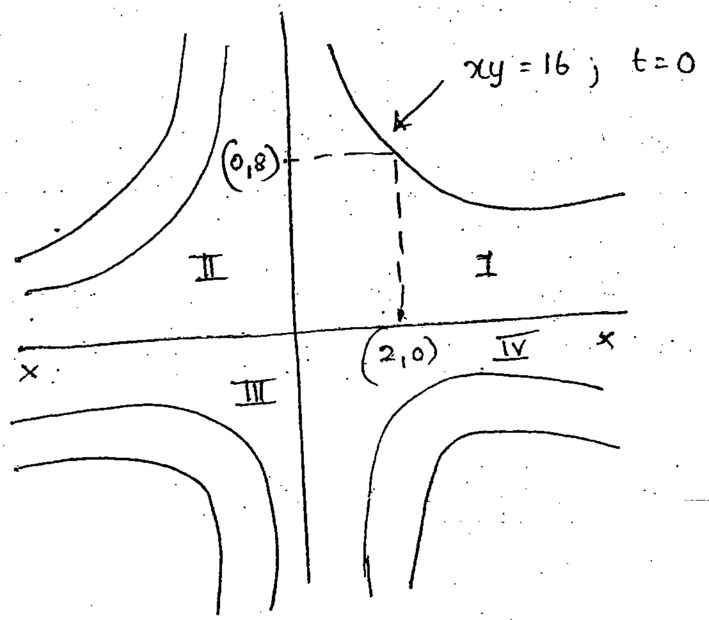
$$\frac{\frac{dx}{dt}}{\frac{dy}{dt}} = \frac{+0.3x}{-0.3y}$$

$$\frac{\partial x}{\partial y} = -\frac{x}{y}$$

Seperating variables and integrating

$$\int \frac{dx}{x} = -\int \frac{dy}{y}$$

$$\boxed{xy = C}$$



$t = 0 \quad x_0 = 2$

$y_0 = 8$

$t = 6\text{ sec} \quad x_6 = ?$

$y_6 = ?$

$$u = 0.3x$$

$$\frac{dx}{dt} = 0.3x$$

$$\frac{\partial x}{x} = 0.3\,\partial t$$

$$\int_{x_0=2}^{x} \frac{\partial x}{x} = 0.3 \int_{t=0}^{t=6} \partial t$$

$$\left[\log_e x\right]_{x_0}^{x} = 0.3\left[t\right]_0^t$$

$$\log\left(\frac{x}{x_0}\right) = 0.3(t-0)$$

$$\frac{x}{x_0} = e^{0.3t}$$

$$\boxed{x = x_0 e^{0.3t}}$$

(Q9) $\vec{V}_{(2,8)} = \cancel{(0.3\times 2)\hat{i} + (-0.3\times 8)\hat{j}}$

$= \cancel{0.6T}$

$x = x_0 e^{\cdots}$

$x = 2\times e^{0.3\times 6}$

$= 2e^{1.8} = \underline{12.1\,m}$

$y = 8\times e^{-0.3\times 6}$

$= 8e^{-1.8} = \underline{1.32\,m}$

$$\vec{V} = (0.3x)\hat{i} + (-0.3y)\hat{j}$$

$$\vec{V}_{(12.1,\,1.32)} = (0.3\times 12.1)\hat{i} + (-0.3\times 1.32)\hat{j}$$

$$= \sqrt{(\quad)^2 + (\quad)^2} = \underline{3.65\ m/sec}$$

Q9) $\vec{V}_{2,8} = (0.3\times 2)\hat{i} + (-0.3\times 8)\hat{j}$

$= (0.6)\hat{i} + (-2.4)\hat{j}$

$= \sqrt{(0.6)^2 + (-2.4)^2}$

$=$ ______

Representation of Flow feild through functions:-

velocity potential function and streamline function

→ **Velocity potential function (ϕ)** ←

It is a scalar function and it Represents the fluid particle in space through functions.

$$\phi = f(x, y, z)$$

It is valid for 3-D and it has the following properties.

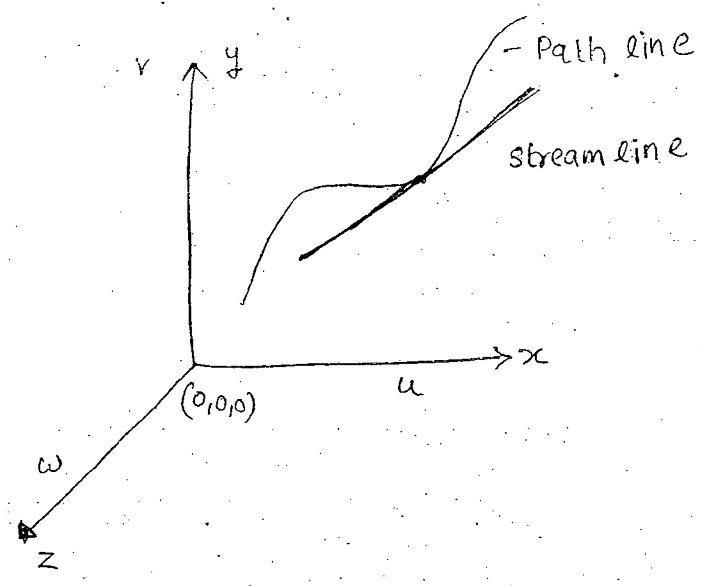
1. velocity potential fn satisfies Law of conservation of mass
2. It provides the condition of flow for irrotation
3. It satisfies Laplace Transform

$$\frac{\partial^2\phi}{} + \frac{\partial^2\phi}{} + \frac{\partial^2\phi}{} = 0$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 v}{\partial x \partial y}$$

$$\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 w}{\partial y \partial z}$$

$$\frac{\partial^2 w}{\partial z \cdot \partial x} = \frac{\partial^2 u}{\partial z \cdot \partial x}$$



$$\phi = F(x, y, z)$$

$$\frac{\partial \phi}{\partial x} = -u$$

$$\frac{\partial \phi}{\partial y} = -v$$

$$\frac{\partial \phi}{\partial z} = -\omega$$

-ve sign indicates that one value increases, other parameter decreases

ex: pressure change b/w 2 points.

Stream function (ψ)

Stream function is also a scalar function in space which provides 2-D fluid flow velocity components into a function w.r.t to normal planes / normal axis.

It is Represented by ψ.

Stream function exists which means that law of conservation of mass is satisfied.

Stream function is valid for Rotational and Irrotational flow.

Laplace transform should be valid.

Physical Meaning :-

Provide discharge b/w 2 points of flow.

Mathematically :

$$\frac{\partial \psi}{\partial y} = -u$$

$$\frac{\partial \psi}{\partial x} = +V$$

Summary

$\frac{\partial \phi}{\partial x} = -u$	$\frac{\partial \psi}{\partial y} = -u$
$\frac{\partial \phi}{\partial y} = -V$	$\frac{\partial \psi}{\partial x} = V$
$\frac{\partial \phi}{\partial z} = -\omega$	—

Page 7

$$\psi = x^2 - y^2$$

$$\frac{\partial \psi}{\partial y} = -2y$$

$$\frac{\partial \psi}{\partial x} = 2x$$

$$u = -\frac{\partial}{\partial y}(x^2 - y^2)$$

$$u = 2y$$

$$V = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}(x^2 - y^2)$$

$$= 2x$$

$$\vec{V} = u\hat{i} + v\hat{j}$$

$$\vec{V} = (2y)\hat{i} + (2x)\hat{j}$$

$\vec{A}$ = Local Accln + Convective Accln.

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + 2x \times 2$$

$$= 4x$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 2y \times 2 + 2x \times 0$$

$A = a_x \hat{i} + a_y \hat{j}$

$\vec{A} = 4x \hat{i} + 4y \hat{j}$

$|\vec{A}| = \sqrt{4^2+4^2}$

$|\vec{A}| = \sqrt{16+16} = \sqrt{32} = \sqrt{4\times 8}$

$= 2\sqrt{4\times 2}$

$= \underline{4\sqrt{2}\ m/s^2}$

$$\begin{array}{r} 1.732 \\ 4 \\ \hline 6.928 \end{array}$$

(19) $\varphi = 2xy$

$\frac{\partial \varphi}{\partial y} = 2x = -u$

$\boxed{u = -2x}$

$\frac{\partial \varphi}{\partial x} = 2y$

$\boxed{V = 2y}$

$\vec{V} = (-2x)\hat{i} + (2y)\hat{j}$

~~$|\vec{V}| = \sqrt{4+4} = 4\sqrt{2}$~~

$\vec{V}_{(+2,-2)} = (-4)\hat{i} + (2\times 2)\hat{j}$

$[(-2\times 2)\hat{i} + (2\times 2)\hat{j}]$

$\vec{V}_{(2,-2)} = -8\hat{i} + 8\hat{j}$

$|\vec{V}| = \sqrt{4^2+4^2} = \underline{4\sqrt{2}}$

(23)

(25) $\phi = x^3 - y^3$

(24)

$\boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0}$ Laplace eqn

$\frac{\partial^2 \phi}{\partial x^2} = 6x$

$-3y^2$

$-6y$

$\frac{\partial^2 \phi}{\partial y^2} = -6y$

$\underline{x = y}$

$-2y$

$\phi = x^2 - y^2$

$\frac{\partial^2 \phi}{\partial x^2} = 2$, $\frac{\partial^2 \phi}{\partial y^2} = -2$

$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$

$\underline{\underline{2 - 2 = 0}}$

(b.)

(29.) Flow Rate = Discharge (Q)

Discharge = $|\partial \psi|$

$Q = |\psi_A - \psi_B|$, $|0 - 4| = \underline{\underline{4\,m^3/sec}}$

$\psi_{A\,(3,0)} = 2 \times 3^2 \times 0 + (3+1) \times 0 = 0$

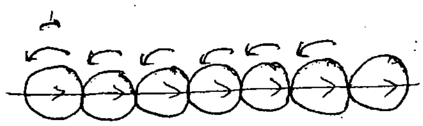
$\psi_B\,(0,2) = 2 \times 0 \times 2 + (0+1) \times 2^2$

$= \underline{\underline{4}}$

→ Rotational and Irrotational Flow: ←

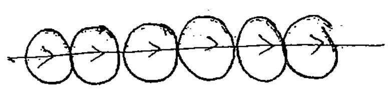
Rotational Flow is a Type of fluid flow in which the fluid particles a' moves along the stream line or stream path with its own axis rotation also

ex:



→ Irrotational Flow:

Type of Flow in which fluid molecules moves in stream line without rotating about its own axis



$(\omega x, \omega y, \ldots)$

$\omega x \quad \omega y \quad \omega z$

$$\text{circulation} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & v & w \end{vmatrix}$$

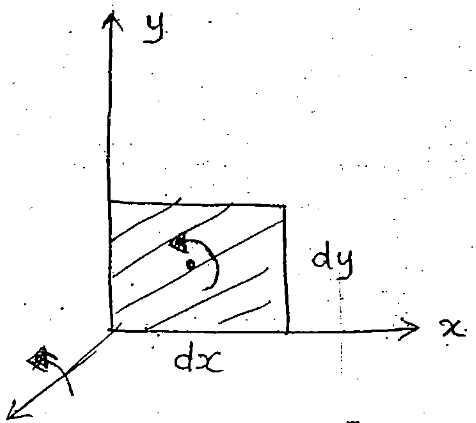
$\downarrow$

2ω

$$\omega x = \frac{|\text{circulation}|}{2} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\omega y = \frac{|\text{circulation}|}{2} = \frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)$$

$$\omega z = \frac{|\text{circulation}|}{2} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$



$$\omega z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$\vec{V} = (u)\hat{i} + (v)\hat{j} + (w)\hat{k}$$

If $\omega z = 0$ {Irrotational flow}

$\omega z \neq 0$ {rotational flow}

Q.18 ~~Sol~~

$$\vec{V} = (x + 2y + 2)\hat{i} + (4 - y)\hat{j}$$

$u = x + 2y + 2$

$v = 4 - y$

$$\omega z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$\frac{\partial v}{\partial x} = 0 \; ; \quad \frac{\partial u}{\partial y} = 2$

$$\omega z = \frac{1}{2}\left[0 - 2\right]$$

$= \underline{-1}$ So rotational

$-1 \neq 0$ (d)

(17) $u = ax + by$

$v = cx + dy$

$\frac{\partial v}{\partial x} = c$

$$\omega z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$\frac{\partial u}{\partial y} = b$

$\frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = 0$ for irrotation

$\frac{1}{2}\left[c - b\right] = 0$

$\underline{c = b}$

2001 GATE

(18) A 2-D flow velocity vector is given as

$\vec{V} = (x + 2y + 2)\hat{i} +$

→ SHEAR STRAIN RATE: ←

$\tau_x = \mu \frac{du}{dy}$ → Rate of shear strain

$\tau_y = \mu \frac{dv}{dx}$

$$\boxed{\varphi_{xy} = \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]}$$

$$\boxed{\varphi_{zx} = \left[\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right]}$$

(23.) $\bar{V} = x^2y\hat{i} + 2xy^2z\hat{j} - yz^3\hat{k}$

$$\boxed{\varepsilon_{yz} = \frac{1}{2}\left[\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right]}$$

$$u = x^2y$$
$$v = 2xy^2z$$
$$w = yz^3$$

$$\frac{\partial v}{\partial z} = 2xy^2$$

$$\frac{1}{2}\left[2xy^2 - z^3\right]$$

At $x = -2$, $y = -1$, $z = 2$

$$\varepsilon_{yz} = \frac{1}{2}\left[2\times(-2)(-1)^2 - (2)^3\right]$$

$$\frac{1}{2}\left[-4-8\right]$$

$$\varepsilon_{yz} = \frac{1}{2}\left[-12\right] = -12$$

$$\underline{\varepsilon_{yz} = -12}$$

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(11.)

$$u_x = \frac{Vy}{h}$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$\frac{1}{2}\times\left[0 - \frac{V}{h}\right]$$

$$= \underline{\frac{-V}{2h}}$$

$$\omega_y = \frac{1}{2}\left[\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right]$$

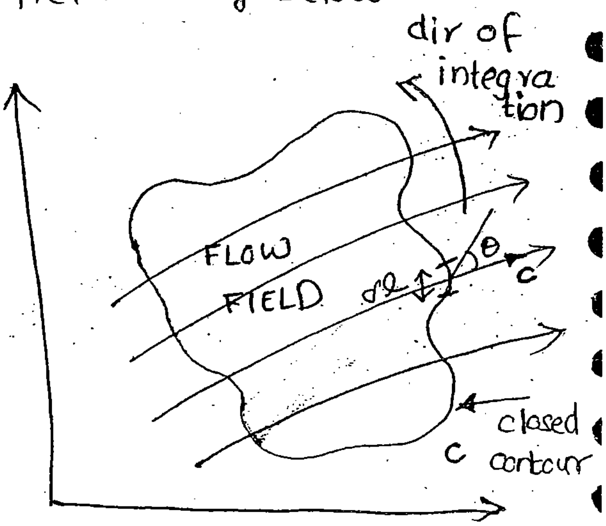
$$\underline{= 0}$$

Conclusions of kinematics of fluid:-

In fluid rotational system, the particle traces the path is expressed in terms of circulation and vorticity. circulation will have 3 components about abt 3 different planes namely ωx, ωy and ωz. A particle in xy plane can rotate about 3rd plane axis ωz. Similarly remaining 2 planes also provide two angular speed components. Circulation is the fluid flow in a a closed curve. which is mathematically line integral of tangential velocity component

CIRCULATION

Circulation is defined as the line integral of velocity around a closed contour in a flow field. Fig Below shows a closed contour C.



velocity vector c at an elemental length "dl" is inclined at an angle θ to the curve. The line integral is $c\cos\theta \cdot dl$ and circulation is $\Gamma = \oint_C c\cos\theta \, dl$.

$\oint_C$ represents integration around closed contour c.

Vorticity (ξ)

It is the limiting value of fluid circulation and the contour covered by the fluid particle tends to be zero. Hence Vorticity is Given by circulation per unit Area.

$$\text{Vorticity } \xi = \frac{\text{circulation}}{\text{Area}}$$

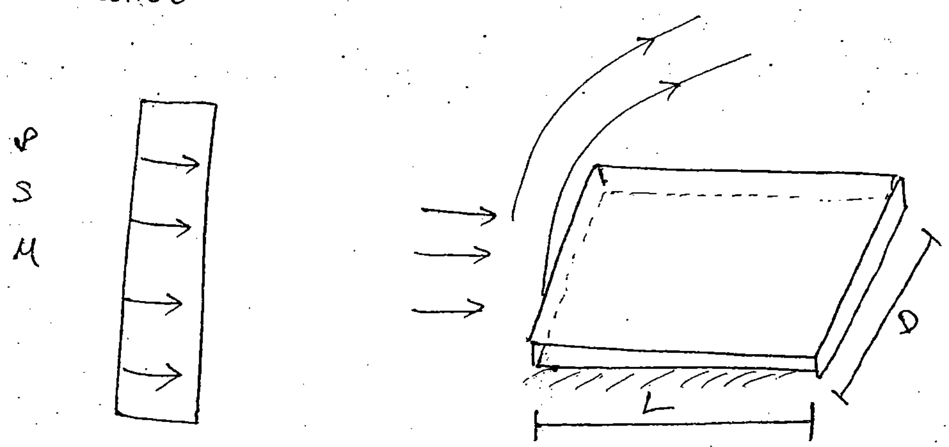
Introduction:-

In Nature two observations. one is a fluid passing over the stationary Body. Next is a Body moves in static fluid. In the above 2 cases the pressure gradient is zero which means flow is external.

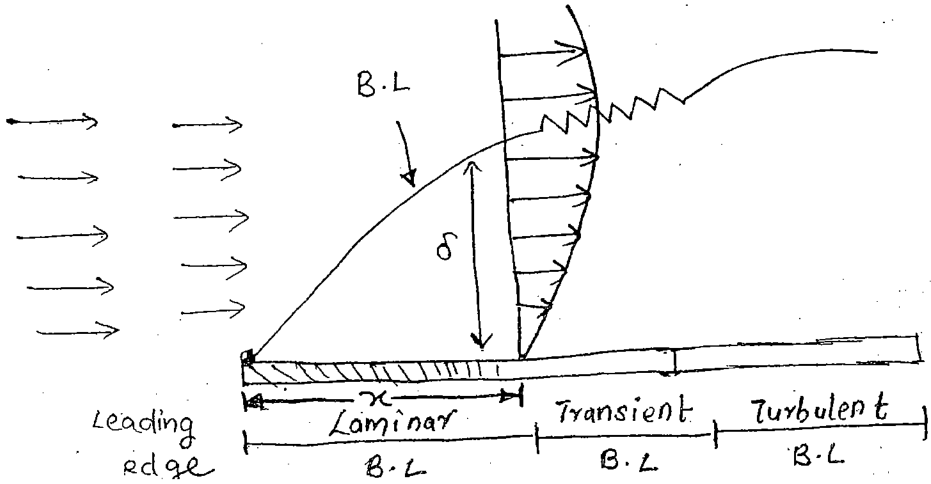
eg: Aeroplane motion in motion, submarine in water, Gas Tourbine Blade in chamber, steam Turbine Blades, High speed Rotary equipment.

When fluid is passing over the Body a thin layer is formed around the Body due to which they bound to be mass flow Rate loss, momentum loss, and kinectic energy of the moving fluid effects the surface of the Body under consideration. eg: Nose of the Aeroplane.
wings of space vehicle.
Blade tips of turbine.

T = const



B.L
Boundary Layer



$Re = 5\times10^{+5}$

$$\underset{\text{(Critical)}}{Re_x} = \frac{u_\infty \cdot x}{\nu} = 5\times10^5$$

~~Ma~~ Boundary Layer is a Thin layer around the solid surface which takes place due viscosity, Resistive force and to some extend due due to ~~circ~~ pressure forces (circular ~~β~~ surface). Boundary Layer is Measured from Leading edge in 2D ~~of~~ till the Fluid reach its original ~~ve~~ uniform velocity. within the Boundary Layer the velocity profile exits ~~at~~ in different forms, at particular distance along the plate, the ~~far~~ ⊥r distance is named as Boundary Layer thickness. Boundary Layer thickness (δ) is a fn of distance 'x' and Reynolds No: within the ~~Boundary Layer thickness~~

Laminar Layer, $\delta = \dfrac{5x}{\sqrt{Re_x}}$

$$\delta = \frac{5x}{\sqrt{\dfrac{u_\infty \cdot x}{\nu}}}$$

$$\delta \propto \frac{x}{\sqrt{x}} \quad , \quad \delta \propto (x)^{1/2}$$

$$\boxed{\frac{\delta_1}{\delta_2} = \sqrt{\frac{x_1}{x_2}}}$$

Q) A ~~p~~ flat plate of length 5m and width 2m subjected to an external air flow at 10m/s. the properties of air are $\rho = 1.2\ kg/m^3$ and $\nu = 0.15$ stoke. Determine the following

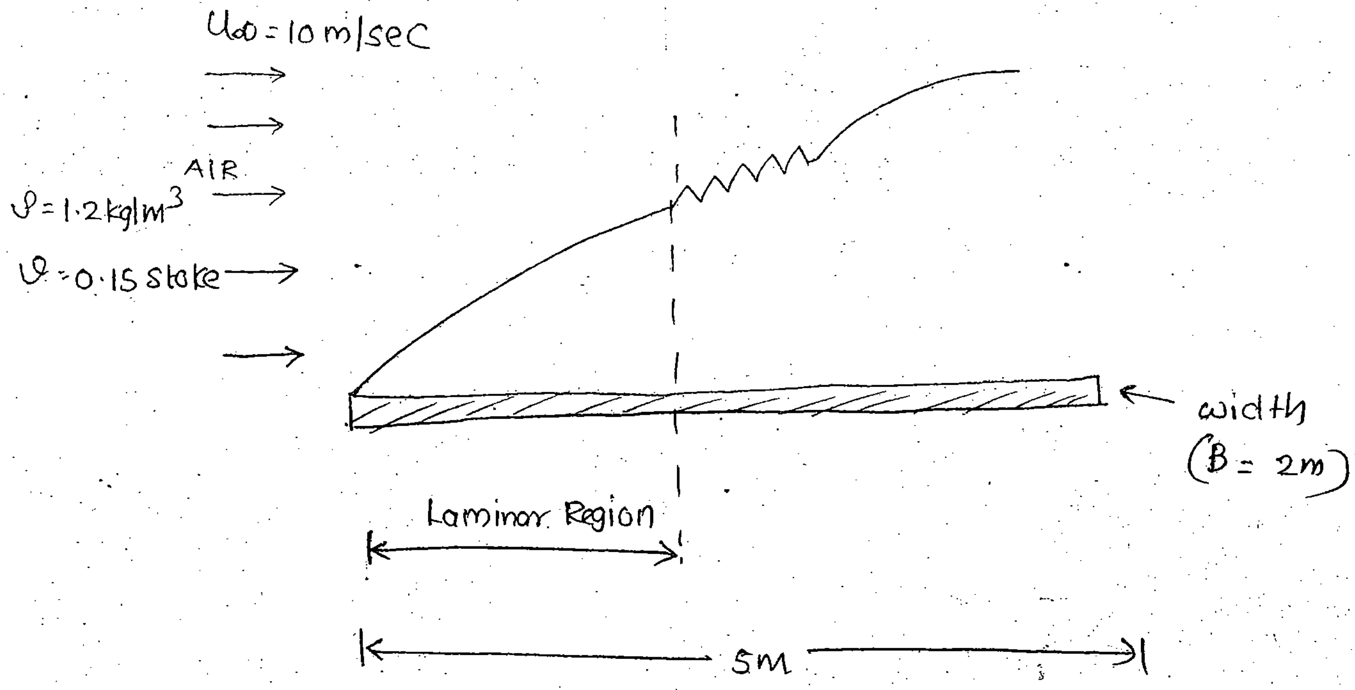
(i) The distance at which Laminar flow is seized.

0.5m

(3) Boundary Layer thickness downstream of the point where the initial Boundary thickness estimated (0.5m) down (4) The mass flow Rate through the specified Boundary Region in kg/s.

(5) Shear stress at wall of the plate.

(6) shear force at the surface of the laminar Region.



(1)

$$Re_x = \frac{U_\infty \cdot x}{\nu}$$

$$5 \times 10^5 = \frac{10 \times x}{0.15 \times 10^{-4}}$$

$$\therefore \underline{\underline{x = 0.75 \text{ m}}}$$

(2) NOTE:

$$\boxed{Re = \frac{V \cdot L_c}{\nu}}$$

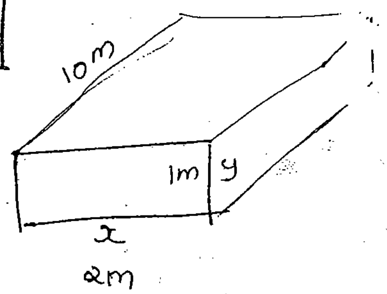
L_c = characteristic length

$$\boxed{Re_{plate} = \frac{V \cdot x}{\nu}}$$

$$\boxed{L_c = \frac{4A}{P}}$$

$$L_c = \frac{4(xy)}{2(x+y)}$$

$$L_c = \frac{4(2 \times 1)}{2(2+1)}$$



(b) Boundary Layer thickness at 0.5 m

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

where $x = 0.5\,m$ $\qquad Re_x = \frac{u_\infty x}{\nu}$

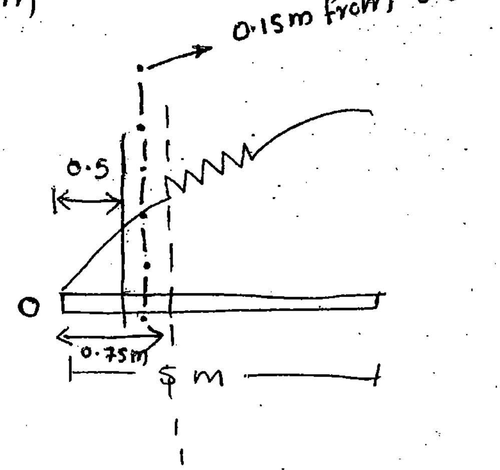
$$= \frac{10 \times 0.5}{0.15 \times 10^{-4}}$$

$$= 3.33 \times 10^5$$

$$\therefore \delta_{x=0.5\,m} = \frac{5 \times 0.5}{\sqrt{3.33 \times 10^5}}$$

$$\delta_{x=0.5\,m} = 0.0043\ m = 4.3\ mm$$

(c) Boundary layer thickness downstream at 0.15 m

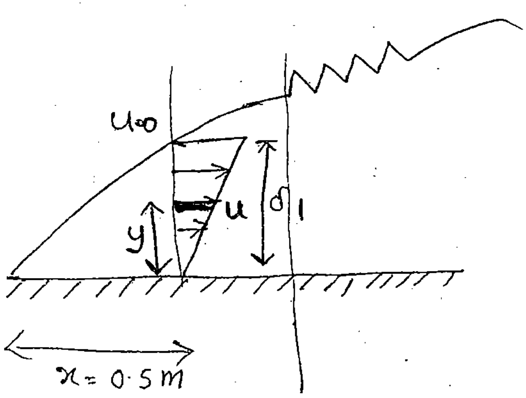


$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{x_1}{x_2}}$$

$$\frac{4.3}{\delta_2} = \sqrt{\frac{0.5\ (m)}{(0.5 + 0.15)\ m}}$$

$$\delta_2 = 4.9\ mm$$

(d) Mass flow Rate ($\dot{m}$)



$$d\dot{m} = \rho\, dQ$$

$$d\dot{m} = \rho . dA . u$$

$$= \rho . B\, dy . u$$

$$\int d\dot{m} = \rho B \frac{u_\infty}{\delta} y$$

$$\int d\dot{m} = \rho B \frac{u_\infty}{\delta} \int_0^{\delta_1} y\, dy$$

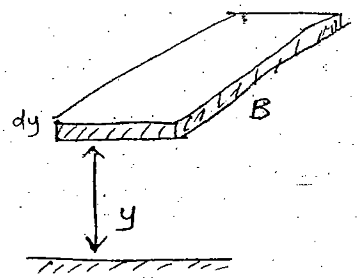
$$\dot{m} = \rho B \frac{u_\infty}{\delta} \left[\frac{y^2}{2} \right]_0^{\delta_1}$$

$$\boxed{\dot{m} = \frac{1}{2} \rho B u_\infty \delta}$$

LINEAR CURVE

$$\frac{u}{y} = \frac{u_\infty}{\delta_1}$$

$$u = u_\infty \cdot \frac{y}{\delta_1}$$



$$\therefore \frac{1}{2} \times 12 \times 2 \times 10 \times 0.0043$$

$$\underline{\dot{m} = 0.0516 \text{ kg/sec}}$$

(e) Shear stress $(\tau) = \mu \frac{du}{dy}$

$$= \mu \frac{d}{dy} \left[u_\infty \cdot \frac{y}{\delta} \right]$$

$$= \mu \frac{u_\infty}{\delta} \qquad \nu = \frac{\mu}{\rho}$$

$$= \nu \times \rho \times \frac{u_\infty}{\delta}$$

$$= (0.15 \times 10^{-4}) \times 12 \times \frac{10}{0.0043}$$

$$\underline{= 0.0418 \text{ N/m}^2}$$

(F) <u>Drag/~~Fe~~ shear force For laminar Region</u>

$$F_D = F_S = \tau \times A$$

$$= 0.0418 \times (\text{laminar length} \times \text{width})$$

$$0.0418 \times (0.75 \times 2)$$

$$F_D \underline{= 0.06 \text{ (N)}}$$

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Q16

$u_\infty = 280$ kmph

$= 280 \times \frac{5}{18} = \underline{77.77 \, m/sec}$

$\nu = 0.15$ stokes

$= 0.15 \times 10^{-4} \frac{m^2}{s}$

$Re_x = 3 \times 10^5$

(a) distance at which Laminar Layer terminates :

$$Re_x = \frac{u_\infty \cdot x}{\nu}$$

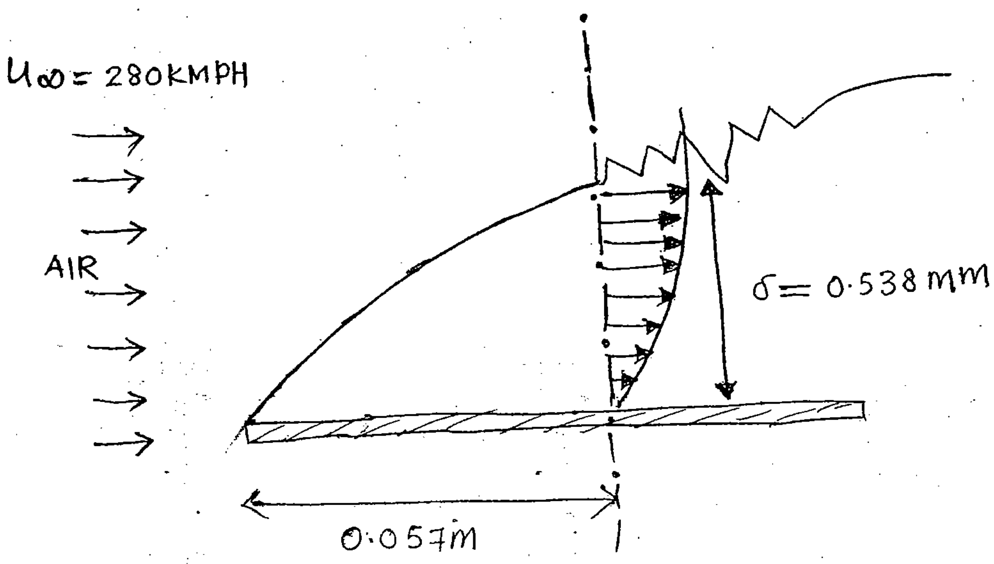
$$3 \times 10^5 = \frac{77.77 \times x}{0.15 \times 10^{-4}}$$

$$\underline{\underline{x = 0.057 \, m}}$$

(b) thickness of Boundary Layer

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 0.057}{\sqrt{3 \times 10^5}}$$

$$\underline{= 0.538 \, mm}$$



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Q2)

$$Re_x = \frac{Vx}{\nu} \qquad \delta = \frac{5x}{\sqrt{Re_x}}$$

$$\delta \propto \frac{1}{\sqrt{Re_x}} \qquad \underline{x = \text{const}}$$

$$\delta_1 \sqrt{(Re_x)_1} = \delta_2 \sqrt{(Re_x)_2}$$

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{Re_{2x}}{Re_{1x}}} = \sqrt{\frac{256}{100}} = \underline{1.6}$$

Q3)

$$\tau = \mu \frac{du}{dy}$$

$$\tau \propto \frac{1}{y}$$

$$\tau \propto \frac{1}{\delta}$$

$$\tau \propto \frac{1}{\sqrt{x}}$$

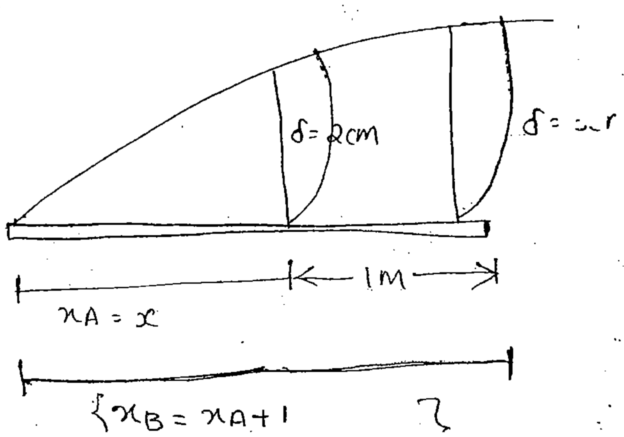
$$\boxed{\frac{\tau_1}{\tau_2} = \sqrt{\frac{x_2}{x_1}}}$$

Q4) a

Q5) a

Q6) c

Q7)



$$\frac{\delta_A}{\delta_B} = \sqrt{\frac{x_A}{x_B}}$$

$$\frac{2\ cm}{3\ cm} = \sqrt{\frac{x\ (metres)}{(x+1)\ metres}}$$

$$\frac{4}{9} = \frac{x}{x+1}$$

$$\therefore x = 0.8m$$

8.) D

9.) $\tau = \mu \frac{du}{dy}$

$$\frac{u}{u_m} = 1.5 \times \eta$$

$$= 1.5 \times \frac{y}{\delta}$$

$$\tau = \mu \frac{d}{dy}\left[u_m \cdot 1.5 \times \frac{y}{\delta}\right]$$

$$= \mu \cdot u_m \times \frac{1.5}{\delta}[1]$$

$$\frac{1.5\,\mu\, u_m}{\delta}$$

$$\therefore \tau = K\left(\frac{\mu u_m}{\delta}\right)$$

$$\therefore K = 1.5$$

(ISRO) Q.) The transition Reynold No for Flow over a flat plat is 5 lakhs. what is the distance from the leading edge at which transition will occur for the flow of fluid with a uniform vel of 1m/s ?

~~unifor~~ ($\nu = 0.86 \times 10^{-6}\ m^2/s$)

(a) 0.43 m (b) 1m (c) 43 m (d) 103 m

$$Re_x = \cancel{\frac{V x}{}} \frac{Vx}{\nu} = \frac{1 \times x}{0.86 \times 10^{-6}}$$

$$5\times10^5 = \frac{1\times x}{0.86\times10^{-6}}$$

$$\underline{x = 0.43\,m}$$

(Q.) GATE 04 ME

For Air flow over a flat plate the velocity 'u' and Boundary layer thickness δ can be expressed Resp as

$$\frac{u}{u_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \quad \text{and} \quad \delta = \frac{4.64x}{\sqrt{Re_x}}$$

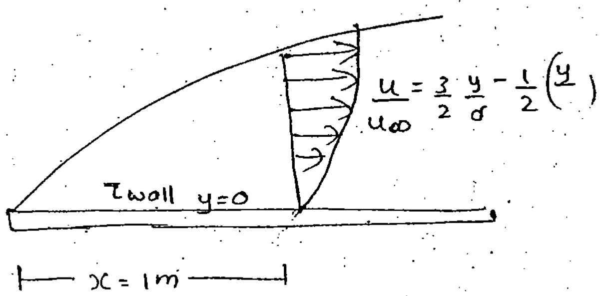
The free stream vel of Air is 2m/s and air properties are $\rho = 1.23\,kg/m^3$ and $\nu = 1.5\times10^{-5}\,m^2/s$. The magnitude of wall shear stress at $x = 1m$.

(a) $2.36\times10^{-2}\,N/m^2$

(b) $43.6\times10^{-3}\,N/m^2$

c) $436\times10^{-3}\,N/m^2$

(d) $2.18\times10^{-3}\,N/m^2$



Ans: $u_\infty = 2m/s$

$\rho = 1.23\,kg/m^3$

$\mu = 1.5\times10^{-5}$

$\tau_{(x=1m)}$

$$\tau = \mu\frac{d(u)}{dy} = \mu\frac{d}{dy}\left[u_\infty\left(\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right)\right]$$

$$= \mu u_\infty \frac{3}{2}\times\frac{1}{\delta} - \frac{1}{2}\times3\left(\frac{y}{\delta}\right)^2\times\frac{1}{\delta}$$

$$\tau_{\substack{x=1\\y=0}} = \mu u_\infty\left[\frac{3}{2\delta} - \frac{3}{2}\frac{y^2}{\delta^3}\right]$$

$$\mu u_\infty\left[\frac{3}{2\delta} - \frac{3}{2}\frac{0^2}{\delta^3}\right] = \mu u_\infty\frac{3}{2\delta}$$

$$\tau_{x=1} = \frac{1.5\,\mu\, u_\infty}{\delta}$$

$$\delta_{x=1} = \frac{4.64\,x}{\sqrt{\frac{u_\infty \cdot x}{\nu}}} \qquad \frac{4.64 \times 1}{\sqrt{\frac{2\times 1}{1.5\times 10^{-5}}}}$$

$$\underline{\underline{\delta = 0.0127\,m}}$$

$$\tau_{x=1} = 1.5 \times (\nu \times \rho) \times \frac{u_\infty}{\delta} \qquad \text{res } \mu/\rho$$

$$= 1.5 \times (1.5\times10^{-5} \times 1.23) \times 2 \times \frac{1}{0.0127}$$

$$\underline{\underline{= 4.36\times10^{-3}\,N/m^2}}$$

Q.) A smooth ~~pl~~ flat plate with a sharp leading edge is placed along a gas stream flow. $u_\infty = 10\,m/s$. The thickness of the Boundary layer at the section R-S is 10mm. The breadth of the plate is 1m (into the paper of plane). The density of the Gas is $1\,kg/m^3$ and Assume that the Boundary layer is thin, 2-D flow which follows a linear velocity distribution $\frac{u}{u_\infty} = \frac{y}{\delta}$ at the section R-S; where y is the Height measured from plate.

Soln:- $u_\infty = 10\,m/s$.

$\delta = 10\,mm$

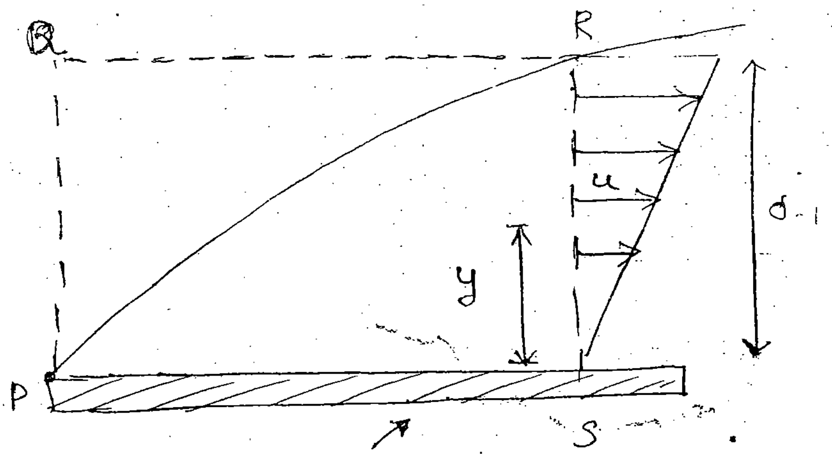
$\rho = 1\,kg/m^3$

$$\delta = \frac{5x}{\sqrt{Re_x}} \qquad \frac{5x}{\sqrt{}}$$

(1.) The Mass flow Rate in (kg/sec) across the Section Q-R is

(a) 0 (b) 0.05 kg/sec

(c) 0.10 kg/sec (d) 0.15 kg/sec



Flate plate

$$\dot{m} = \frac{1}{2}\rho \cdot B \cdot u_\infty \cdot \delta$$

$$\frac{1}{2}\times 1\times 10\times 1\times 0.01$$

$$\dot{m} = 0.05\ kg/sec$$

$$\boxed{\frac{u}{u_\infty} = \frac{y}{\delta}}$$

$$\dot{m} = \rho\times Q$$

$$= \rho A u$$

$$= \rho A u_\infty \cdot \frac{y}{\delta}$$

$$\frac{\rho u_\infty}{\delta} B \int_0^\delta y\,dy$$

$$\frac{\rho u_\infty \cdot B}{\delta}\left[\frac{y^2}{2}\right]_0^\delta$$

$$= \frac{\rho u_\infty \cdot B}{\delta}\times\frac{\delta^2}{2}$$

$$\dot{m} = \frac{1}{2}\times\rho\times B\times u_\infty\times\delta$$

(2.) Drag ~~[illegible]~~ force on the plate b/w P and S.

$$F_D = F_s = \tau\times Area$$

$$= \mu\frac{du}{dy}\times Area$$

$$F_D = \mu\frac{d}{dy}\left[u_\infty\cdot\frac{y}{\delta}\right]\times Area$$

$$F_D = \mu\cdot\frac{u_\infty}{\delta}\times\left[PS\times Breadth\right]$$

Assume

$$\mu = 18\times 10^{-6}\ \frac{N\text{-}sec}{m^2}$$

$$F_D = \frac{18\times10^{-6}\times 10}{0.01}\left[1\times 1\right]$$

$$0.018\ \frac{N}{m^2}\times 1\times 1$$

$$0.018\ \frac{N}{m}$$

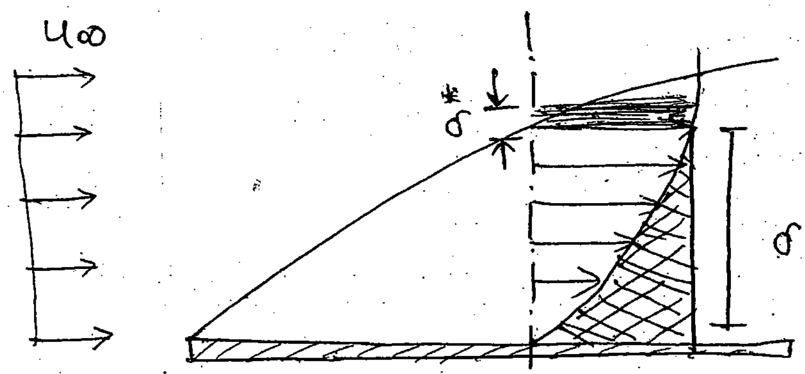
ILL EFFECTS OF BOUNDARY LAYER FORMATION

The following are ill effects

1. Loss of Mass flow Rate (displacement thickness δ^*)
2. Loss of Momentum (Momentum thickness θ)
3. Loss of kinectic energy [energy thickness δ^{**}]

→ **Displacement Thickness:-**

It is the distance measured ⊥r to the surface of plate upto which, where the ~~fre~~ velocity reaches free stream velocity, the Boundary shifted to compensate loss of mass, due to boundary layer formed.



δ^* : Displacement thickness
(compensation of mass flow Rate)

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{u_\infty}\right) dy$$

→ **Momentum thickness:-**

It is the distance measured ⊥r to flat plate surface upto which the Boundary layer is shifted to compensate loss in momentum due to Boundary layer formation. It is given by θ = Momentum thickness

$$\int_0^{\delta} \frac{u}{u_\infty}\left[1 - \frac{u}{u_\infty}\right] dy$$

→ Energy thickness: (∪ /

It is the distance ⊥r to the flat surface upto which the Boundary layer shifted to compensate the loss of K.E of Moving Fluid due to Boundary layer formed.

Mathematically it is given by (δ^{**})

$$\delta^{**} = \int_0^{\delta} \frac{u}{u_\infty}\left[1-\left(\frac{u}{u_\infty}\right)^2\right] dy$$

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① (c)

List-II

2. Net main mass flow

② $\frac{u}{U} = \frac{y}{\delta}$

$$\frac{u}{u_\infty} = \frac{y}{\delta}$$

$$\delta^* = \int_0^{\delta}\left[1-\frac{u}{u_\infty}\right] dy \qquad u = u_\infty \cdot \frac{y}{\delta}$$

$$\delta^* = \int_0^{\delta}\left[1-\frac{u_\infty \cdot y}{u_\infty \cdot \delta}\right] dy$$

$$\int_0^{\delta}\left[1-\frac{y}{\delta}\right] dy = \left[y-\frac{y^2}{2\delta}\right]_0^{\delta}$$

$$\delta - \frac{\delta^2}{2\delta}$$

$$\delta - \frac{\delta}{2} = \frac{\delta}{2}$$

$$\frac{\delta^*}{\delta} = \frac{\delta}{2\delta} = \frac{1}{2}$$

$$\frac{\theta}{\delta} = 6$$

$$\theta = \int_0^{\delta} \frac{u}{u_\infty}\left[1-\frac{u}{u_\infty}\right] dy$$

$$\int_0^{\delta} \frac{u_\infty \cdot y}{u_\infty \, \delta}\left[1-\frac{u_\infty . y}{\delta \, u_\infty}\right] dy$$

$$\int_0^{\delta} \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy$$

$$\int_0^{\delta} \frac{y}{\delta} - \frac{y^2}{\delta^2}$$

$$\left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right]_0^{\delta} = \frac{\delta^2}{2\delta} - \frac{\delta^3}{3\delta^2}$$

$$= \frac{\delta}{2} - \frac{\delta}{3}$$

$$= \frac{\delta}{6}$$

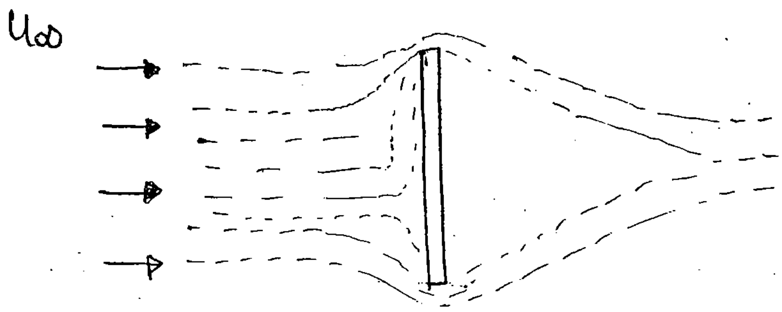
$$\frac{\theta}{\delta} = \frac{\delta}{6\delta} = \frac{1}{6}$$

(3) $\boxed{\text{SHAPE FACTOR}(H) = \dfrac{\delta^*}{\theta}}$ $= \dfrac{\text{Displacement thickness}}{\text{Momentum thickness}}$

Ratio of displacement thickness to Momentum thickness

$$H = \frac{\frac{\delta}{2}}{\frac{\delta}{6}} = 3$$

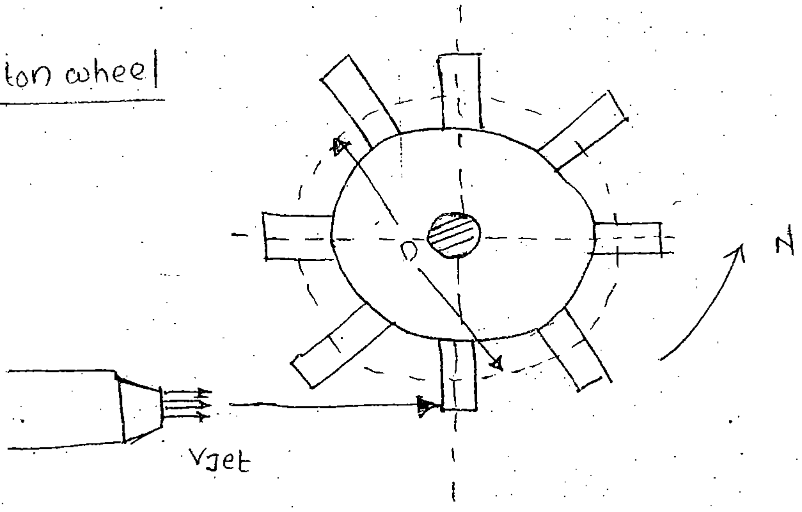
(8) WAKE:-



disturbed Region downstream of Seperation

Tangential Flow

ex: pelton wheel



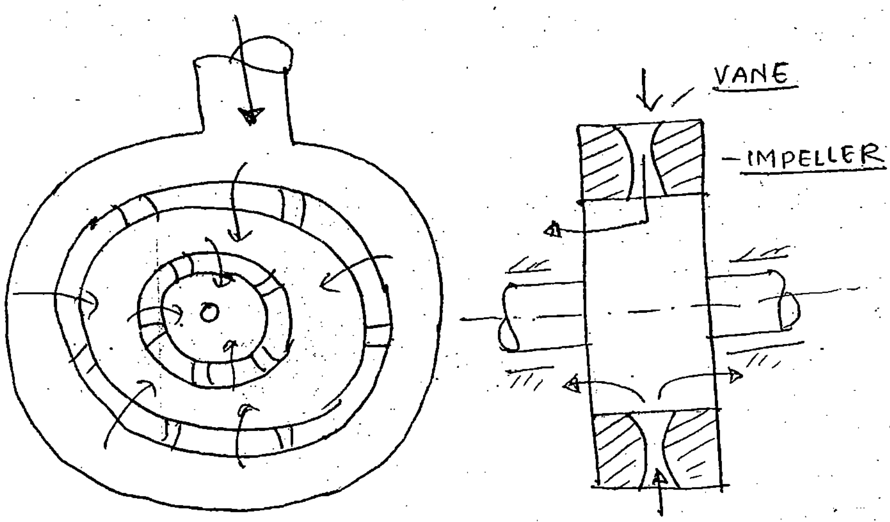
$V_{Jet} = \sqrt{2gH}$

$u = \frac{\pi d N}{60}$ (m/sec)

$u = \omega \times R$ (m/sec)

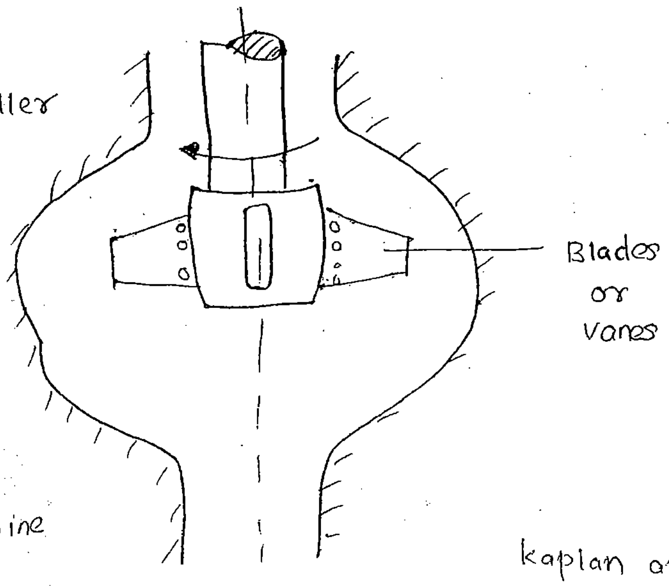
Radial Flow:-

Francis Turbine (Inward flow)



Axial Turbine

kaplan and Propeller



Note:

① Vanes are adjustable in kaplan

② Vanes are Not adjustable in propeller

High Head

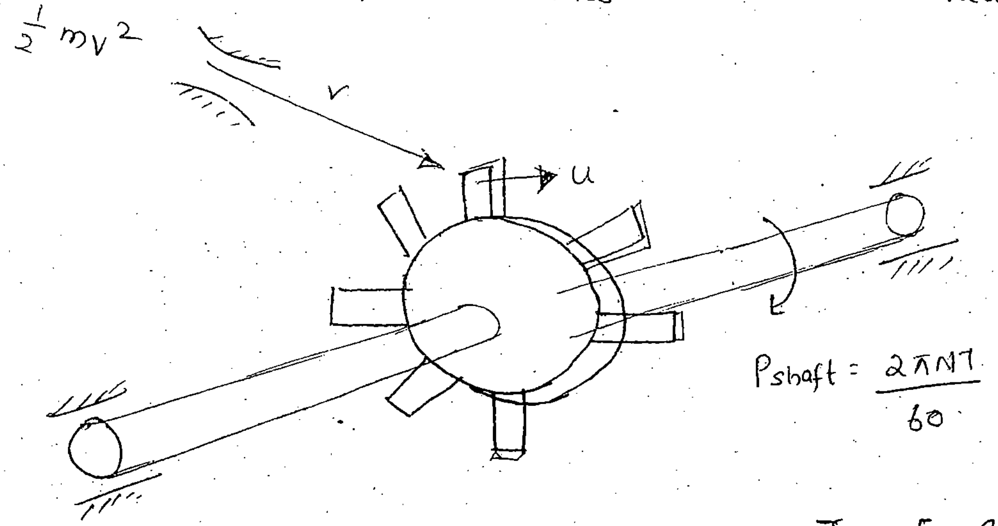
ex: Pelton wheel
Francis Turbine

kaplan and Propeller } Low Head

~~Impelle~~

Water Action

Impulse Turbines | Reaction Turbines



$$P_{shaft} = \frac{2\pi N T}{60}$$

$$T = F \times R$$
$$= m a \times R$$
$$= m a \times \frac{D}{2}$$
$$T = (\dot{m} \times \Delta V) \times \frac{D}{2}$$

③ No draft Tube ~~Requid~~ Required

$$\boxed{T = \text{Rate of change of Momentum} \times \text{Radius of wheel}}$$

Eulers Turbo Machine

(Turbine)

1. works in open atmosphere
2. Input energy pure impulse action
3. No draft tube is Required
4. High Head reqd
5. Low discharge of water
6. Tangential flow

7) Newton's 2nd Law

Reaction Turbine :-

1) works under pressure
2) Input energy is pure potential or pressure before Turbine ...

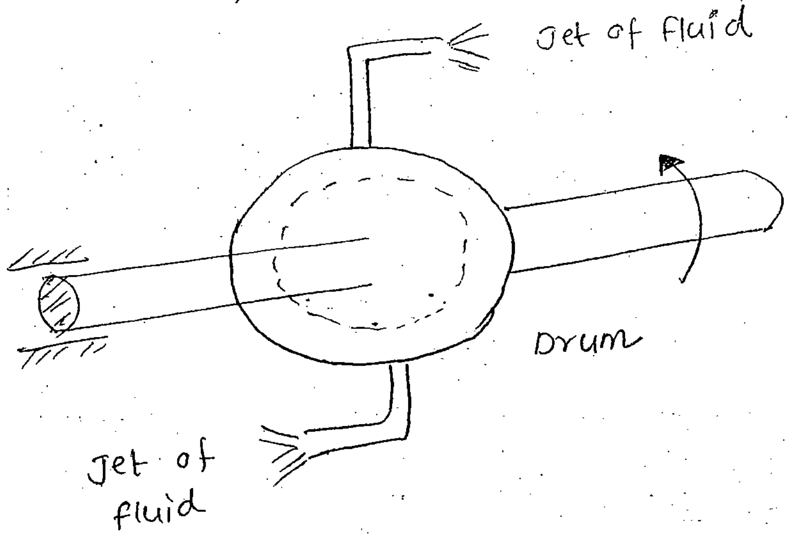
Larger the turbine partly Potential and partly kinetic energy

(3) Draft Tube in essential element

(4) Medium and Low Heads

(5) Medium and large discharge

(6) Newton's 3rd Law of Motion



eg: Francis Turbine
Kaplan Turbine
propeller Turbine

(iv) Based on Specific speed :- (N_s)

Specific speed is a relative term used in Turbines to the use of three operating parameters.

(i) head on Turbine

(ii) Power produced by Turbine

(iii) Speed of Turbines

$$\boxed{N_{s\,turbine} = \frac{N\sqrt{P}}{(H)^{5/4}}}$$

where $N \rightarrow$ rpm

$P \rightarrow$ kilowatt (kW)

$H \rightarrow$ metres

$$\boxed{N_s \text{ pump} = \frac{N\sqrt{Q}}{(H)^{3/4}}}$$

$N \rightarrow$ rpm

$Q \rightarrow m^3/sec$

$H \rightarrow$ metres

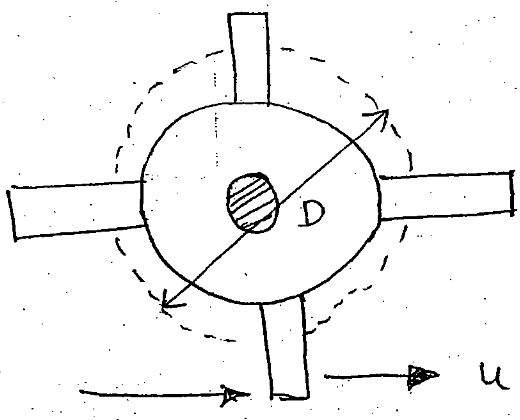
$P = \dot{m} g H$

$P = \rho Q g H$

$\boxed{P \propto Q \cdot H}$ — (1)

$Q = \underset{(c/s)}{\text{Area}} \times \text{velocity}$

$\boxed{Q \propto D^2 \cdot V}$ — (2)



$u = \frac{\pi D N}{60}$

$V = \sqrt{2gH}$

$\cancel{P = \frac{K H^{5/2}}{N^2}}$

$P \propto D^2 V \cdot H$

$P \propto D^2 \sqrt{H} \cdot H$

$\cancel{P \propto D^2 H}$ $P \propto \frac{H^1}{N^2} \cdot \sqrt{H} \cdot H^1$

$$P \propto \frac{(H)^{5/2}}{N^2}$$

$$P = \frac{K H^{5/2}}{N^2}$$

$K =$ const

Let $P =$

Dimension

$\rightarrow M^a L^b T^c$

$\rightarrow F^x L^y T^z$

$$N_s = \frac{N\sqrt{P}}{(H)^{5/4}} = \frac{rpm\sqrt{kW}}{(m)^{5/4}}$$

$$= \frac{\left(\frac{1}{Time}\right)\sqrt{\frac{Joule}{sec} = \frac{N-m}{sec} = \frac{kg-\frac{m}{sec^2}\cdot m}{sec}}}{(m)^{5/4}}$$

$$\frac{M^{1/2} \cdot L^1}{T \cdot T^{3/2} \cdot L^{5/4}}$$

$$= M^{1/2} \, L^{-1/4} \, T^{-5/2}$$

$a = 1/2$

$b = -1/4$

$$C = \frac{-5}{2}$$

$$\Rightarrow M^{1/2} L^{-1/4} T^{-5/2}$$

x ———————— x

$$= \frac{F^{1/2} \; L^{1/2}}{T \times T^{1/2} \times (L)^{5/4}}$$

$$= F^{1/2} \; L^{-3/4} \times T^{-3/2}$$

$$x = 1/2, \; y = -\frac{3}{4}, \; z = -\frac{3}{2}$$

$$\Rightarrow F^{1/2} \; L^{-3/4} \; T^{-3/2}$$

Dimensionless specific speed = Shape Number

$$N_s = \frac{N' \sqrt{\frac{P'}{\rho}}}{(gH)^{5/4}}$$

N' = rps = $\frac{N}{60}$ rpm

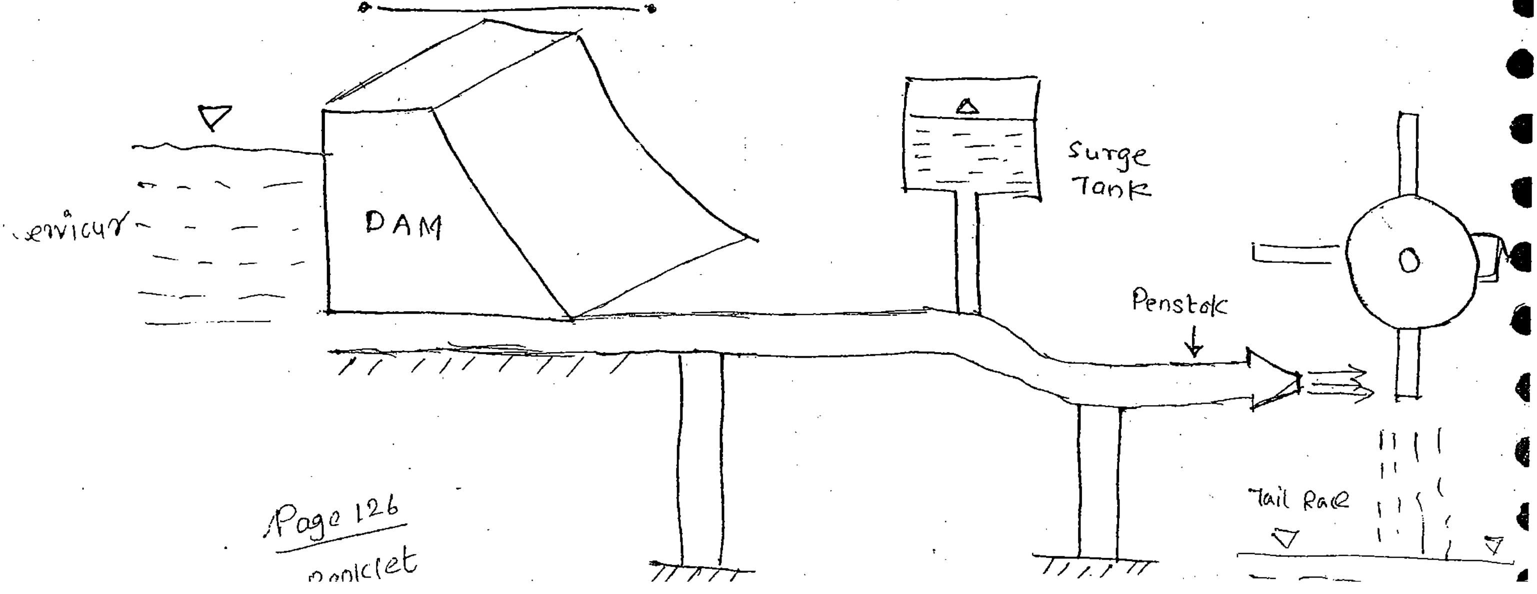
P' = in watt

$= \frac{P}{1000}$ kW

ρ = density

$a = b = c = 0$

Line diagram of Hydro electric Power plant



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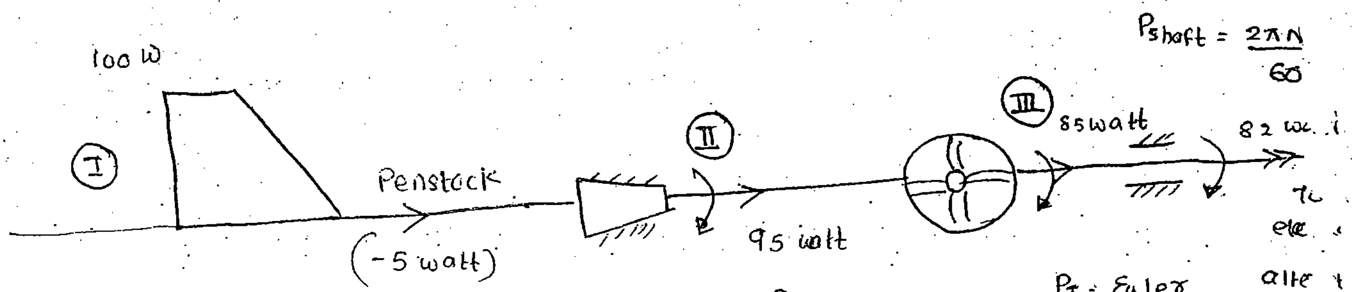
Power flow diagram:-



$$P_{water} = \dot{m} g H = \rho g Q H$$

$$K.E = \frac{1}{2}\dot{m}v^2$$

P_T = Euler turbine work $= \dot{m}\,\Delta V \times u$

Water Power

$$P_{water} = \dot{m} g H$$
$$= \rho g Q H$$

Kinectic energy

$$K.E = \frac{1}{2}\dot{m}v^2$$

Turbine power

$$P_T = \dot{m}\,\Delta V \times u$$

Shaft Power:-

$$P_{shaft} = \frac{2\pi N T}{60}$$

$$\eta_{nozzle} = \frac{II}{I} = \frac{\frac{1}{2}\dot{m}V_1^2}{\dot{m}gH}$$

$$\eta_{Turbine} = \frac{III}{II} = \frac{P_T}{\frac{1}{2}\dot{m}V_1^2}$$

$$\eta_{Hydraulic} = \eta_{nozzle} \times \eta_{Turbine}$$

$$\eta_{mechanical} = \frac{P_{shaft}}{P_{Turbine}}$$

$$\eta_{overall} = \frac{P_{shaft}}{P_{water}} = \eta_{Nozzle} \times \eta_{Turbine} \times \eta_{mech}$$

$$= \eta_{Hydraulic} \times \eta_{mechanical}$$

Model Testing :-

It is an experimental approach with due Support calculation of the Turbine. Before manufacture, erection and Commission it is necessary to apply model Analysis and Model Testing. In ~~model~~ model Analysis of Turbine four Numbers are used.

(i) specific speed
(ii) Head quantity
(iii) discharge number or qty
(iv) Power number or qty.

Model Turbine :- (small Turbine or Laboratory Turbine)

Tcy Turbine.

P- Prototype Turbine (Large scale Turbine)

or

(*) Scale Ratio or Model Ratio $= \frac{D}{D} = \frac{D_{model}}{D_{prototype}}$

ex: 1:2
1:4
1:10
1:20

(I) $(N_s)_m = (N_s)_p$

$$\frac{N_m\sqrt{P_m}}{(H_m)^{5/4}} = \frac{N_p(\sqrt{P_p})}{(H_p)^{5/4}}$$

(II) Head Number.

water jet velocity $\propto$ Tangential vel of wheel

$$V_{Jet} \propto u$$

$$\sqrt{2gH} \propto \frac{\pi DN}{60}$$

$$\sqrt{H} \propto DN$$

$$\frac{\sqrt{H}}{DN} = \text{constant}$$

Squaring Both sides

$$\frac{H}{D^2N^2} = \text{const}$$

$$\frac{\sqrt{H_m}}{D_m \cdot N_m} = \frac{\sqrt{H_p}}{D_p \cdot N_p}$$

III Discharge Number:-

Discharge = Area (c/s) × vel of fluid

$$Q = D^2 V$$

$$Q = D^2 \cdot \sqrt{2gH}$$

$$\boxed{Q \propto D^2 \sqrt{H}}$$

$$\boxed{\frac{Q}{D^2\sqrt{H}} = \text{constant}}$$

IV Power Number.

$$\text{Power} = \dot{m} g H$$

$$= \rho g Q H$$

$$P \propto QH$$

$$P \propto D^2 \sqrt{H} \cdot H$$

$$P \propto D^2 H^{3/2}$$

$$\frac{P}{D^2 H^{3/2}} = \text{constant}$$

$$\boxed{\frac{P_m}{D_m^2 (H_m)^{3/2}} = \frac{P_p}{(D_p)^2 (H_p)^{3/2}}}$$

UNIT QUANTITIES :-

they are used for checking the performance of one individual turbine for which scale Ratio is 1 (same ~~sca~~ size of Turbine)

they are three unit quantities.

1. unit ~~Head~~ speed (Nu)
2. unit discharge (Qu)
3. unit Power. (Pu)

unit speed (Nu)

$$\sqrt{2gH} \propto \frac{\pi D N}{60}$$ (for same Turbine)

$$\sqrt{H} \propto N$$

$$N \propto \sqrt{H}$$

$$N = N_u \sqrt{H}$$

$$N_u = \frac{N}{\sqrt{H}}$$

ie $$\boxed{\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}}$$

2)

$$Q = A \cdot V$$

$$Q = \frac{\pi}{4} D^2 \cdot \sqrt{2gH}$$

For same turbine

$$Q \propto \sqrt{H}$$

$$Q = Q_u \sqrt{H}$$

$$Q_u = \frac{Q}{\sqrt{H}}$$ $\therefore$ $$\boxed{\frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}}$$

3.)

$$P = \dot{m} g H$$

$$P = \rho g Q H$$

$$P \propto Q \cdot H$$

$$P = P_u \cdot \sqrt{H} \cdot H$$

$$P_u = \frac{P}{\sqrt{H} \cdot H}$$

$$\boxed{P_u = \frac{P}{H^{3/2}}}$$

2010 GATE ME

A Hydraulic Turbine producing 1000 kW at 40m Head. If Head is Reduced to 20m then power delivered by Same Turbine would be

(a) 177 kW (b) 354 kW (c) 500 kW (d) 707

$$\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

$$\frac{1000}{(40)^{3/2}} = \frac{P_2}{(20)^{3/2}}$$

$P_2 =$ ______ kW

Q.) A hydraulic Turbine is Reqd to produce 25 MW at 50m Head and 90 rpm. The Laboratory facilities available to test 25 kW model turbine at 50m Head. What should be the model Turbine ~~at 50~~ runner speed and the scale Ratio of model & prototype? 107 101

Soln Given

$P_p = 300$ kW

$N_p = 100$ rpm

$H_p = 40$ m

$\frac{D_m}{D_p} = 1:4 = \frac{1}{4}$

$H_m = 10$ m

$P_m = ?$

we have $\frac{P_m}{\ldots^{3/2}} = \frac{P_p}{\ldots^{3/2}}$

$$P_m = \left(\frac{1}{4}\right)^{2} \cdot \left(\frac{10}{40}\right)^{-12} \cdot 300$$

$$\underline{\underline{2.34KW}}$$

$$\frac{\sqrt{H_m}}{N_m D_m} = \frac{\sqrt{H_p}}{N_p D_p} \quad ; \quad \sqrt{\frac{H_m}{H_p}} \times \frac{D_p}{D_m} \times N_p = N_m$$

$$\sqrt{\frac{5}{50}} \times \frac{6.3}{1} \times 90 = N_m$$

Q) A Model hydraulic Turbine is Tested at a Head of $\frac{1}{4}$th of Head with which Full scale Turbine operates. The Dia of model Turbine is 50% of full scale Turbine dia. If N is the rpm of the full scale turbine, the rpm of model will be

(a) $\frac{N}{4}$ (b) $\frac{N}{2}$ (c) N (d) 2N

Ans: Given

$H_m = \frac{1}{4} H_p$

$D_m = 0.5 D_p$

$N_p = N$

$N_m = ?$

Head Ratio

$N_p = N$

$$\frac{\sqrt{H_m}}{N_m D_m} = \frac{\sqrt{H_p}}{N_p D_p}$$

$$N_m = ? = \sqrt{\frac{H_m}{H_p}} \times \frac{D_p}{D_m} \times N_p = \sqrt{\frac{1}{4}} \times$$

$$= \underline{\underline{N}}$$

Q.) At a Hydroelectric power plant Available Head and Flow Rate are ~~25.3 m~~ 24.5 m and 10.1 m^3/s resp. if the Turbine is to Run at 4 revolutions per second with overall efficiency of 90% then the suitable Type of Turbine for the given conditions is

a) Pelton wheel ⓑ Francis Turbine ⓒ Kaplan Turbine ⓓ propellor Turbine

ANS:

$$N_s = \frac{N\sqrt{P}}{(H)^{5/4}}$$

where $N = 4$ rev/sec

$N = 4 \times 60$ rev/min $= 240$ rpm

$H = 24.5$ (m)

$P = ?$

$$\eta_o = \frac{P_{o/p}}{P_{water}} = \frac{P}{\dot{m} g H} = \frac{P}{\rho Q g H}$$

$$0.9 = \frac{P}{1000 \times 10.1 \times 9.81 \times 24.5}$$

$P = 2184$ kW

$$N_s = \frac{240\sqrt{2154.4 \text{ (kW)}}}{\left(\frac{24.5}{m}\right)^{5/4}}$$

$N_s = 205.8$ m

$30 < N_s < 275 \rightarrow$ Francis Turbine

$N_s > 275 \rightarrow$ Kaplan Turbine

$N_s < 30$: Low speed Turbine (Pelton wheel with single Nozzle)

Q.) In a Hydroelectric power station water is available at the rate of $175 m^3/s$ and a Head of 18 m. The Turbines run at speed of 150 rpm with overall efficiency 82%. Find the No of Turbines to be installed if they have Maximum Specific Speed of 460

Soln:

$Q = 175 \, m^3/s$

$H = 18 m$

$N = \cancel{0.82}\ 150 \, rpm$

$N_s = 460$

$\eta_o = 0.82$

No of Turbines = ?

$$N_s = \frac{N\sqrt{P}}{(H)^{5/4}} = \frac{150\sqrt{P}}{(18)^{5/4}}$$

$$460 = \frac{150\sqrt{P}}{(18)^{5/4}} = \quad \therefore P = 12927 \, kW$$

$$\underline{P = 12.93 \, MW}$$

$$\eta_o = \frac{P_{o/p}}{P_{water}} : \frac{P_{shaft}}{P_{water}}$$

$$0.82 = \frac{P_{shaft}}{\rho g Q H}$$

$$P_{shaft} = 0.82 \times 1000 \times 9.81 \times 175 \times 18$$

$$= 25.34 \times 10^6 \text{ watt}$$

$$= \underline{25.34 \, MW}$$

* No of Turbines Reqd = $\frac{\text{Total Power}}{\text{Power / Turbine}}$

$$= \frac{25.34}{12.93} = 1.98 \simeq \underline{2 \text{ Turbines}}$$

Q) A water Turbine producing 10 MW power is to be tested with a geometrically Similar 1:8 model which runs at same speed as that of prototype

ⓐ find the Power developed by the Model Turbine assuming efficiencies of model and prototype are equal

ⓑ find the Ratio of Heads and

ⓒ find the Ratio of Mass flow Rates b/w prototype and Model Turbines.

$P_p = 10\ MW = 10,000\ kW$

$\frac{D_m}{D_P} = \frac{1}{8}$

$N_m = N_p$

$\eta_m = \eta_p$

a) $P_m = ?$

b) $\cancel{\frac{H_m}{H_B}}\ \frac{H_p}{H_m} = ?$

c) $\frac{\dot{m}_p}{\dot{m}_m} = ?$

$\frac{\sqrt{H}}{N^2 D} = \text{const}$

$\frac{Q}{D^2 \sqrt{H}} = \text{const}$

$\frac{P}{D^2 (H)^{3/2}} = \text{const}$

$\frac{N \sqrt{P}}{(H)^{3/4}} = \text{const}$

$\dot{m} = \rho Q$

$\frac{\dot{m}_p}{\dot{m}_m} = \frac{\rho Q_p}{Q_m} = ?$

$\frac{\sqrt{H_m}}{\cancel{N_m^2} D_m} = \frac{\sqrt{H_p}}{\cancel{N_p^2} D_p}$

$\cancel{\frac{\sqrt{H_m}}{3}}\ \frac{\sqrt{H_p}}{\sqrt{H_m}} = \frac{D_p}{D_m} = 8$

$H_p = 64$

$$\frac{P_m}{D_m^2 (H_m)^{3/2}} = \frac{P_p}{D_p^2 (H_p)^{3/2}}$$

$$P_m = 10{,}000 \times \left[\frac{1}{8}\right]^2 \times \left[\frac{H_m}{H_p}\right]^{3/2}$$

$$10{,}000 \times \left[\frac{1}{8}\right]^2 \times \left[\frac{1}{64}\right]^{3/2}$$

$$\underline{P_m = 305 \text{ watt}}$$

c.) $$\frac{\dot m_p}{\dot m_m} = \frac{\rho Q_p}{\rho Q_m}$$

Discharge No:

$$\frac{Q}{D^2 \sqrt{H}} = \text{const}$$

$$\frac{Q_m}{D_m^2 \sqrt{H_m}} = \frac{Q_p}{D_p^2 \sqrt{H_p}}$$

$$\frac{Q_p}{Q_m} = \left[\frac{D_p}{D_m}\right]^2 \times \sqrt{\frac{H_p}{H_m}}$$

$$= [8]^2 \times \sqrt{64}$$

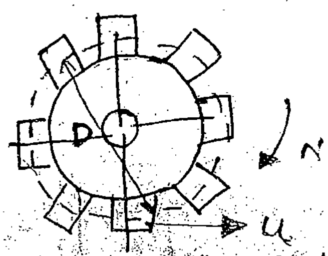
$$\frac{Q_p}{Q_m} = 8^3 = 512$$

Hydraulic Analysis of an Impulse Turbine

Velocity Triangles

- Water velocity
- wheel velocity
  - ↓ Bucket velocity / speed
  - ↓ Tangential velocity of wheel

$\left.\right\} u$

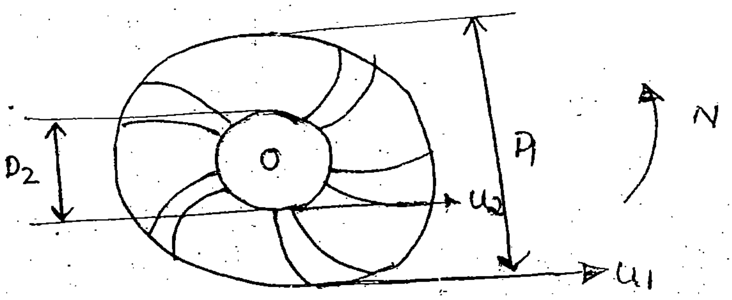


$u = \pi D N$

For impulse (Pelton wheel)

$u_1 = u_2 = u$

For ~~other~~ Francis Turbine



$u_1 = \omega \cdot R_1$

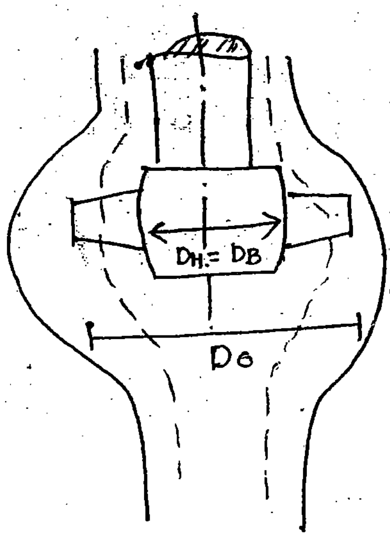
$$u_1 = \frac{\pi D_1 N}{60}$$

$u_2 = \omega \cdot R_2$

$$u_2 = \frac{\pi D_2 N}{60}$$

Kaplan Turbine :-

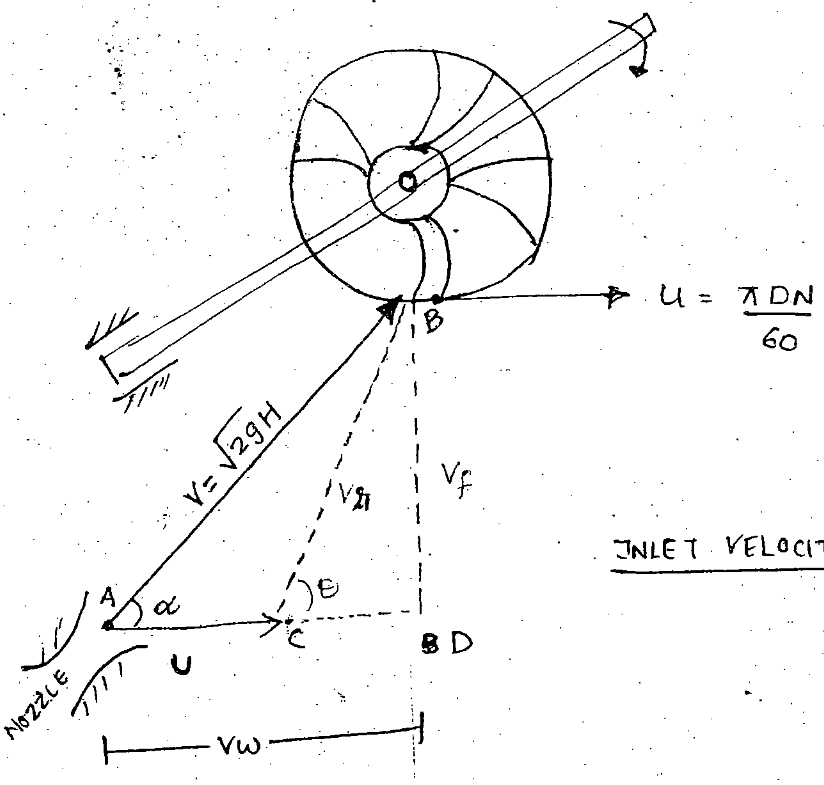
$$u = u_0 = \frac{\pi D_0 N}{60} \text{ (m/sec)}$$



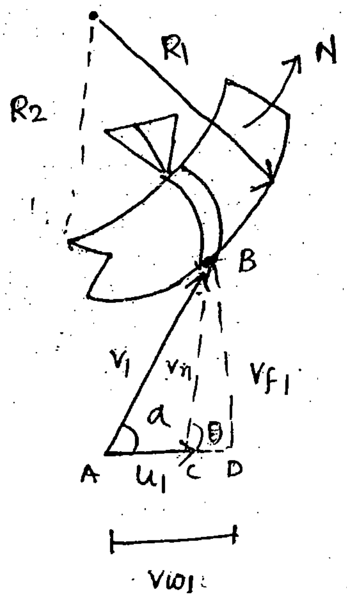
$$\boxed{D_{Hub} = \frac{1}{3} D_0}$$

Water Velocity

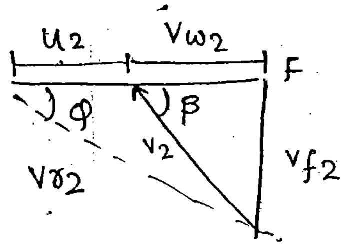
- Absolute velocity [V]
- Relative velocity [Vr]
- Flow velocity [Vf]
- whirl velocity [Vw]



INLET VELOCITY Δ



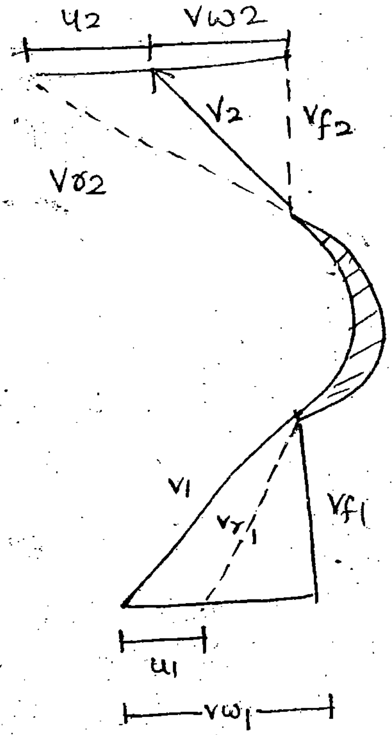
outlet vel Δ



Pelton wheel :-

V_1 = Absolute vel at inlet
u_1 = Tangential vel
V_{r1} = Relative vel water
V_{f1} = Flow vel of water
V_{w1} = whirl vel of water

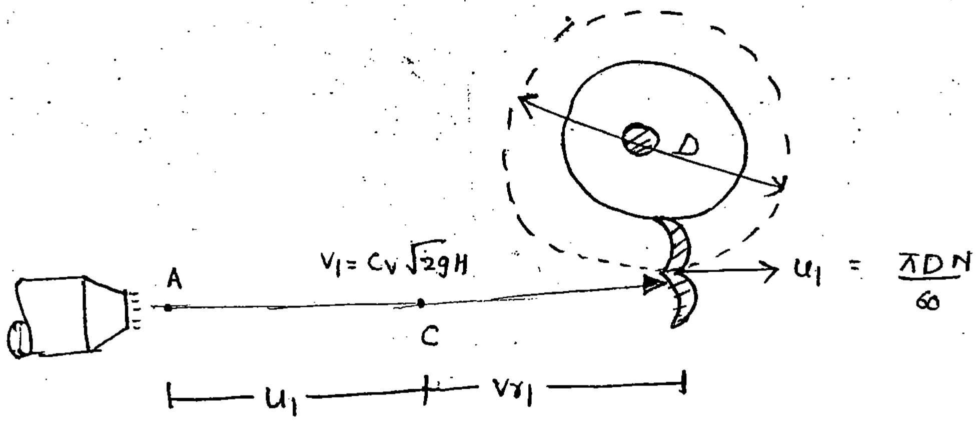
[
α = Nozzle angle at inlet
θ = Blade angle at inlet
]

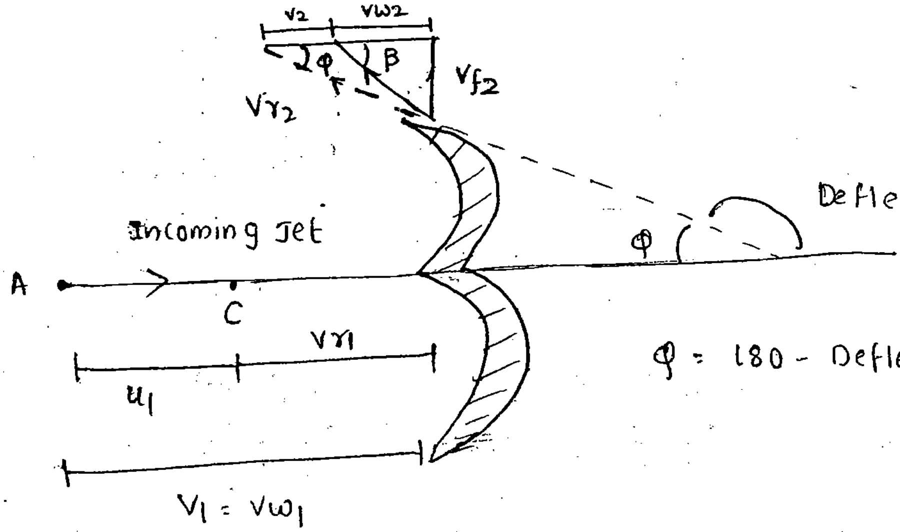


General Turbine

Pelton wheel

GATE 2008





Deflected angle 165° to 175°

ϕ = 180 - Deflected angle

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2011 XE Q 1 (d)

Q 2

3

4

5

6

7 correction Point B, 1 ~~cm~~ m down stream

8

Q-10 conventional

P19

Unit 2

QNO: 1 C
2 B
3 D
4 A
5 A
6 A
7 B
8 C
9 B
10 C
11 A

Page 21

Q12 - D
13 - A
14 - B
16 B
17 A
18 B
19 C (XE 2011)

Page 22

Q 01: D
02 C
3 C
4 C
5 A
6 A
7 B
8 C
9 B
10 B
11 D
12 B
13 A
14 A
15 B

Page 23

Q-1 A
2 D
3 B
4 A
5 C
6 A
7 A
~~7~~ 8 B — 2
9 B
10 C

Unit 4

Fluid dynamics

Page No: 55

Q NO: 1 C
2 C
3 B
4 C
5 Ans: $F = 4.999 N \approx 5N$
6 A
7 A
8 B
9 D
10 D
11 D
12 B
13 D
14 C

Q 15 D
Q 16 D
Q 17 A
Q 18 B
Q 19 Avg 264 (N)
Q 20 (a) $P_A = P_{atm} - \rho g h_2$
(b) $V_D = \sqrt{2g(h_1 - h_3)}$

Page No 75 & 74

Q-1 A
Q 2 A
3 C
4 C
5 B
6 C — 5.8×10^{-5}
Q 7 C
8 A
9 C
10 D
Q 11 B
Q 12 C