MACHINE

DESIGN

Design - is a process plan to satisfy a human need.

It is a mechanism to convert an idea into reality

It is a solution to the problem.

Machnine - A m/c is a combination of resistant bodies with Successfully constrained motion (relative motion) to other both of energy into mechanical energy.
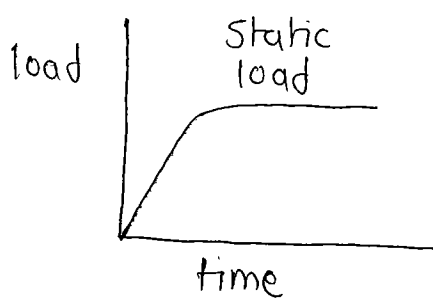
OR

To transmit and modify the available energy to do some useful work.

Machine design involve the selection of suitable material and optimum dimentions to withstand the load without Failure.
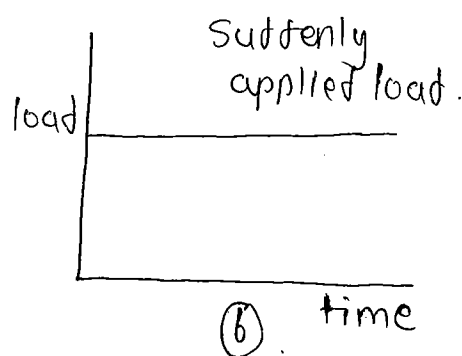
Loads - Externally applied force or moment acting on the member

loads

Static   Variable load

Cont. { Magnitude
w.r to { Direction
time { Point of application

load | Static load

time
(a)

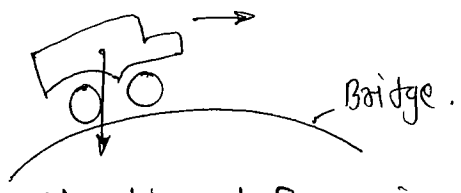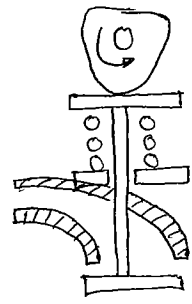load | Suddenly applied load.

time
(b)

Gradually applied load
can also called as static load.

① Magnitude changes — magnitude is fluxtuating but direction remain same

EX:-

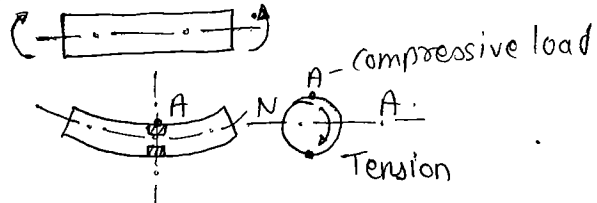the direction of force is cont. but magnitude is changing while vehicle crossing the bridge.

Bridge.

valve spring.

② Direction Changes — Constant magnitude
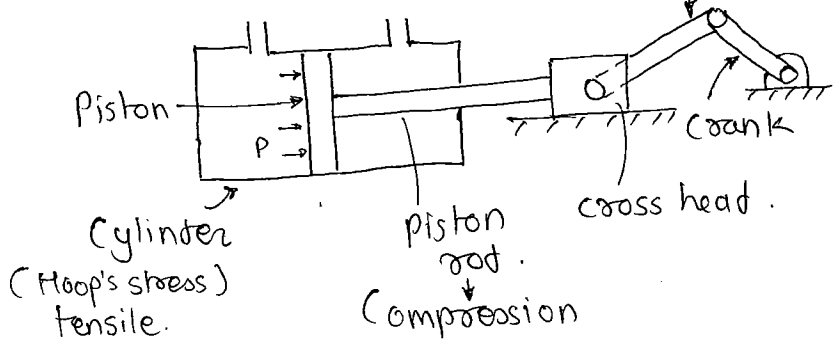
EX:- Rack pinion., gear m/c sm.

A rotating shaft carrying a const. bending movement.

A - compressive load

A - Tension

$$\bar\sigma_b = \frac{M}{I} \times y.$$

$$\sigma_b = \frac{32 M}{\pi d^3}.$$

connecting rod = tensile load ≠ comp load

Piston

P

Cylinder (Hoop's stress) tensile.

piston rod. Compression

Crank

cross head.

$-x \longleftarrow \longrightarrow +x$

Pressure P

$$\vec{F} = \text{Pres} \times \text{Area}$$
$$= P \times \pi/4 \, D^2.$$
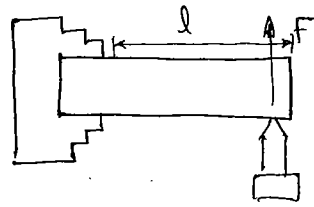$$\overleftarrow{F} = P \times \frac{\pi}{4} (D^2 - d^2)$$

$d < < D$.

$$\vec{F} = \overleftarrow{F}$$

Piston rod = $\vec{F}$ - tensile load $\vec{F}$ - comp. load.

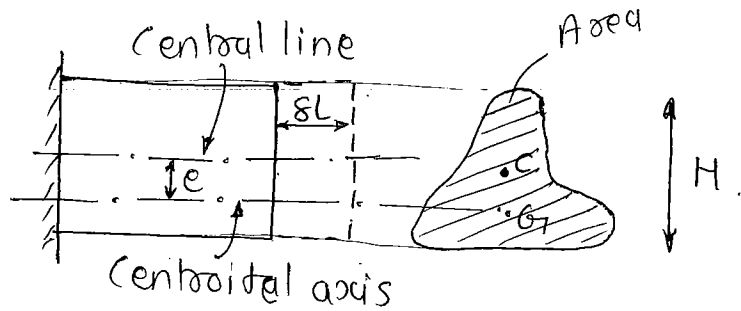hence piston rod come under this catagory.

③ Point of application.

$$M_{max} = F \times l$$
$$M_{min} = F \times 0$$

Stress (σ)

$$\sigma = \frac{load}{area} = \frac{F}{A}$$

centroidal axis

$$\sigma_{central\ line} = \frac{F}{A} + \frac{M}{I} y$$

Strain (ε)

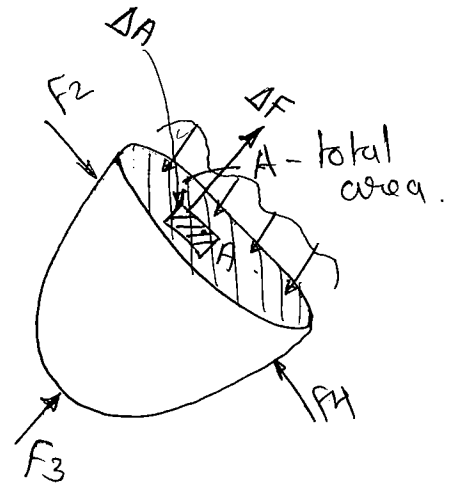$$\varepsilon = \frac{\delta L}{L}$$
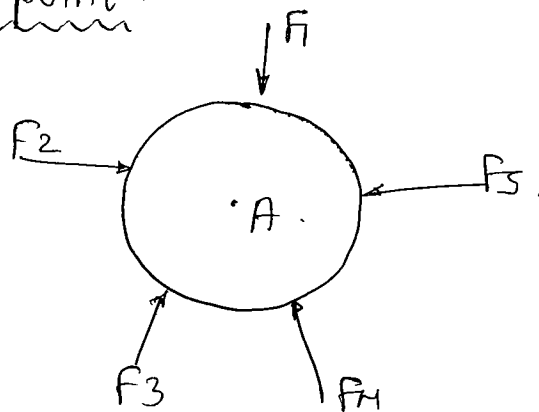
Hook's law

$$\sigma \propto \varepsilon \quad (with\ in\ the\ proportional\ limit) \quad (No\ elastic\ limit).$$

Stiffness (K)

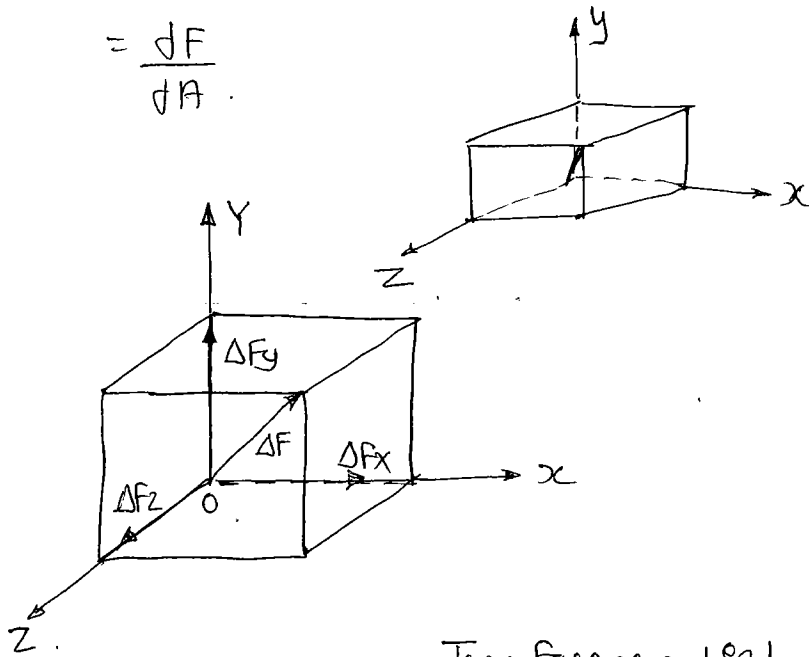$$K = \frac{load}{deflection}$$

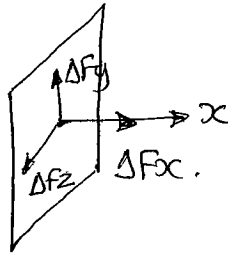$$= \frac{F}{\frac{Fl}{AE}} = \frac{AE}{L}.$$

Stress at a point.

$$\sigma_f = \underset{\Delta A \to 0}{Lt} \frac{\Delta F}{\Delta A}$$

$$= \frac{dF}{dA}.$$

→ Force $\perp^{er}$ to x axis

Two Face are $\perp^{er}$ to x axis.

Tensor
(vector quant.) $\sigma_{xx} = \underset{\Delta A \to 0}{Lim} \frac{\Delta Fx}{\Delta A}$.

Force Normal   Force
to co-ordinate direction

$$\sigma_{xy} = \underset{\Delta A \to 0}{Lim} \frac{\Delta Fy}{\Delta A}.$$

$$\sigma_{xz} = \underset{\Delta A \to 0}{lim} \frac{\Delta Fz}{\Delta A}.$$

→ Force $\perp^{er}$ to Y axis (same).
  "   "   " Z axis   "

Stress tensor

Direct stress
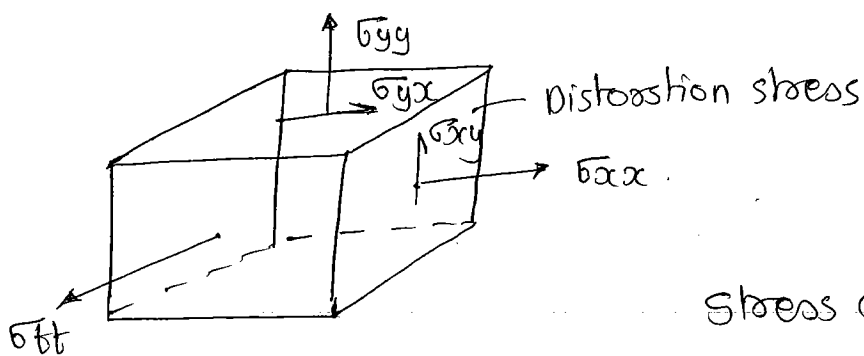Normal stresses (volume change)

Distortion
Stress

$$\sigma = \begin{array}{c} x \\ y \\ z \end{array} \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

Normal stress — It changes the size (volume changes)
It is called as direct stress (No shape change)

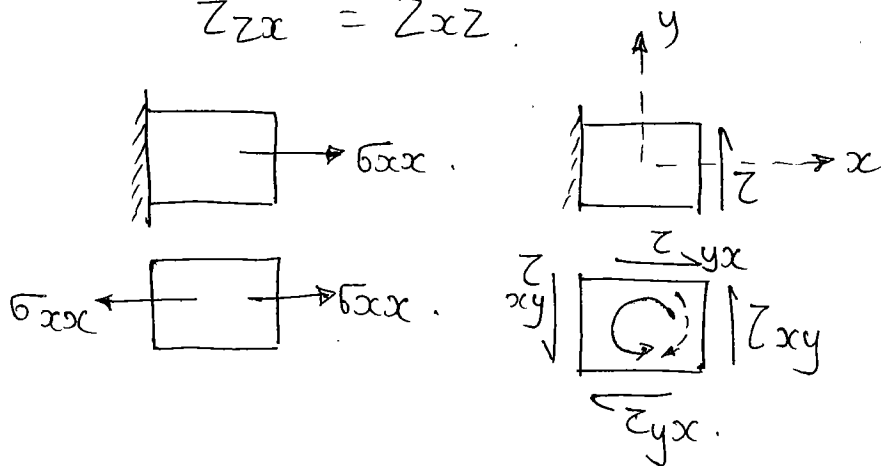Distortion stress

σ_{xx}

Stress component-g.

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

Complementary shear stress are

$\tau_{xy} = \tau_{yx}$.

$\tau_{yz} = \tau_{zy}$

$\tau_{zx} = \tau_{xz}$.

6 stress components are req to define the total stress
at a point in the space.

σ_{xx} τ_{xy}
σ_{yy} τ_{yz}
σ_{zz} τ_{zx}

Two dimention stress. (Plane stress)

Ex:- ① Thin sheets (laminar)

$$\uparrow y \quad \rightarrow x \quad \text{(thickness is very less)}$$

② Thin cylindrical shell

σ_b, σ_l and p (pressure) is neglected.

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix}$$

3 stress compents are required to define the stress

σ_{xx}, σ_{yy}, $\tau_{xy} = \tau_{yx}$.

Principle Stress.

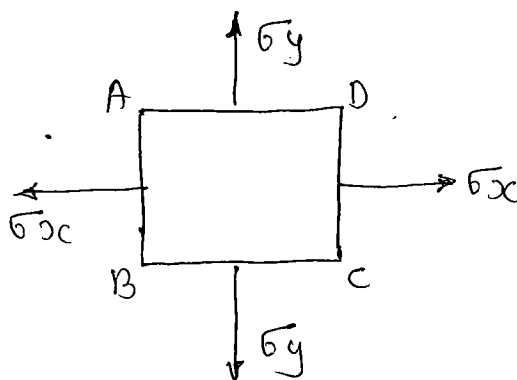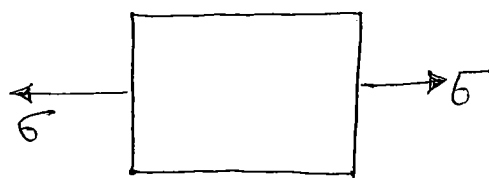Principle plain - A plain carrying zero ~~principle~~ shear stress.

Principle stress - The stress (normal) acting on the principle plain.

Principle palanes and stresses.

Principle plane.

zero shear stress $\begin{cases} AB \text{ or } CD & \sigma_1 = \sigma \\ AD \text{ or } BC & \sigma_2 = 0. \end{cases}$

Principle plane.

AB or CD $\sigma_1 = \sigma_y$ Major

AD or BC $\sigma_2 = \sigma_x$ Minor

$\sigma_y > \sigma_x$.

Biaxial time

$\longrightarrow$

AB or CD $\quad \sigma_2 = \sigma_x$

AD or BC $\quad \sigma_1 = \sigma_y$

$\sigma_y > \sigma_x$.

$\longrightarrow$

$\sigma_y > \sigma_x$
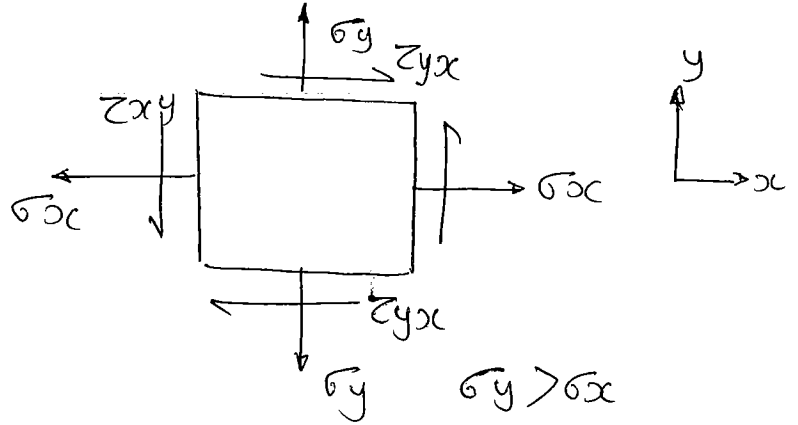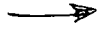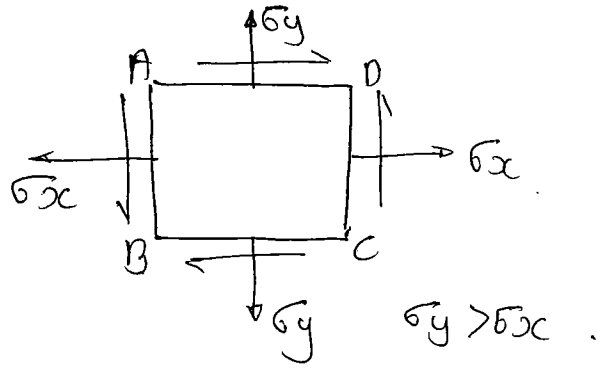
Principle ~~planes~~ stress
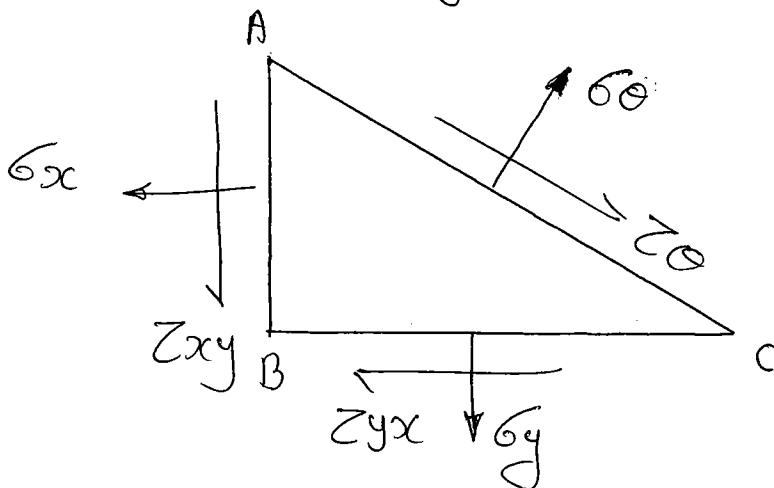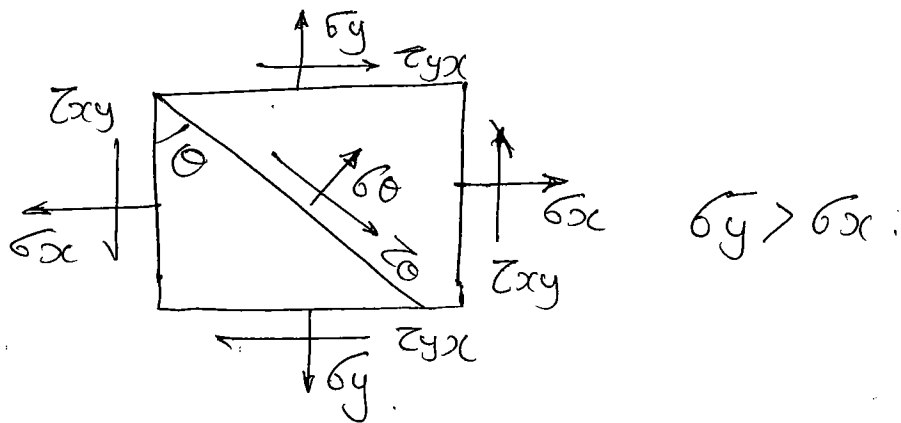
$$\sigma_1 \& \sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\frac{(\sigma_x - \sigma_y)^2}{2} + \tau_{xy}^2}.$$

Principal planes

$$\tan 2\phi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}.$$
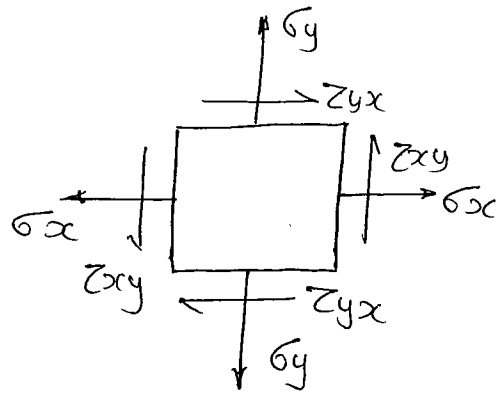
$\longrightarrow$

$\sigma_y > \sigma_x$.

$$\sigma\theta = \left(\frac{\sigma x + \sigma y}{2}\right) + \left(\frac{\sigma x - \sigma y}{2}\right)\cos 2\theta + \tau xy \sin 2\theta$$

$$\tau\theta = \left(\frac{\sigma x - \sigma y}{2}\right)\sin 2\theta - \tau xy \cos 2\theta.$$

downward shear stress
positive direction

Mohr's Circle.

$$\sigma = \begin{bmatrix} \sigma x & \tau xy \\ \sigma yx & \sigma y \end{bmatrix}$$

Assume

$\sigma x > \sigma y$.

$OA = \sigma x$

$OB = \sigma y$.

$AD = -\tau xy$
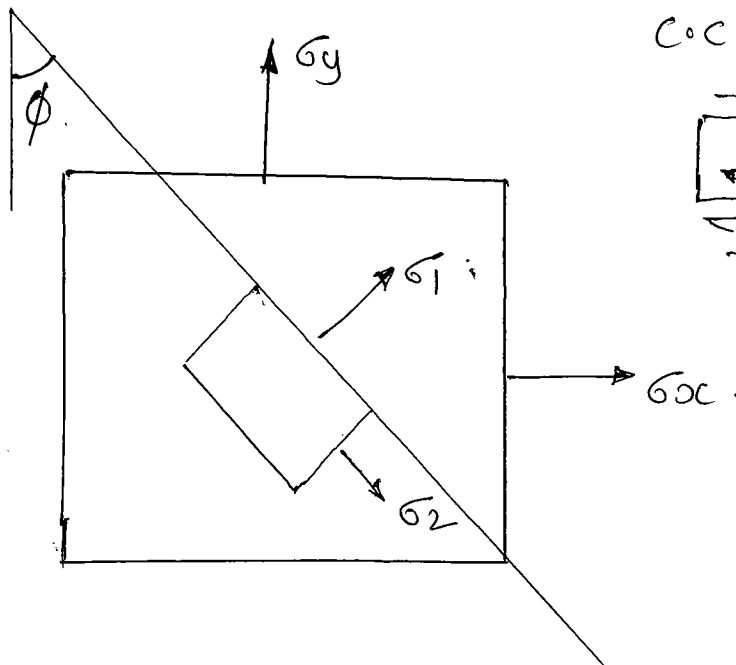
$BE = +\tau yx$.

Sign Convention

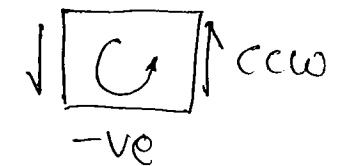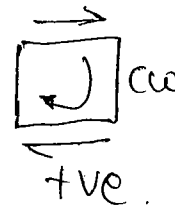→ Normal stress — abscissa
Tensile ----- +ve
Compressive --- -ve

→ Shear stress — ordinate
Clockwise +ve
C.C.w   -ve

+ve.   -ve

2ϕ - hat twise the actual value.
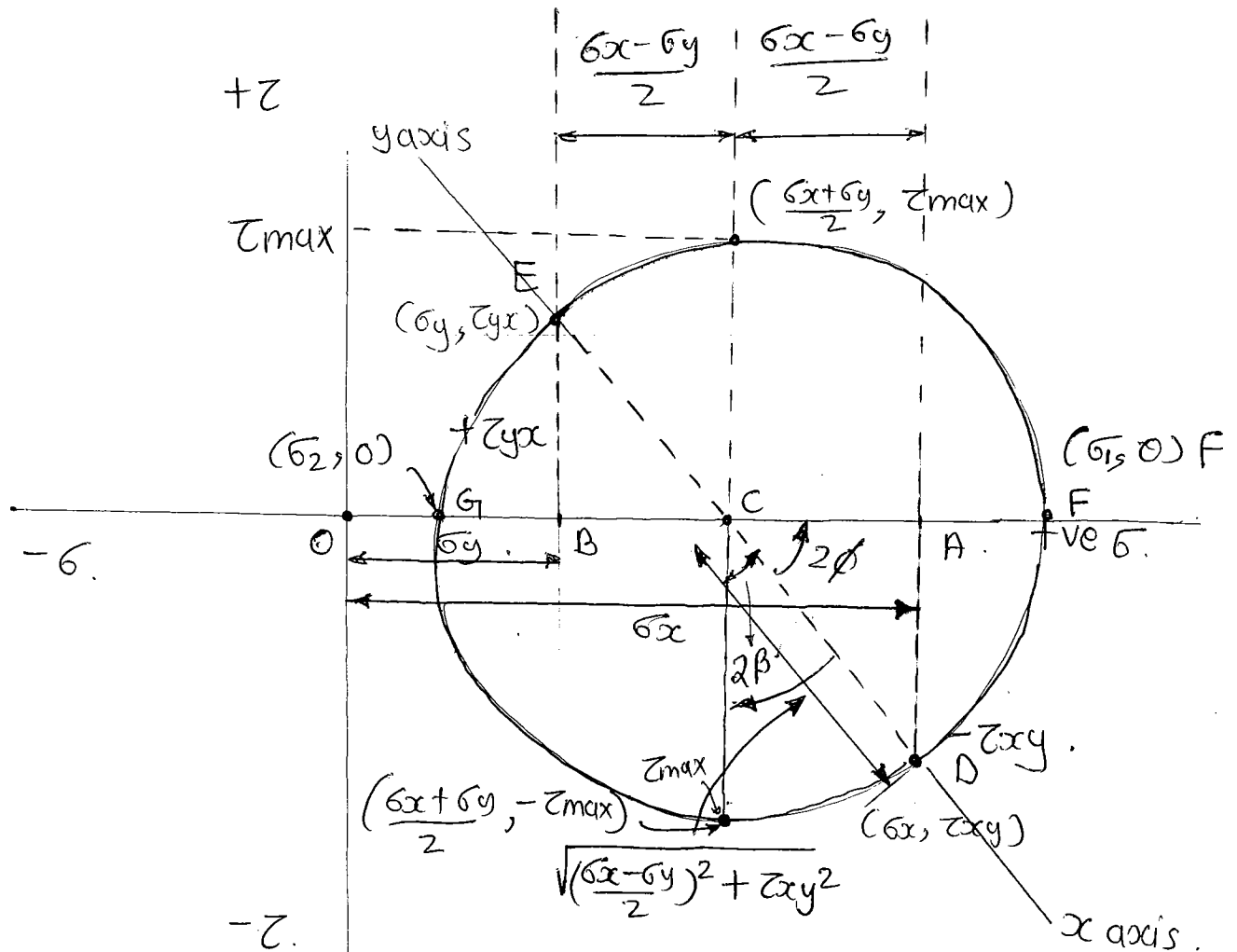
$\sigma_1 = OF$ — Maximum stress with zero shear stress. Hence Max principle stress.

$\sigma_2 = OG$ — Min stress with zero shear stress. Hence Min principle stress.

$$AC = BC = \frac{AB}{2} = \frac{\sigma x - \sigma y}{2}$$

$$AB = \sigma x - \sigma y$$

$$OC = \sigma y + \frac{\sigma x - \sigma y}{2} = \frac{\sigma x + \sigma y}{2} \quad (OB + OC)$$

$$OF = \sigma_1 = OC + CF \quad (CF = CD = R)$$
$$= OC + CD.$$

$$\sigma_1 = \frac{\sigma x + \sigma y}{2} + \sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau x y^2}$$

$$\sigma_2 \quad - \quad \sigma x + \sigma y - R.$$

$$\tau_{max} = \text{Radius of mohr's circle} = R.$$

$$= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
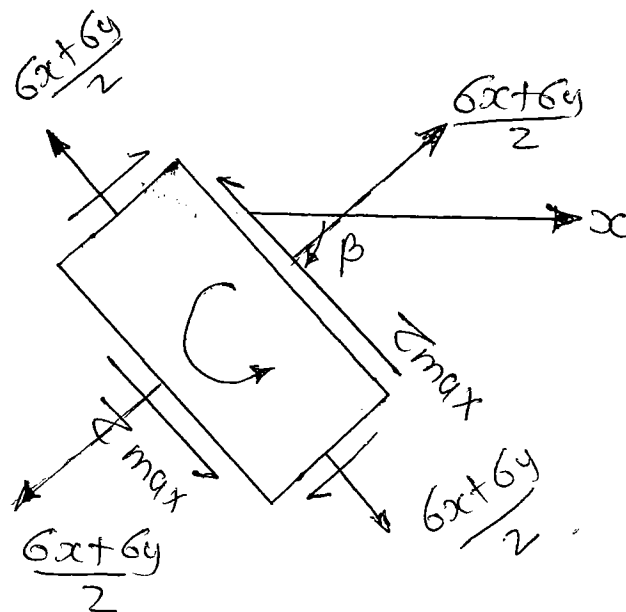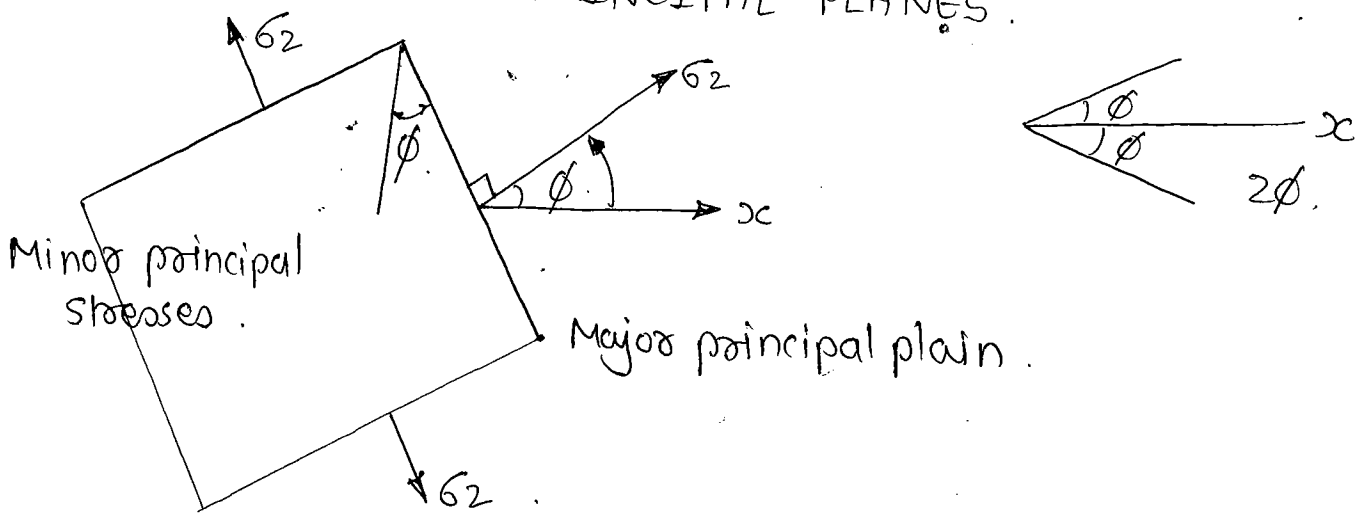
$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \tau_{max}.$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \tau_{max}.$$

ORIENTATION OF PRINCIPAL PLANES.

Minor principal Stresses.

Major principal plain.

Design for static strength.

Q1] The strength of stress at a point is given, in Mpa is

$$\sigma = \begin{bmatrix} 40 & 0 \\ 0 & -30 \end{bmatrix}$$ The maximum shear stress.

a] 20 MPa   b] 30 MPa   c] 35 ✓   d] 40 MPa.

$\sigma_1 = 40$ $\sigma_2 = -30$. $\sigma_3 = 0$

$$\tau_{max} = \max \text{ of } \begin{cases} \left|\dfrac{\sigma_1-\sigma_2}{2}\right| \\ \left|\dfrac{\sigma_2-\sigma_3}{2}\right| \\ \left|\dfrac{\sigma_3-\sigma_3}{2}\right| \end{cases} = \begin{cases} \dfrac{40-(-30)}{2} \\ +\dfrac{40}{2} \\ \dfrac{-30}{2} \end{cases} \quad \begin{array}{l} \dfrac{70}{2} = 35 \\ \dfrac{40}{2} = 20 \\ \dfrac{-30}{2} = 15 \end{array}$$

$$\sigma = \begin{bmatrix} 40 & 0 \\ 0 & 30 \end{bmatrix} \qquad \tau_{max} = \dfrac{40-30}{2} = \dfrac{10}{2} = 5$$

$$\dfrac{40}{2} = 20 \quad ✓.$$

$$\dfrac{30}{2} = 15$$

2] The stress at a point in MPa is. $\sigma = \begin{bmatrix} 50 & 10 \\ 10 & 30 \end{bmatrix}$
The principle stress.

$$\sigma = \begin{bmatrix} 50 & 10 \\ 10 & 30 \end{bmatrix} \qquad \sigma_1 = \dfrac{\sigma x + \sigma y}{2} + \sqrt{\left(\dfrac{\sigma x - \sigma y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_1 = 50$ $\sigma_2 = 30$ $\sigma_3 = 0$ Principal stress.

$$\sigma_A = \dfrac{50+30}{2} + \sqrt{\left(\dfrac{50-30}{2}\right)^2 + 10^2} = 54.1421 \text{ Mpa}.$$

$$\sigma_B = \dfrac{50+0}{2} + \sqrt{\left(\dfrac{50}{2}\right)^2 + 10^2} = 51.92 \text{ MPa}.$$

$$\sigma_C = \dfrac{30}{2} + \sqrt{\left(\dfrac{30}{2}\right)^2 + 10^2} = 33.02 \text{ MPa}$$

Shear stress

$$\sigma_A \sqrt{\left(\dfrac{\sigma x - \sigma y}{2}\right)^2 + \tau^2} = 14.14$$

$$\sigma_B = 26.92 \text{ MPa}. ✓$$
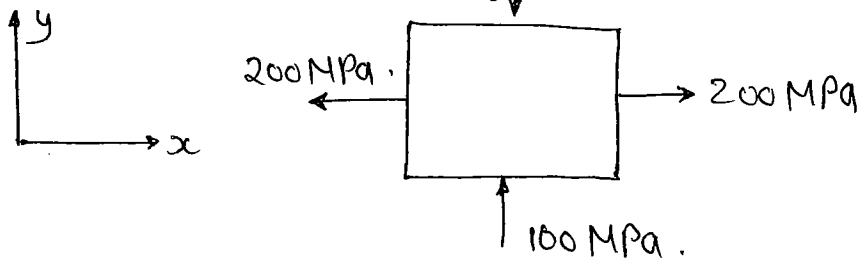
③ The stress tensor at a a point is given by by (in MPa)

$$T = \begin{bmatrix} 10 & 5 & 0 \\ 5 & 20 & 0 \\ 0 & 0 & -20 \end{bmatrix}$$

The principal stresses are

$$T = \begin{bmatrix} 10 & 5 & 0 \\ 5 & 20 & 0 \\ 0 & 0 & -20 \end{bmatrix} \quad \sigma_3 = -20 \, MPa.$$

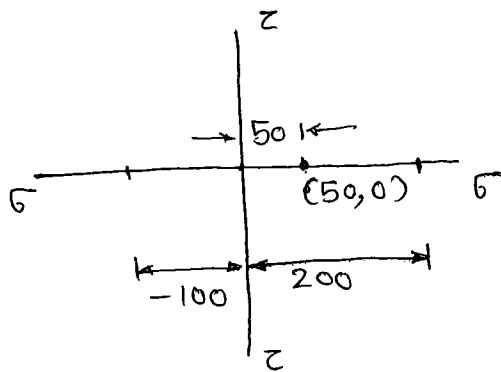$$\sigma_1 = \frac{10+20}{2} + \sqrt{\left(\frac{10+20}{2}\right)^2 + 5^2} = 22.071$$

$$\sigma_2 = \frac{10+20}{2} \mp \sqrt{\left(\frac{10+20}{2}\right)^2 + 5^2} = 7.93 \, MPa.$$

④ Consider a two dimention state of stress given form element as shown in Fig.

What are the co-ordinate centre of mohr's circle.

a) (0,0)   b) (100,200)   c) (200,100)   d) (50,0) ✓

⑤ In a strained material one of the principal stresses is twise the other. The maximum shear stresses in the same case is τ_{max}, then what is the max value of principal stress

a] τ_{max} b] $2\tau_{max}$ ✓   c] $4\tau_{max}$ d] $8\tau_{max}$.
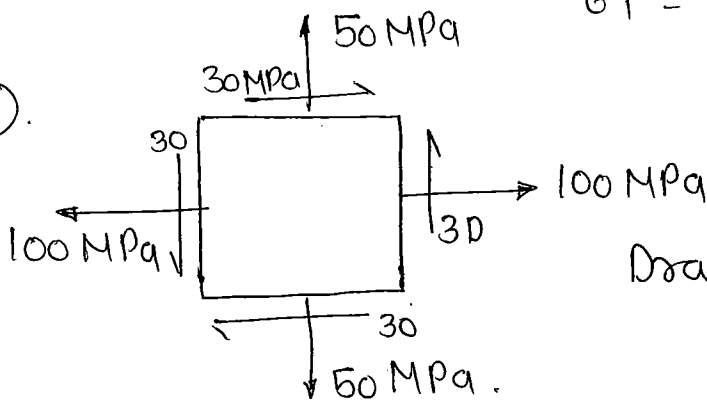
$$\sigma_1 = 2\sigma_2$$

$$\tau_{max} = \frac{2\sigma_1 - \sigma_2}{2} = \frac{2\sigma_2 - \sigma_2}{2}$$
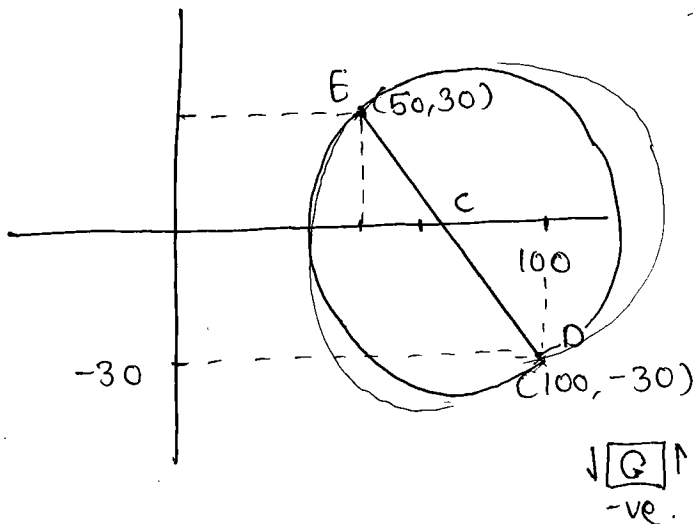
$$\sigma_2 = 2\tau_{max}$$

$$\sigma_1 = 4\tau_{max} \checkmark$$

⑥.

50 MPa
30MPa
30
100 MPa
30
100 MPa
30
50 MPa .

Draw the mohr's circle.

$1\,cm = 10\,MPa$

E (50,30)
c
100
-30
D (100,-30)
↓ Q ↑
-ve .

$$\tau_{max} = \text{max of} \begin{cases} \dfrac{\sigma_1 - \sigma_2}{2} \\[4pt] \dfrac{\sigma_1}{2} \\[4pt] \dfrac{\sigma_2}{2} \end{cases}$$

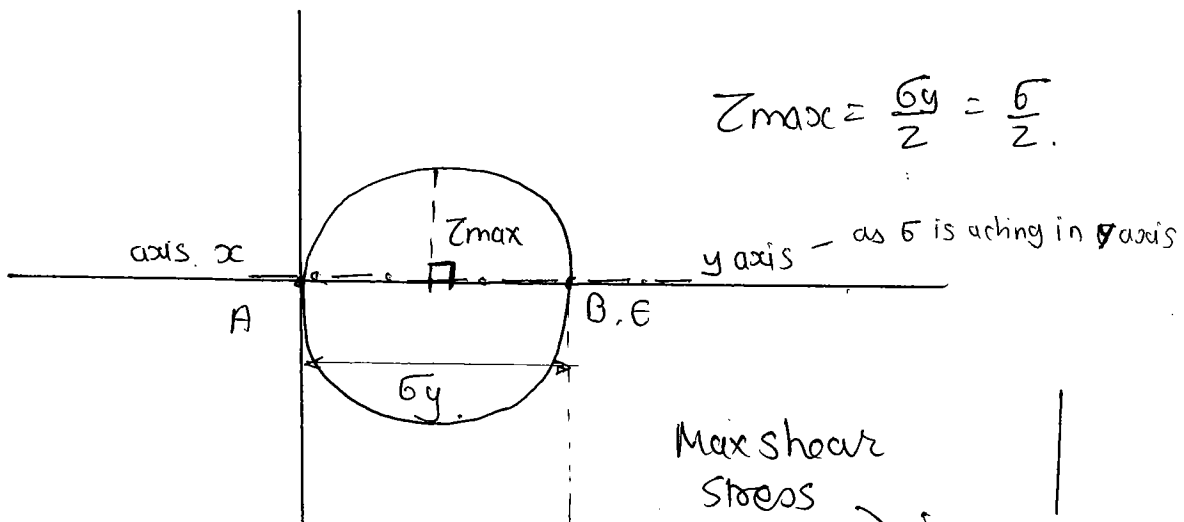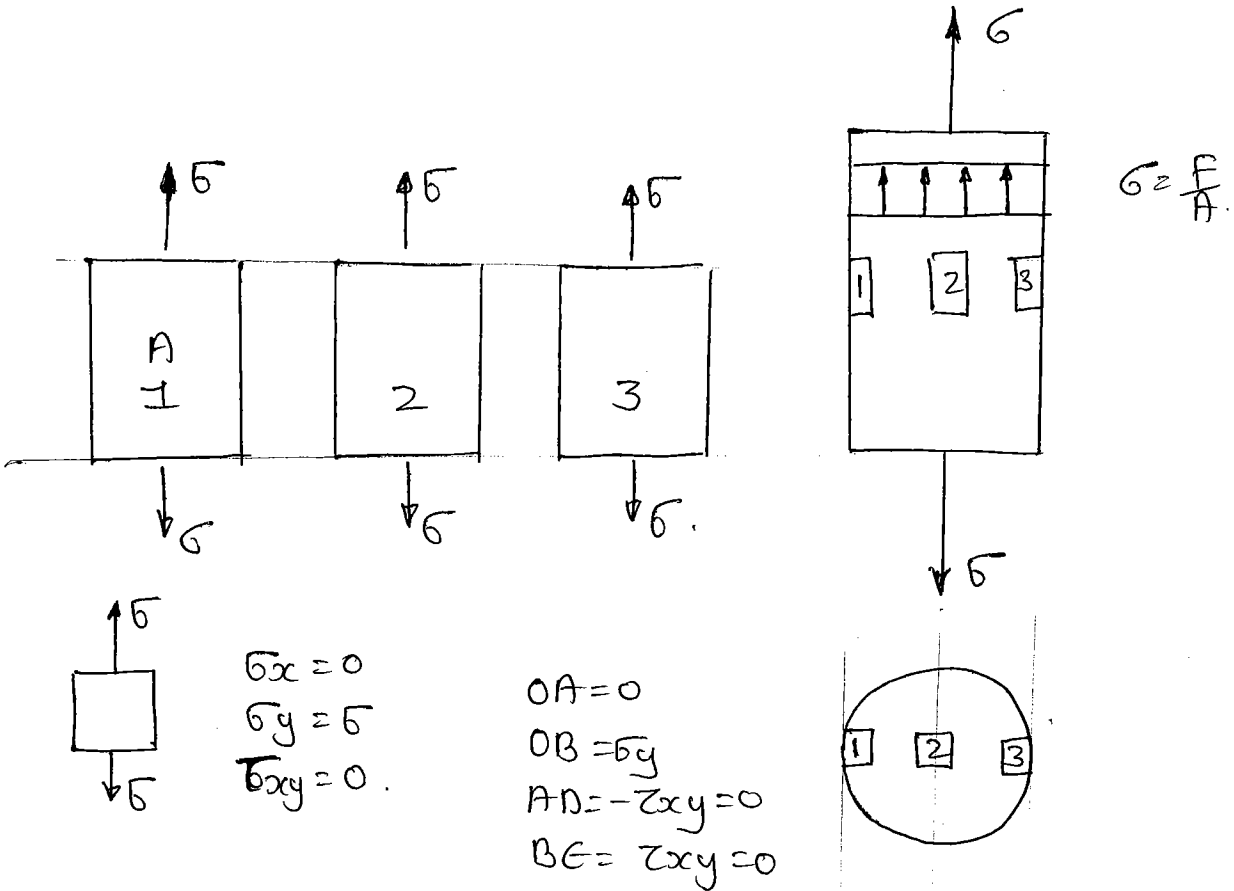$$= \frac{2\sigma_2 - \sigma_2}{2} \qquad = \frac{\sigma_2}{2}$$

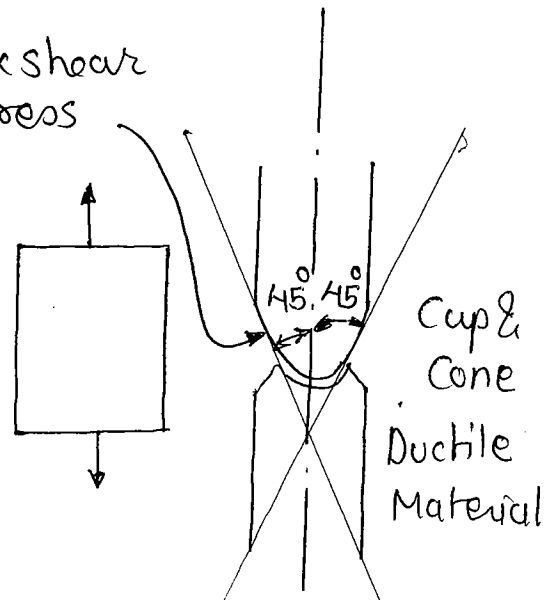$$= \frac{2\sigma_2}{2} \qquad\qquad = \sigma_2 = \tau_{max}.$$

$$= \frac{\sigma_2}{2}$$

$$\sigma_1 = 2\tau_{max}$$

⑦. Draw a mohr's circle under uniaxial state of stress.

Mohr's circle for simple tension

$$6x = 0$$
$$6y = 6$$
$$6xy = 0$$

$$OA = 0$$
$$OB = 6y$$
$$AD = -7xy = 0$$
$$BE = 7xy = 0$$

$$6 = \frac{F}{A}$$

$$Zmax = \frac{6y}{2} = \frac{6}{2}.$$

axis x   y axis — as 6 is acting in y axis

$$Zmax$$

A   B,E

$$6y$$

Max shear Stress
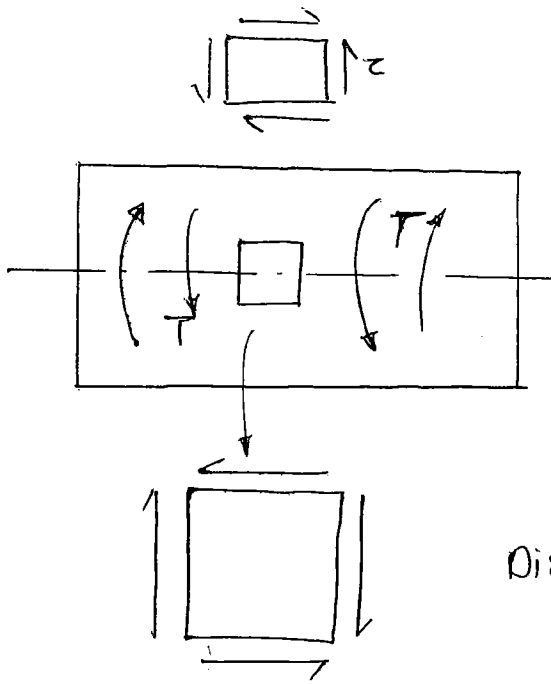
45° 45°

Cup &
Cone
Ductile
Material

Mohr's Circle For Torsion Stresses.
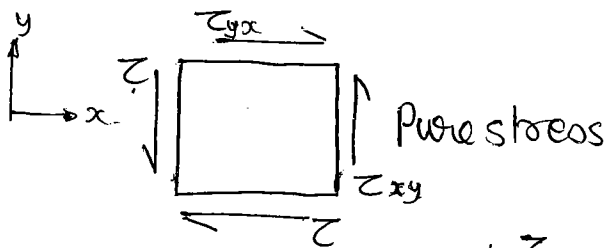
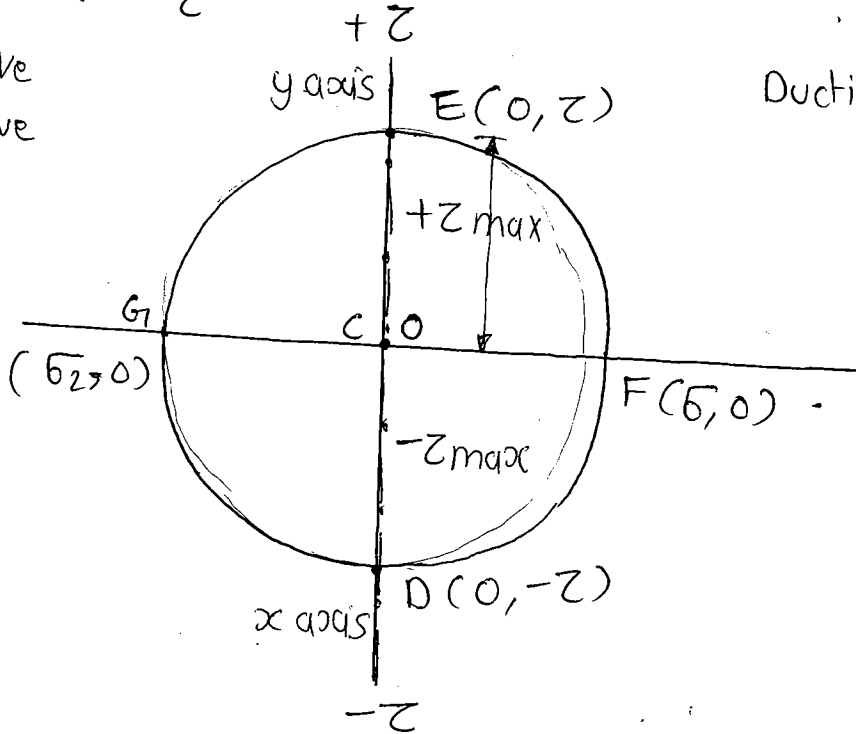Critical element
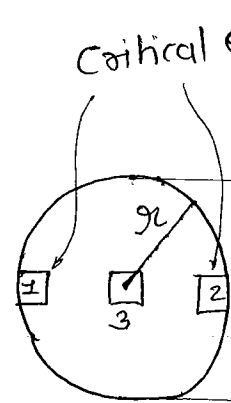
$$\frac{T}{J} = \frac{Z}{R}$$

$$Z \propto R.$$

$$Z \uparrow \quad R \uparrow$$

$$R = 0 \quad Z = 0.$$

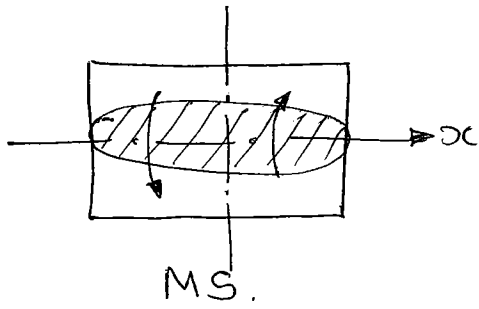Direction of shear stress is depends upon the direction of torque we applied.

Pure stress

Z_{xy} -Ve
Z_{yx} +ve

Brittle material — Seperation of layer
Ductile   "   — Sliding

E (0, Z)

+Z max

G
$(6_2, 0)$

C O

F (6, 0)

-Z max

D (0, -Z)

x axis

+Z
y axis

-Z

MS.

Shear failure under torsion
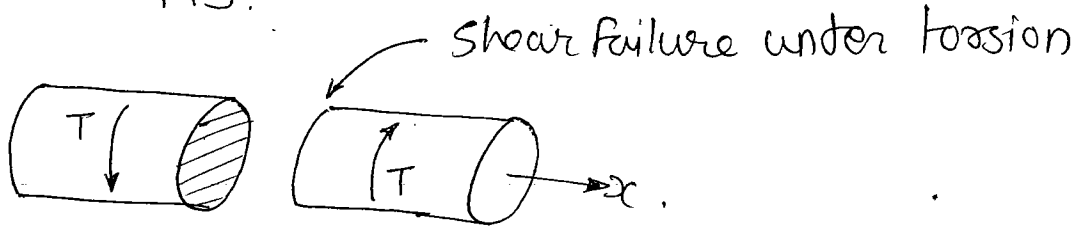
Resistance is less. Hence failure take place at a plane ⊥ to x axis.

σ

C I
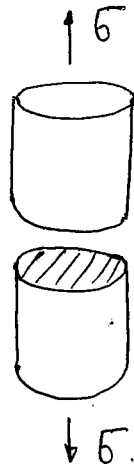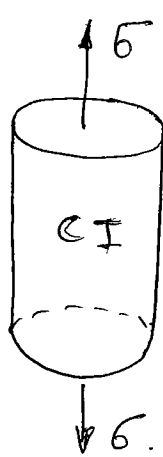
Under Normal stress
Brittle material

Fractured.
Separated.

σ.   σ.

Plane of max Normal stress at an angle of 45°
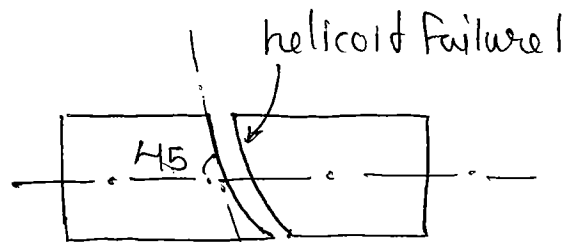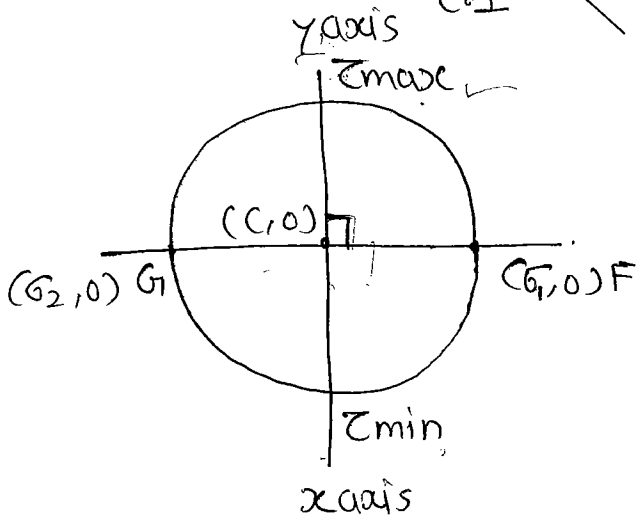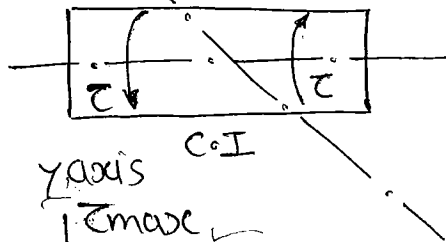
τ   τ

C.I

Plane

y axis
τ max
(C,0)
(σ₂,0) G
(σ₁,0) F
τ min
x axis

helicoid failure

45

CI, concrete, glass
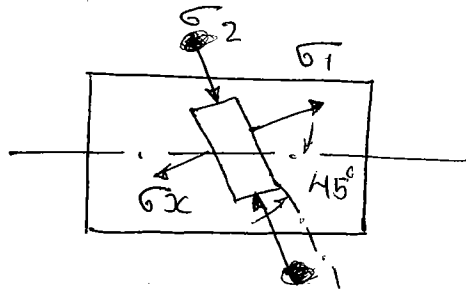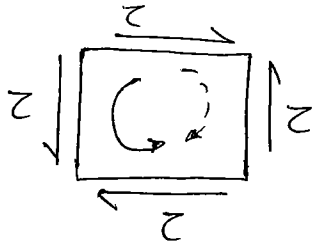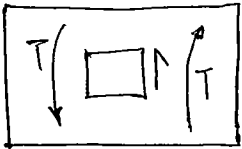
Mohr's circle for Hydrostatic stress.

The stress is same in all direction. in compression or in tension.

Compression

tension.

D (-σ,0)
E (-σ,0)
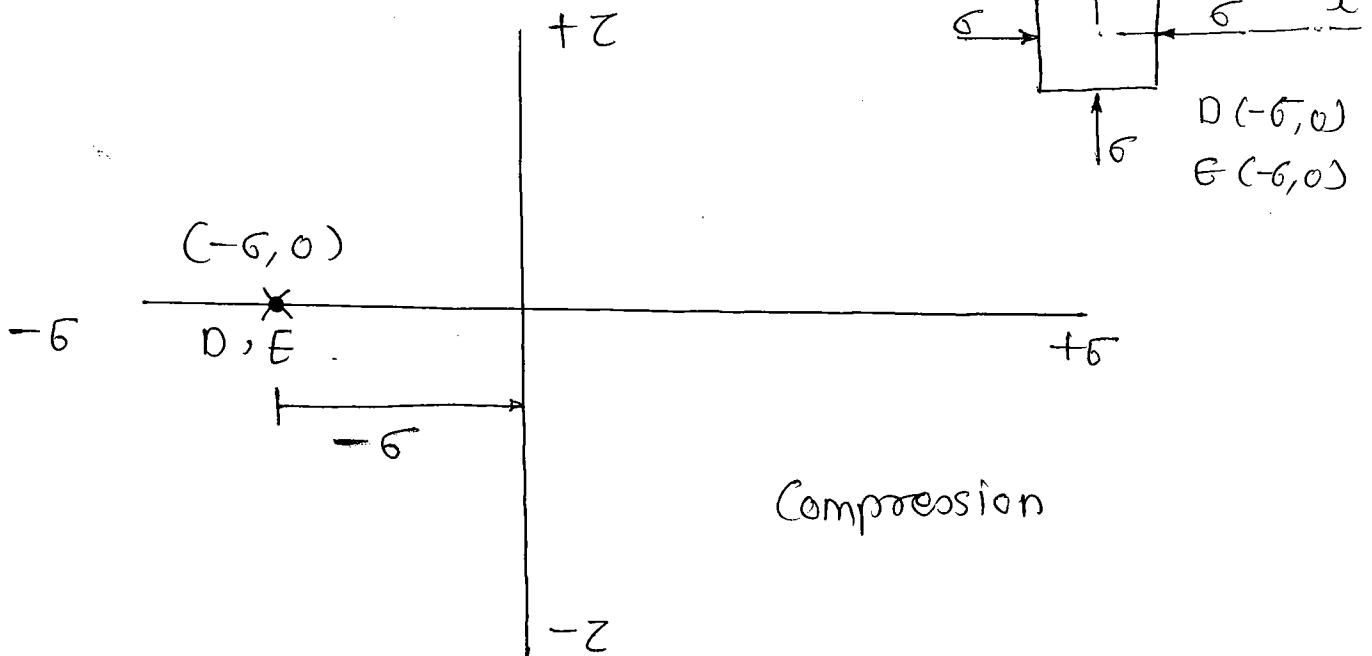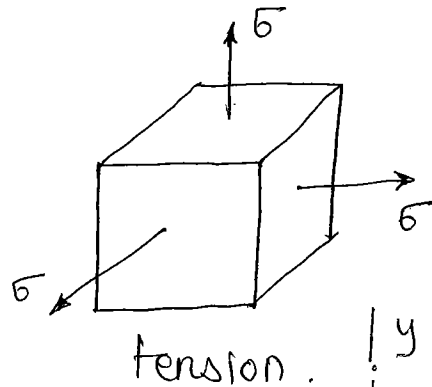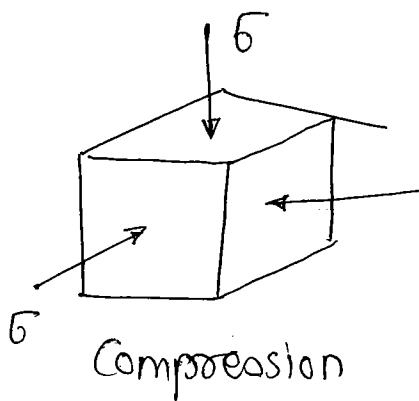
A point -ve side of absisa is called as hydrostatic stress. under compression
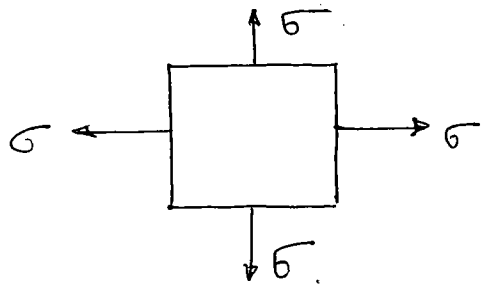
$D(\sigma, 0) - x$ plane
$E(\sigma, 0) - y$ plane
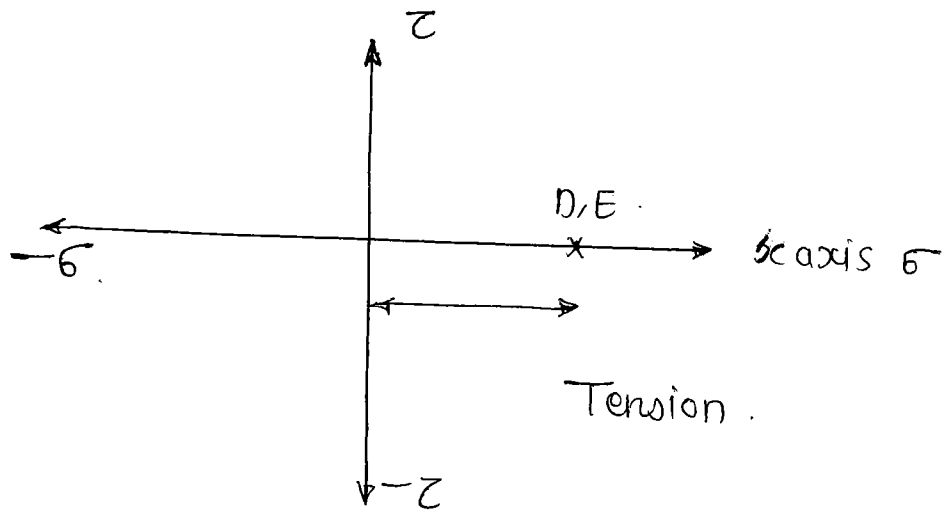
A point on the +ve side of the x axis is called as Hydrostatic stress under tension.

	Ductile Material	Brittle Material.
Failure mode	yielding (Slipping of plane under shear)	Separation or Fracture.
Strengths	$S_{yt} = S_{yc}$ Strength at yield point under tension or comp.	$\cancel{S_{yt}} = \cancel{S_{yc}}$ $S_{ut} \neq S_{uc}$ (Not isotropic)
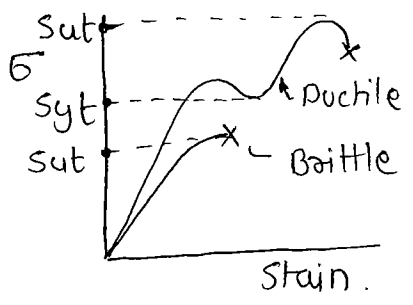
(Isotropic)
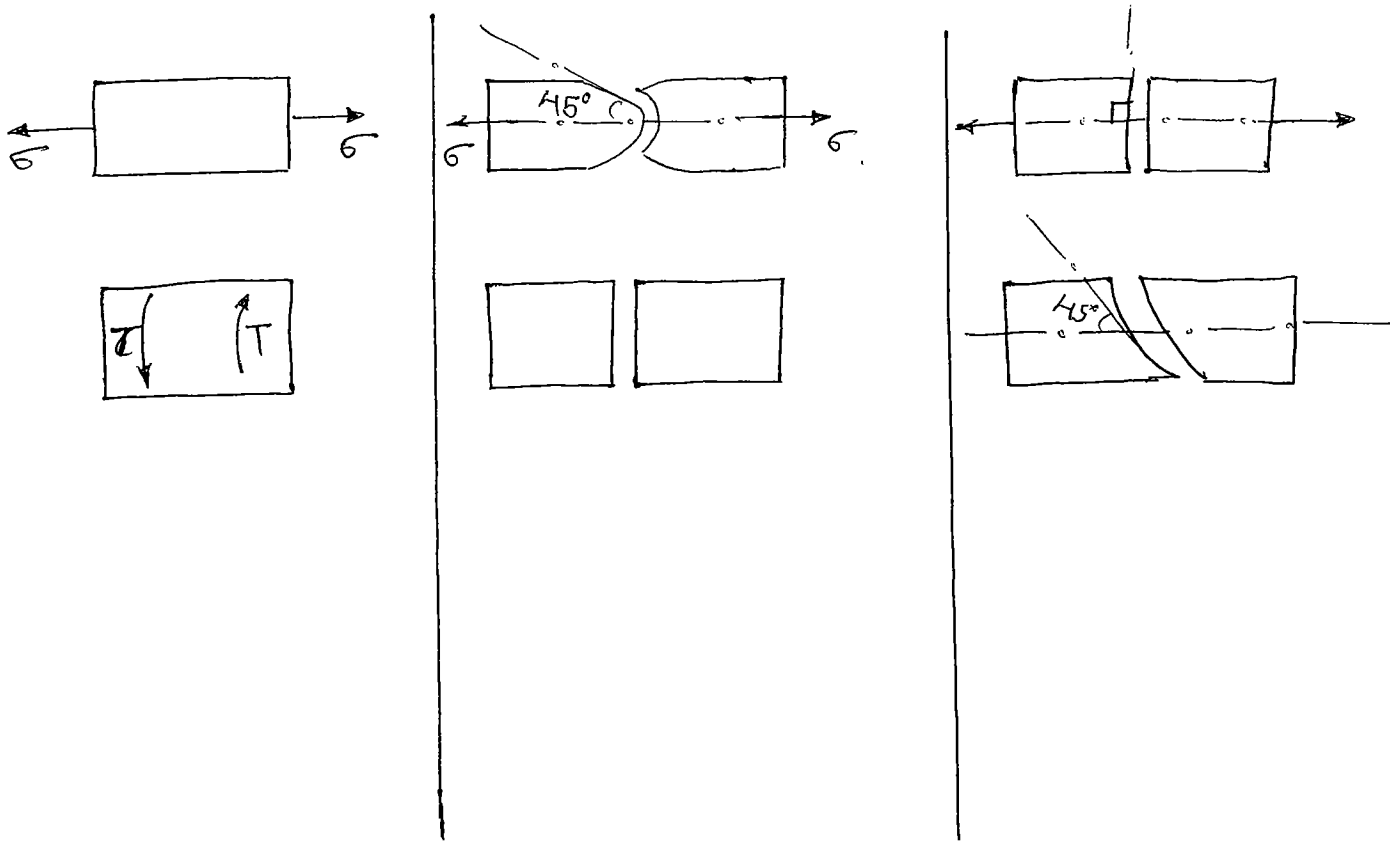
Theories of Failure.
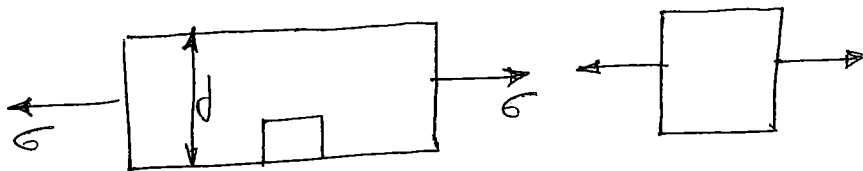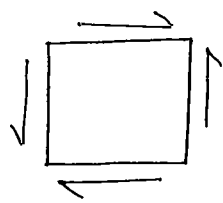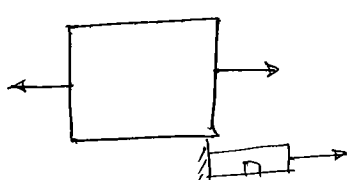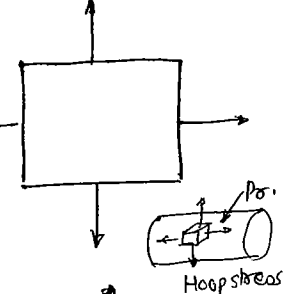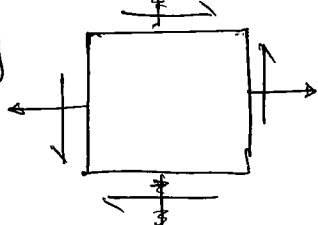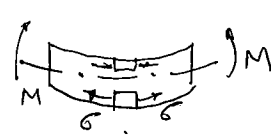
$$\sigma = \frac{F}{\frac{\pi}{4}d^2} = strength.$$

The component may get fail because of Sut, Suc, Syt, Sz
and the max value out of this is consider.

Establishes a relationship between principal stresses, together with some elastic constants and the strength of the component determine by simple torsion.

E, G or C or N, K, μ - cont.

List 1	List 2	Theories of Failure

			A	B	C	D
A]	Combine bending & torsion.	a)	2	1	3	4
B]	Torsion.	b)	3	4	2	1
C]	Thin cylindrical Shells under pressure	c) ✓	2	4	3	1
D]	Tie bar subjected to tensile force	d)	3	1	2	4.

Hoop stress
Po.
M   M
σ   σ

Q1] The maximum shear stress in a bar of c/s are A mm²
and subjected to a tensile load of p newtons is

a] 0   b] P/A c] $P/2A$ ✓   d] $P/4A$.

$\tau_{max} = \sigma/2$
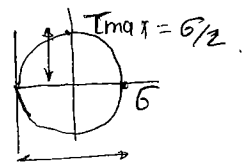
Q2. A steel saw blade 1mm thick is bent into an arc of circle
of 500mm radius. The width of H blade is 15mm and
E = 210 GPa.

1. Fluxtural stress induced is.

a] $\frac{P}{110}$ b] $\frac{P}{210}$ c] $\frac{P}{250}$ d] $\frac{P}{350}$.
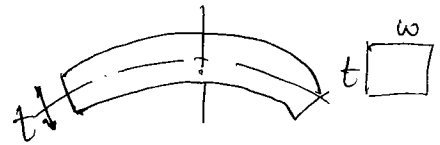
$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}.$$

$$\frac{\sigma_b}{y} = \frac{E}{R}.$$

$$\sigma_b = E \times \frac{y}{R} = 210\times10^{3} \times \frac{t}{2} \times \frac{1}{500}$$

$$\sigma_b = 210\,t \cdot \quad t=1.$$

$$\boxed{\sigma_b = 210}$$

Theory of Failures.

① Rankine's theory (or) Maximum normal stress theory.

If $\sigma_1 > \sigma_2 > \sigma_3$.

Then $\sigma_1 =$ Max normal stress
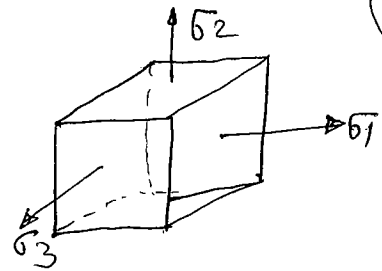
If $\sigma_1 > S_{yt}$ – ductile. $\Big\}$ Component of Failure.
$\sigma_2 > S_{ut}$ – brittle.

$\sigma_1 \leqslant S_{ut}$ – safe
$\sigma_1 \leqslant S_{ut}$ – safe.

Design Equation.

$$\sigma_1 = \frac{S_{yt}}{F.S} \quad - \text{ Ductile} \qquad F.S = \text{factor of Safety.}$$

$$\sigma_2 = \frac{\sigma_{ut}}{F.S} \quad - \text{ Brittle}.$$

Validities — Valid for brittle material and not valid for
    ∴   ductile material.
        — Disregards the effect of $\sigma_2 = 50 MPa$.
          other stresses.

$\sigma_1 = 100 MPa$.

$\sigma_3 = 99.9 mpa$.

Graph.

a] If $\sigma_1 > \sigma_2$

$$\sigma_1 = \frac{Syt}{F\cdot S = 1} \text{(Ductile)}.$$

$\sigma_1 = Syt$

$x = a$.

b] If $\sigma_2 > \sigma_1$.

$\sigma_2 = Syt$

For Ductile material

$\sigma_2 = Syt$

$+\sigma_2$

Syt

$\sigma_1 = Syt$

yielding

Safe

Zone

$2syt$

$-\sigma_1$

$-syt$

syt

$+\sigma_1$

$\sigma_1 = -syt$

C

$-syt$

D

$-\sigma_2$

Square.

Elastic

plastic or
Failure.
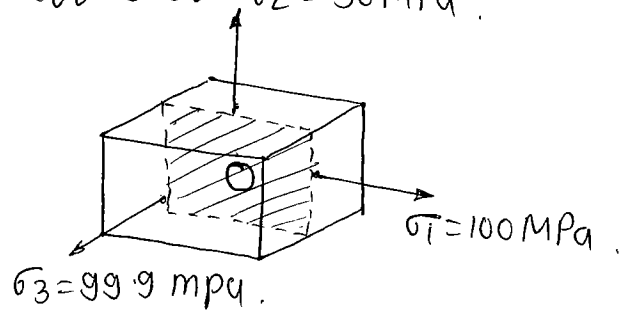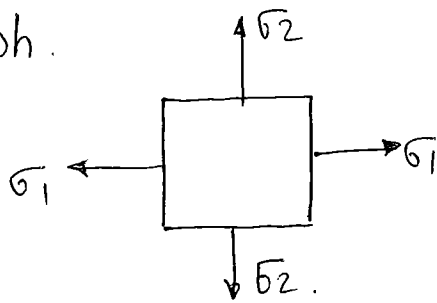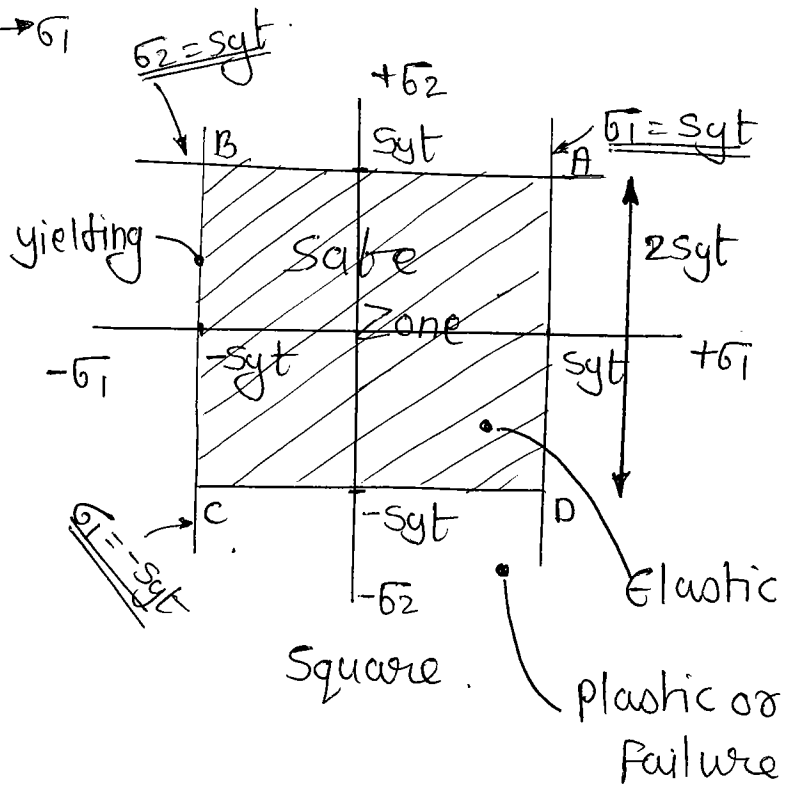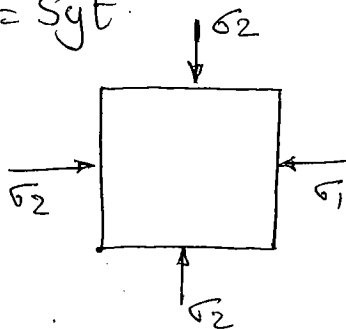
c] If $\sigma_1 > \sigma_2$

$\sigma_1 = Sye = -Syt$ (isotropic
        material)

d] If $\sigma_2 > \sigma_1$

$\sigma_2 = -Syt$

Graph — Brittle Material.
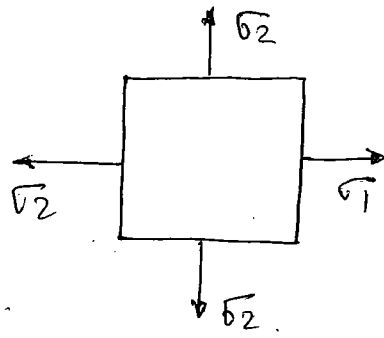
a) If $\sigma_1 > \sigma_2$

$\sigma_1 = Sut$

b) If $\sigma_2 > \sigma_1$

$\sigma_2 = Sut$
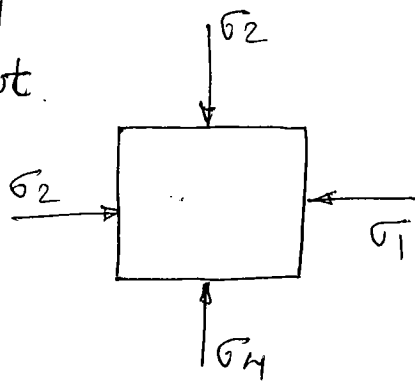
c) If $\sigma_1 > \sigma_2$

$\sigma_1 = Suc$

d) If $\sigma_2 > \sigma_1$

$\sigma_2 = Suc$
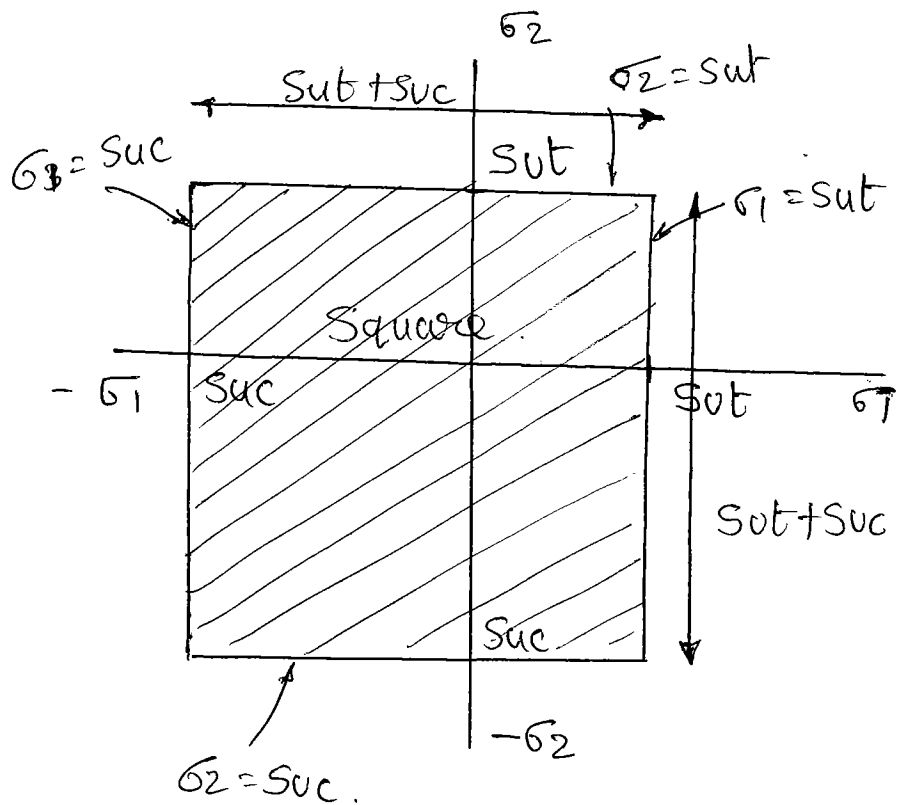
A component subjected to complex state of stress is said to be faild when the maximum normal stress at a point in the material exceeds the strength at yeld point under simple tension.

② Guest's theory or Tresca's theory or Maximum shear stress theory.

Maximum shear stress.

$$\tau_{max} > S_{sy} - \text{Failed}.$$

S_{sy} = shear strength at yield point.

Design

$$\tau_{max} = \frac{S_{sy}}{F.S}$$

If Maximum shear stress

$$\tau_{max} < S_{sy}. - \text{Safe}.$$

$$\tau_{max} = \text{Max of} \begin{Bmatrix} \dfrac{\sigma_1 - \sigma_2}{2} \\ \dfrac{\sigma_1 - \sigma_3}{2} \\ \dfrac{\sigma_2 - \sigma_3}{2} \end{Bmatrix} = \text{max of} \begin{Bmatrix} \left|\dfrac{\sigma_1 - \sigma_2}{2}\right| \\ \left|\dfrac{\sigma_1}{2}\right| \\ \left|\dfrac{\sigma_2}{2}\right| \end{Bmatrix}$$
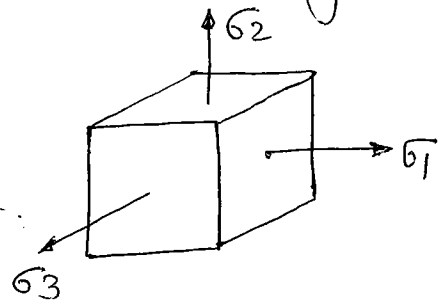
If $\sigma_1 > \sigma_2 > \sigma_3$ and all are like stresses then,

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\frac{\sigma_1 - \sigma_3}{2} = \frac{S_{yt}}{2 \times F.S}.$$

$$\boxed{\sigma_1 - \sigma_3 = \frac{S_{yt}}{F.S}}$$

$$\tau_{max} = \frac{\sigma}{2}$$

$$S_{sy} = \frac{S_{yt}}{2}$$
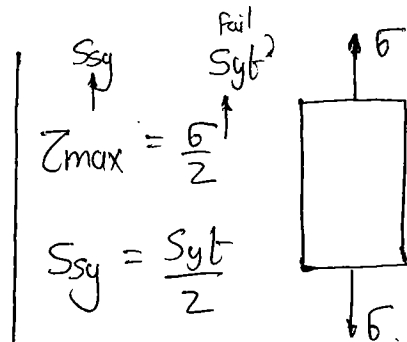
$$\tau_{max} = \frac{S_{st}}{F.S}$$
$$= \frac{S_{st}}{2 \times F.S}.$$

* Validity

① Valid for only ductile material
② Not valid for brittle material
③ Not explained the failure phenomenon under hydrostatic stress

Graph. (Ductile material)

①

$\sigma_1 = -syt$

$\sigma_2 - \sigma_1 = syt$

σ_2

$\sigma_2 = Syt$

syt

$C (0, syt)$

$\sigma_1 = Syt$

$\sigma_1 - \sigma_2 = Syt$

$D \ C(-syt, 0)$

$A(syt, 0)$

$-\sigma_1$

$-syt - \tau$

$45°$

$syt + \sigma_1$

$B (0, -syt)$

$-syt$

$\sigma_2 = -syt$

Tresca's Hexagone

$-\sigma_2$

$\sigma_1 = -\sigma_2$
$\sigma_2 = -\sigma_1$

Shear diagonal

a] If $\sigma_1 > \sigma_2$

then, $\tau_{max} = \dfrac{\sigma_1}{2}$
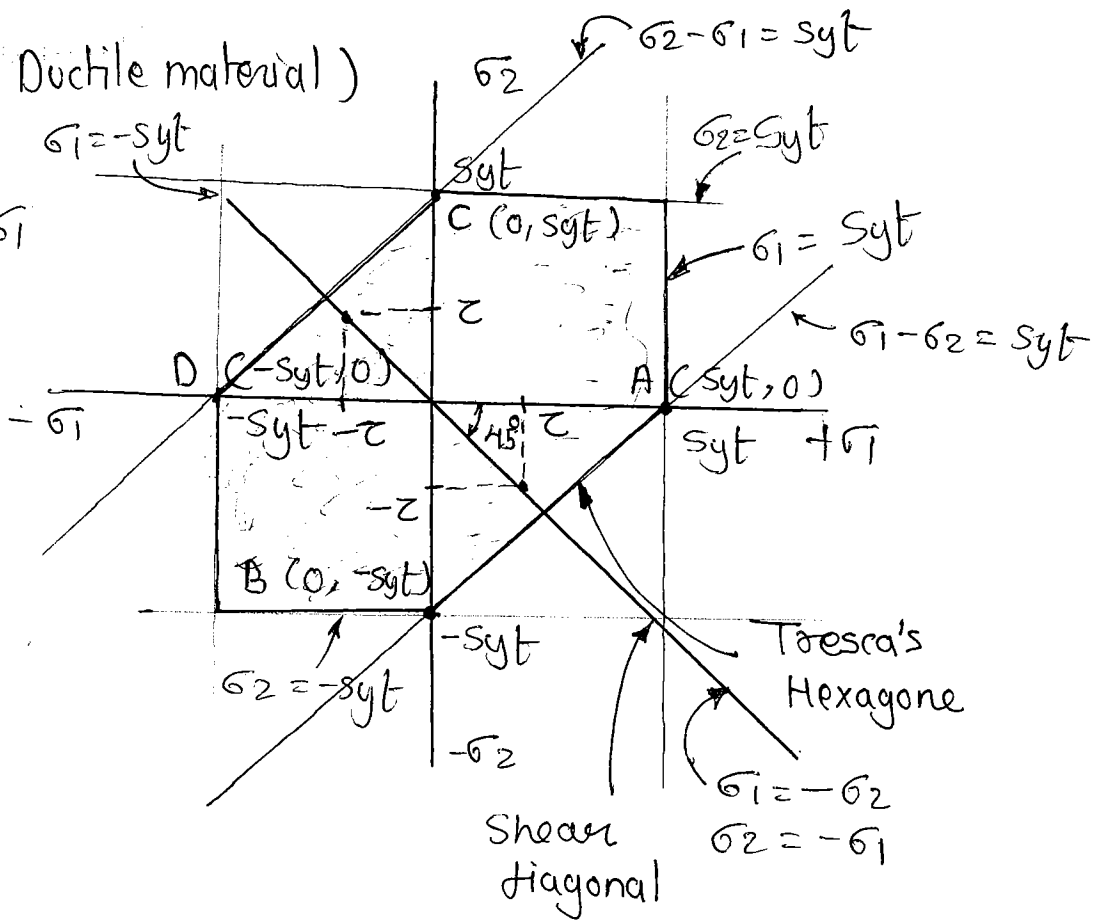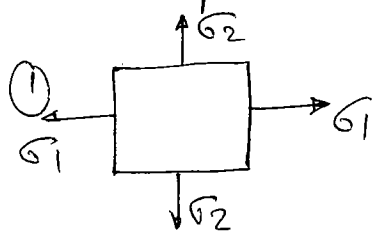
$\tau_{max} = \dfrac{\sigma_1}{2} = \dfrac{Syt}{2}$

$\sigma_1 = Syt$

b] If $\sigma_2 > \sigma_1$

$\tau_{max} = \dfrac{\sigma_2}{2} = \dfrac{Syt}{2}$

$\sigma_2 = Syt$

②

σ_2

σ_1

σ_1

σ_2

c] If $\sigma_1 > \sigma_2$

$\tau_{max} = \dfrac{\sigma_1}{2} = \dfrac{-Syt}{2}$

$\sigma_1 = -Syt$

d] If $\sigma_2 > \sigma_1$

$\tau_{max} = \dfrac{\sigma_2}{2} = \dfrac{-Syt}{2}$

③

σ_2

σ_1

σ_1

σ_2
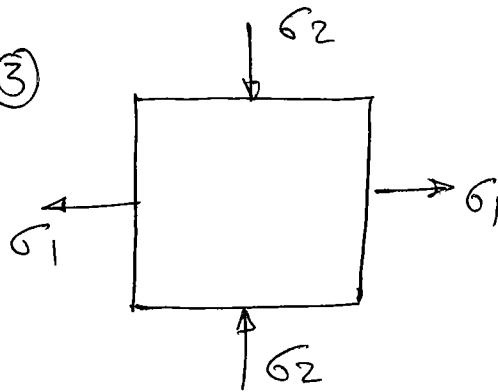
$\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{Syt}{2}$

$\sigma_1 - \sigma_2 = Syt$

If $\sigma_1 = 0$, $A(syt, 0)$

$\sigma_1 = 0$, $B(0, -Syt)$

$\tau_{max} = max \ of \begin{cases} \dfrac{\sigma_1 - \sigma_2}{2} & 40 \\ \dfrac{\sigma_1 - \sigma_1}{2} & 50 \\ \dfrac{\sigma_2 - \sigma_3}{2} & 10 \end{cases}$

20   100

④

$$Z_{max} = \frac{\sigma_2 - \sigma_1}{2} = \frac{S_{yt}}{2}.$$

$$\sigma_2 - \sigma_1 = S_{yt}$$

If,
$\sigma_1 = 0$. $c(0, S_{yt})$
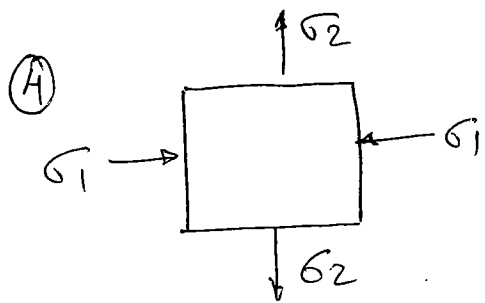
$\sigma_2 = 0$ $D(-S_{yt}, 0)$.

* State of pure shear

$$\sigma = \begin{bmatrix} 0 & \tau \\ \tau & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sigma_1}{2} & \tau_{xy} \\ 0 \\ \tau_{xy} & \frac{\sigma_2}{2} \\ 0 & -\tau \end{bmatrix}$$

$$\sigma = \begin{bmatrix} 0 & \tau \\ -\tau & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\sigma_1}{2} & 0 \\ 0 & \frac{\sigma_2}{2} \end{bmatrix}$$

A component subjected to complex state of stress is said to be failed when the maximum shear stress at a point in the material exceeds the shear strength at yield point under simple tension.

Q   A component is loaded as shown in fig the factor of safety according to

1] Maximum normal stress theory.

a) 1   b) 2   c) 3   d) 4 .

$$Syt = \frac{\sigma}{F.S}$$

100 mm²

100 MPa

Syt = 300 MPa.
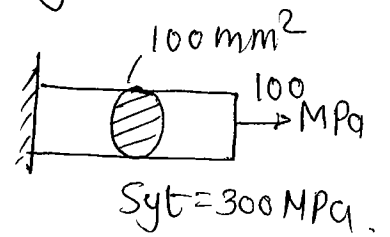
2] Maximum shear stress theory

a) 1   b) 2   c) 3   4) 4

$$Zmax = \frac{\sigma_1}{2} = \frac{Syt}{2 \times F.S}$$

Q A steel bolt is subjected to a direct tensile load of 8.4 kN and a shear load of 48 kN. The cross section area of bolt is 600 mm² and Syt for bolt material is 200 MPa, the F.S according to maximum shear stress theory is

a) 1   b) 2   c) 3   d) 4.

$$\sigma_1 = \frac{8.4 \times 10^3}{600} = 14.80 MPa$$

$$Z = \sigma_s = \frac{18 \times 10^3}{600} = 30 \, MPa.$$
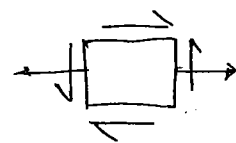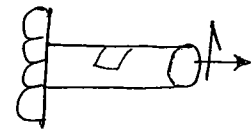
$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + Z^2}.$$

$$= 90 \, MPa.$$

$$\sigma_2 = -10 \, MPa.$$

$$Zmax = \frac{\sigma_1 - \sigma_2}{2} = \frac{90 - (-10)}{2} = 50 \, MPa.$$

$$Zmax = \frac{Syt}{2 F.S}$$

$$50 = \frac{200}{2 F.S} \quad \boxed{F.S = 2}$$

③ Saint venant's theory (or) Maximum Principal strain theory.

If $\sigma_1 > \sigma_2 > \sigma_3$

Then $\varepsilon_1 > \varepsilon_2 > \varepsilon_3$.
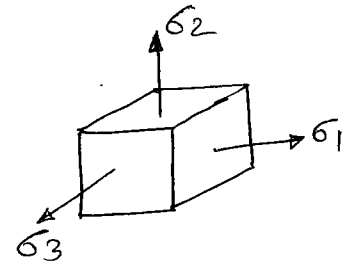
$\varepsilon_1 \rightarrow$ Maximum principal strain

$\varepsilon_1 > \dfrac{S_{yt}}{E}$ — Failed.

Design Equation

$$\varepsilon_1 = \frac{S_{yt}}{E \times F_s} \qquad — ①$$

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E} - \frac{\mu \sigma_3}{E} \qquad — ②$$

$$\frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E} - \frac{\mu \sigma_3}{E} = \frac{S_{yt}}{E \times F \cdot S}$$

$$\boxed{\sigma_1 - \mu \sigma_2 - \mu \sigma_3 = \frac{S_{yt}}{F \cdot S}}$$

$\mu = \dfrac{-\text{lateral strain}}{\text{longitudinal }''}$

$\mu = 0$ for cork

$\mu = 0.25$ to 0.33
  for steel.

$= 0.47$ for rubber

For 2D $\sigma_3 = 0$

$$\sigma_1 - \mu \sigma_2 = \frac{S_{yt}}{F \cdot S}$$

Not valid for Brittle material or for Ductile material.
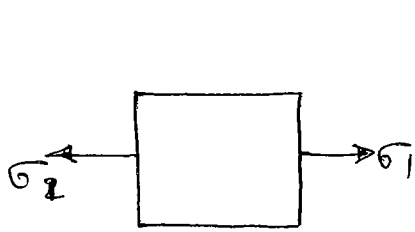
$$\varepsilon_{uni} = \frac{\sigma_1}{E}$$
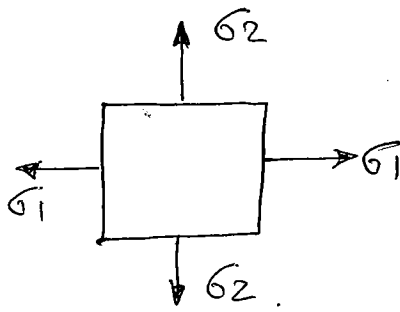
$$\varepsilon_{biaxial} = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E}$$

$$\varepsilon_{uni} > \varepsilon_{bi}$$

Graph
(Ductile)
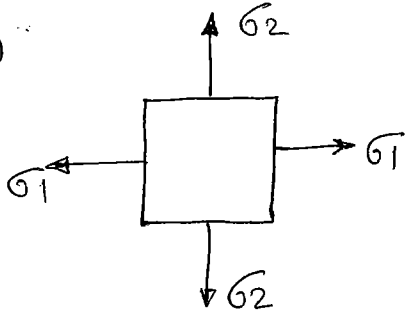
a] If $\sigma_1 > \sigma_2$

$$\varepsilon = \frac{\sigma_1 - \mu\sigma_2}{E} = \frac{Syt}{E}$$

$$\sigma_1 - \mu\sigma_2 = Syt$$

If $\sigma_1 = 0$ $A\left(0, -\frac{Syt}{\mu}\right)$

$\sigma_2 = 0$ $B(Syt, 0)$

b] If $\sigma_2 > \sigma_1$.

$$\sigma_2 - \mu\sigma_1 = Syt.$$

If $\sigma_1 = 0$ $c(0, Syt)$

$\sigma_2 = 0$ $D\left(-\frac{Syt}{\mu}, 0\right)$

$\left(\frac{Syt}{1-\mu}, \frac{Syt}{1-\mu}\right)$

$E\left(0, \frac{Syt}{\mu}\right)$

$\left(\frac{-Syt}{1+\mu}, \frac{Syt}{1+\mu}\right)$

$C(0, Syt)$

$F(-Syt, 0)$

$B(Syt, 0)$

$D\left(-\frac{Syt}{\mu}, 0\right)$

$+\sigma_1$

$H\left(\frac{Syt}{\mu}, 0\right)$

$G(0, -Syt)$

$\left(\frac{Syt}{1+\mu}, \frac{-Syt}{1+\mu}\right)$

$\left(\frac{-Syt}{1-\mu}, \frac{-Syt}{1-\mu}\right)$

$A\left(0, -\frac{Syt}{\mu}\right)$

$-\sigma_2$

②
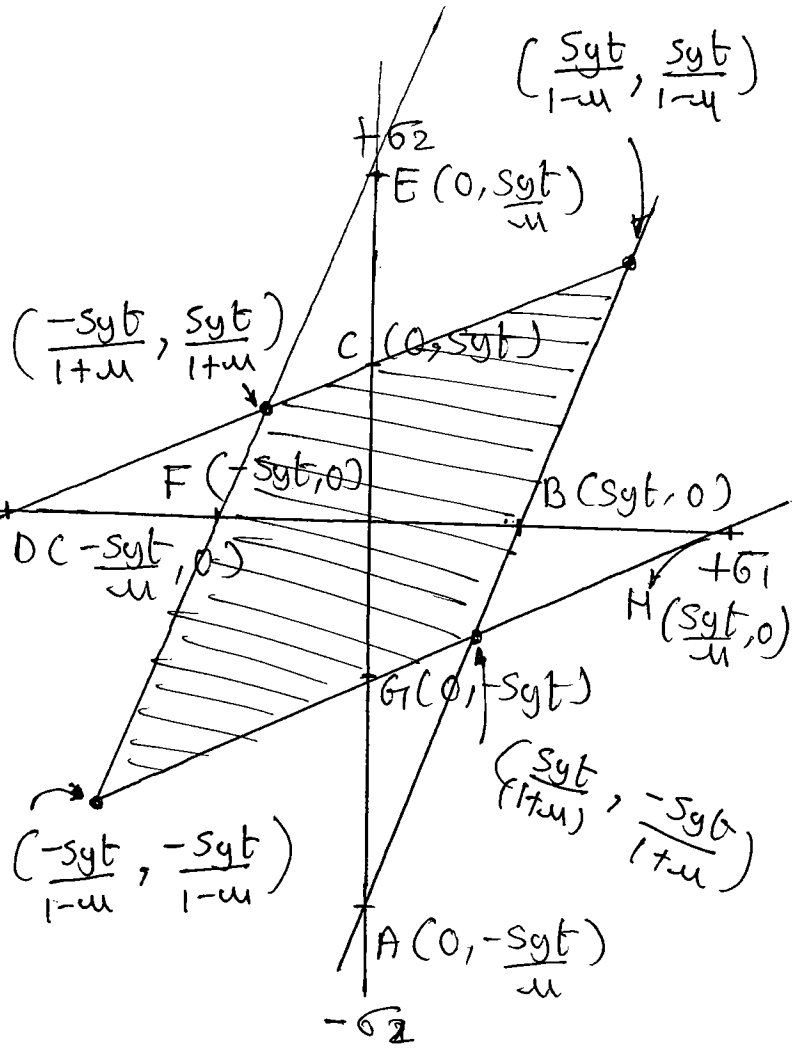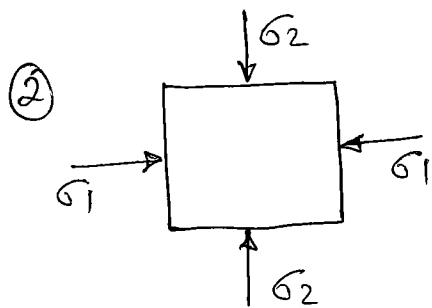
C] If $\sigma_1 > \sigma_2$.

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E} = \frac{-Syc}{E}$$

$$\sigma_1 - \mu \sigma_2 = -Syt$$

If $\sigma_1 = 0$ $E(0, \frac{+Syt}{\mu})$

$\sigma_2 = 0$ $F(-Syt, 0)$

D] If $\sigma_2 > \sigma_1$

$$\sigma_2 - \mu \sigma_1 = -Syt.$$

If $\sigma_1 = 0$ $G(0, -Syt)$

$\sigma_2 = 0$ $H(\frac{Syt}{\mu}, 0)$

A component subjected to complex state of stress is said to be failed when the maximum principal strain at a point in the material exceeds the strain at yield point under simple tension.

(A) Haigh's theory (or) Maximum strain energy theory.

Strain Energy

$$U = \frac{1}{2} \varepsilon \times \sigma$$

$$= \frac{1}{2} \sigma \varepsilon$$

$$= \frac{1}{2} \times \sigma \times \frac{\sigma}{E}$$

$$= \frac{\sigma^2}{2E}$$

stress.

Syt - Proof resilience

σ

ε
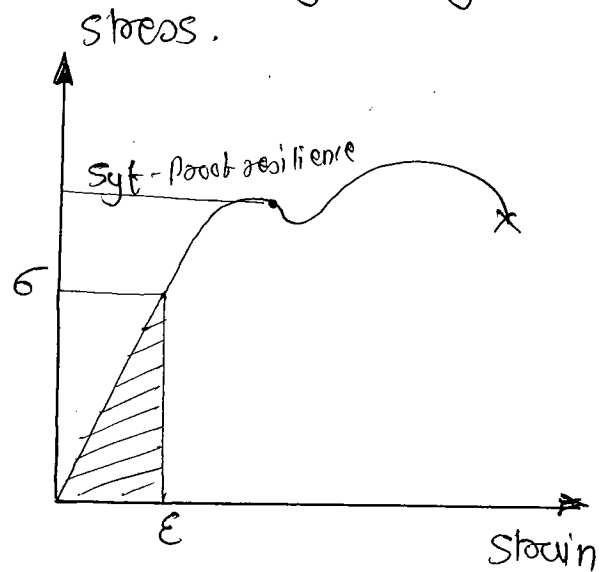
Strain

$$U \text{ at yield point} = \frac{Syt^2}{2E}$$

Strain energy density under complex loading.
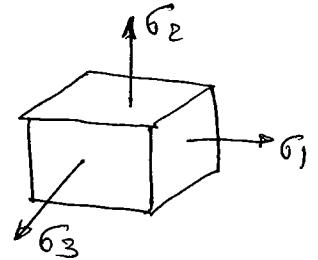
$$U_{complex} = \frac{1}{2}\sigma_1 \varepsilon_1 + \frac{1}{2}\sigma_2 \varepsilon_2 + \frac{1}{2}\sigma_3 \varepsilon_3$$

$$\varepsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E}$$

$$\varepsilon_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$$

$$\varepsilon_3 = \frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$U_{complex} = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

$$U_{complex} \leq U \text{ simple tension} \quad - \quad \text{safe.}$$

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] = \left(\frac{Syt}{F \cdot S}\right)^2 \times \frac{1}{2E}$$

$$\frac{Syt}{F \cdot S} = \sqrt{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}$$

For 2D

$$\frac{S_{yt}}{F \cdot S} = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\mu \sigma_1 \sigma_2}$$

$$x^2 + y^2 - 2bxy = a^2.$$

$$\sigma_1^2 + \sigma_2^2 - 2\mu_1 \sigma_1 \sigma_2 = S_{yt}^2.$$

If $\sigma_1 = 0$ $\sigma_2 = \pm S_{yt}$

     $\sigma_2 = 0$ $\sigma_1 = \pm S_{yt}$
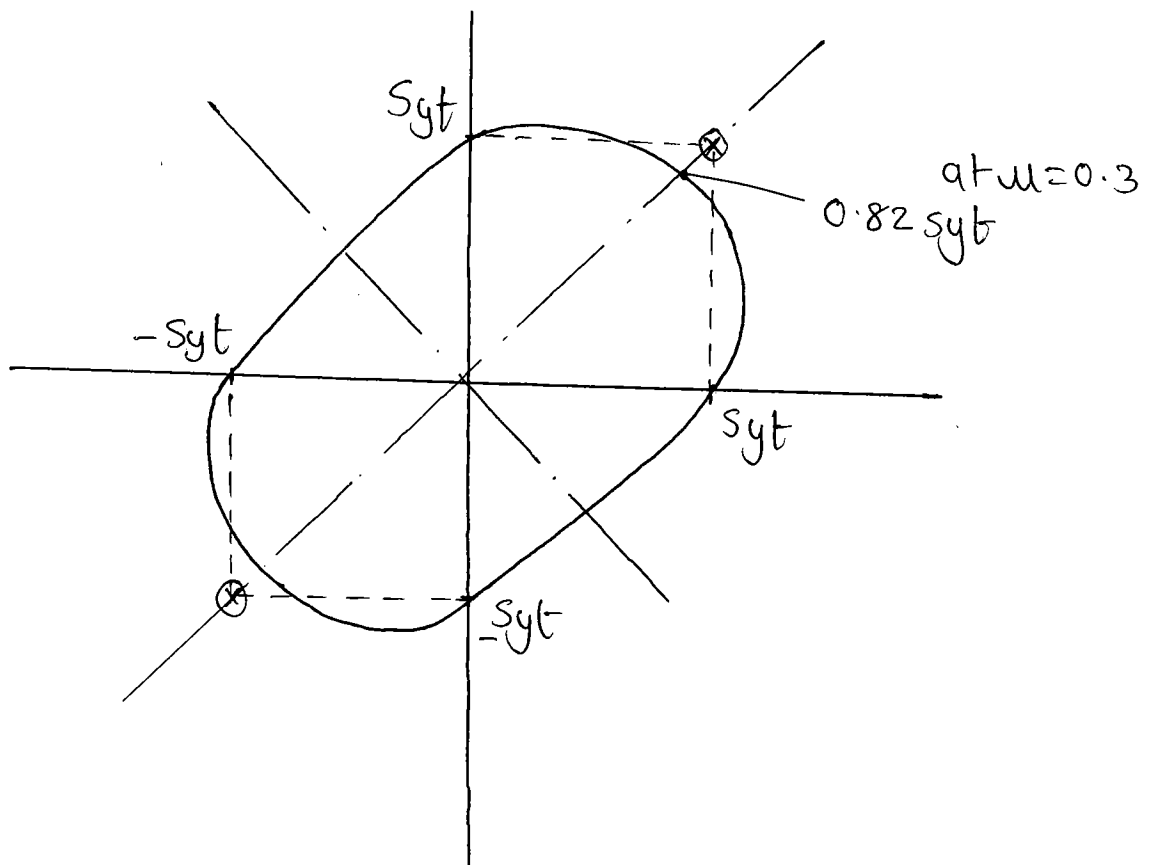
Validity — Valid for Ductile material.

Q) If the principal stresses at point are $\sigma_1 = +2f$
$\sigma_2 = +f$, $\sigma_3 = -\frac{1}{2}f$, and $s_{yt} = 200 \, N/mm^2$, $\mu = 0.3$
The value of 'f' according to Haish's theorem

a) $100 \, N/mm^2$ b) $75 \, N/mm^2$ c) $90 \, N/mm^2$ ✓   d) $150 \, N/mm^2$

$$\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\mu \left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \right) \right]^{1/2} = \frac{s_{yt}}{f \cdot S}$$

5) Von Mises and Henky's theory (or) Shear strain Energy theory (or) Distortion Energy theory.
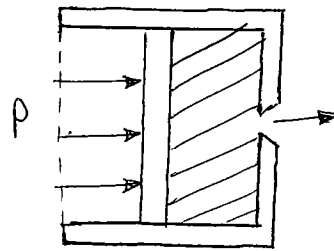
Total Strain Energy (Ut)
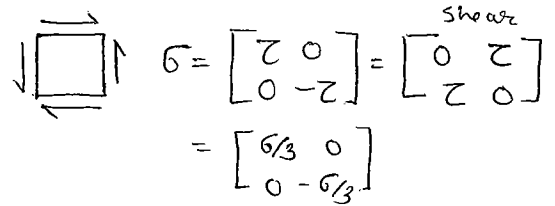
$$U_t = \frac{1}{2E} \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu \left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \right) \right]$$

$$U_t = U_v + U_d$$

= Volume change + Change of shape.
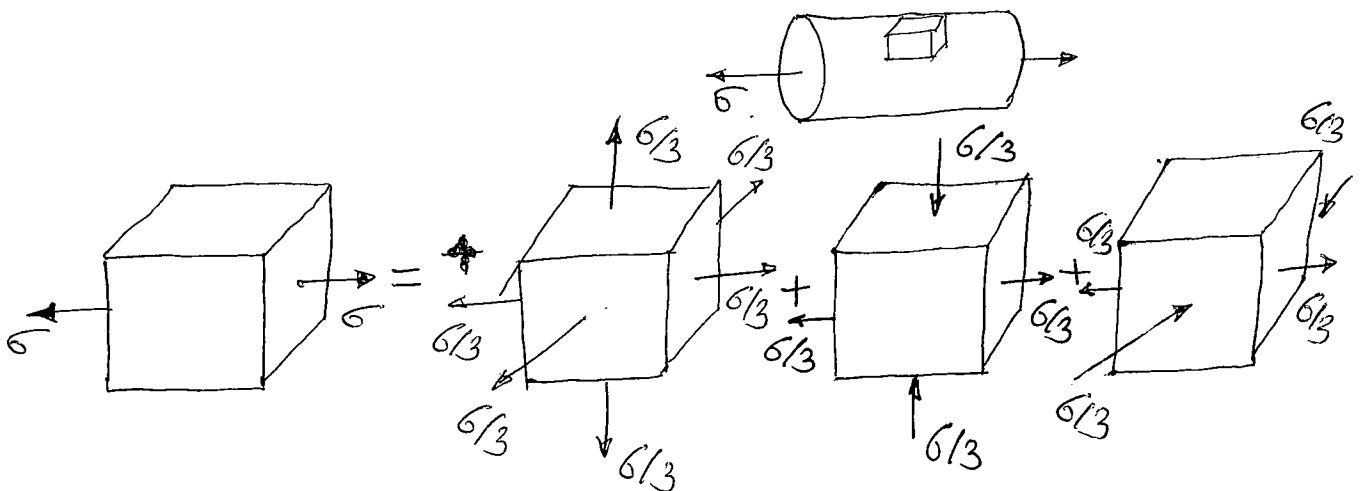
$= $ Volume change $+$ Distortion $+$ Distortion

$$\overrightarrow{\Box} \quad \sigma = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \text{ shear}$$

$$= \begin{bmatrix} \sigma/3 & 0 \\ 0 & -\sigma/3 \end{bmatrix}$$

$$E = 3K(1 - 2\mu)$$

$$= 3 \frac{\sigma}{\varepsilon_V}(1 - 2\mu)$$

$$\varepsilon_V = 3\frac{\sigma}{E}(1 - 2\mu) \quad \cdots \quad \varepsilon_V = \frac{\delta V}{V}$$

$$\delta V = \frac{3\sigma}{E}(1 - 2\mu)V$$

If $\mu = 0.5$, $\delta v = 0$.

$$U_t = \frac{1}{2E}\left[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

If $u = 0.5$ $\quad U_t = U_d$.

$$U_d = \frac{1}{2E}\left[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

$$= \frac{(1 + \mu)}{3E}\left[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

$$E = 2G(1 + \mu)$$

$$= \frac{1}{6G}\left[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

For simple tension.

$$\sigma_1 = \sigma \quad, \quad \sigma_2 = \sigma_3 = 0.$$

$$U_d = \frac{1}{6G}\sigma^2.$$

$$E = 2G(1 + \mu)$$

$$U_d = \frac{1 + \mu}{3E}\sigma^2.$$

Distortion energy under Complex loading $>$ Distortion energy under simple tension at yield point

— Failed

$$= \frac{1}{6G} Syt^2 \quad \cdots \cdots \text{ At yeiled point}$$

$$\frac{1}{6G}\left[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] = \frac{1}{6G}\left(\frac{Syt}{F \cdot S}\right)^2$$

$$\frac{Syt}{F \cdot S} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}$$

For 2D stress.

$$\frac{Syt}{F \cdot S} = \sqrt{(\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2)}$$

$$x^2 + y^2 - xy = a^2 \quad \cdots \text{ Ellipse eq}^n.$$

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = Syt^2$$

$$\sigma_1 = 0 \qquad \sigma_2 = \pm Syt$$
$$\sigma_2 = 0 \qquad \sigma_1 = \pm Syt.$$

Torsion or State of pure Shear stress
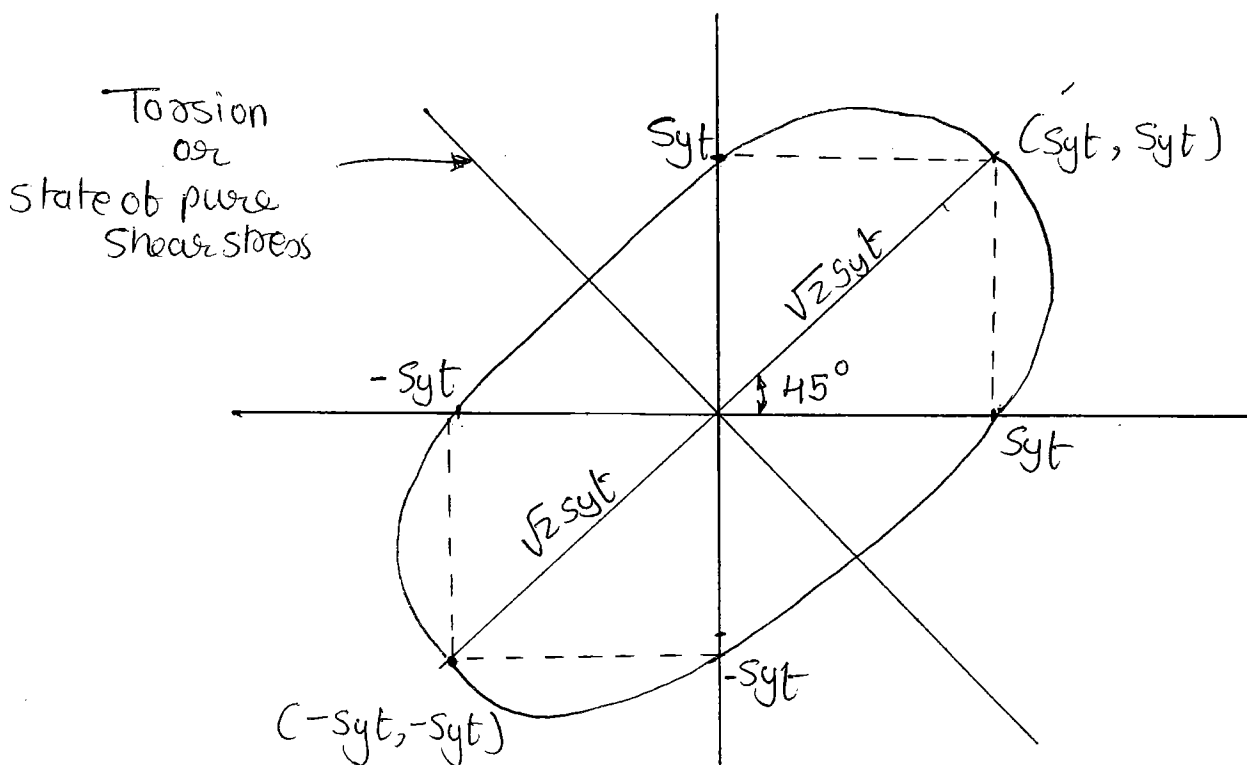
Validity — This theory gives very good results for

① $\sigma_1 = \dfrac{Syt}{F\cdot S}$ — Rankine

② $\dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{Syt}{F\cdot S}$ — Guest

③ $\sigma_1 - \mu\sigma_2 = \dfrac{Syt}{F\cdot S}$ — St venent

④ $\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 = \left(\dfrac{Syt}{F\cdot S}\right)^2$ — Haigh's

⑤ $\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \left(\dfrac{Syt}{F\cdot S}\right)^2$ — Von Moises

Q A component is subjected to torsion, T. Estimate the permisseble value of shear stress z according to different theories of failure in terms of yield strength, Syt Take $\mu = 0.3$ E $= 2.1 \times 10^5$ N/mm²

Ans
① Rankines $\sigma_1 = \dfrac{Syt}{F\cdot S} = $
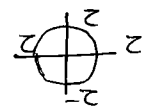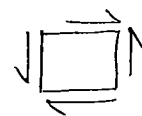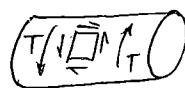
$\sigma = \begin{bmatrix} 0 & z \\ z & 0 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 0 & -z \end{bmatrix}$

$\sigma_1 = Syt$

$z = Syt$

$\dfrac{z}{Syt} = 1$

Exp. value → Ssy = 0.57 to 0.6 of Syt

② Guest   (most conservative theory)

$\dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{Syt}{2 F\cdot S}$

$z - (-z) = Syt$

$2z = Syt$ $z = \dfrac{Syt}{2} = 0.5 \, Syt$

③ St venents theory

$$\sigma_1 - \mu \sigma_2 = S_{yt}$$

$$2 - 0.3 \times (-2) = S_{yt}$$

$$2 = \frac{S_{yt}}{1.3}$$

$$2 = 0.769 \, S_{yt} = 2, \quad \frac{2}{S_{yt}} = 0.769$$

④ Haigh's theory

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 = S_{yt}^2$$

$$2 + (-2)^2 - 2\mu(2)(-2) = S_{yt}^2$$

$$2 = 0.63 \, S_{yt}$$

⑤ Von moises.

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = S_{yt}^2$$

$$2^2 + (-2)^2 - 2(-2) = S_{yt}^2$$

$$2 \cdot 2^2 + 2^2 = S_{yt}^2 \qquad (\text{more economical})$$

$$2 = 0.577 \, S_{yt}$$

Note:-

1 Use of guest theory Increase size hence increse more cost and create bulky parts

Benting Equation.

$$M = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$$

Torsion ↓ $\underbrace{\phantom{M_y\hat{j} + M_z\hat{k}}}$
              Bending.

$$M = M_x i + M_z k$$
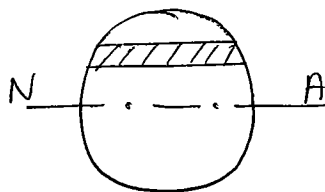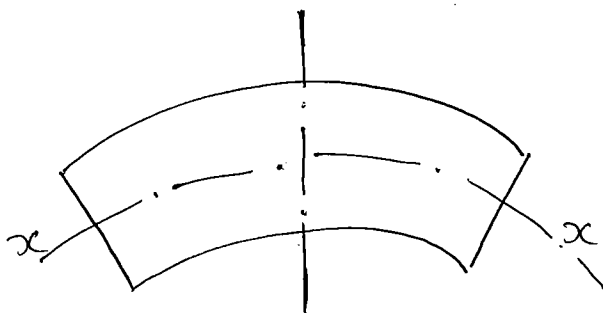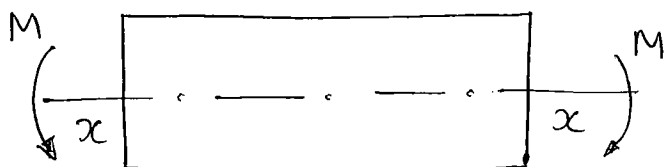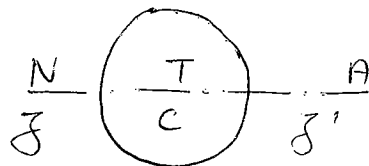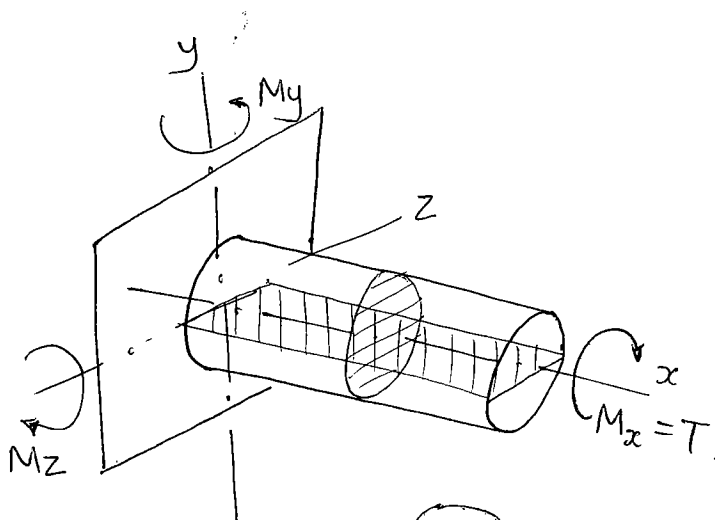
$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}.$$

M = Bending moment

I = M.I of c/s about neutral axis

σ_b = Bending stress

y = Distance measured from Nekubal axis.

R = Radius measured from nekubal axis.

$$I = \frac{\pi}{64} d^4$$

$$= \frac{\pi}{64} (D_o^4 - d_i^4)$$

Max stress $\sigma_{b,max}$

min stress = zero

d

$\sigma_b \propto y$

$\sigma_b = 0$

$\sigma_{b,max}$

If $y = R$ $\sigma_b = \dfrac{R}{R}\sigma_{max}$

$y = R$; $\sigma_b = \sigma_{b,max}$.

N —— A
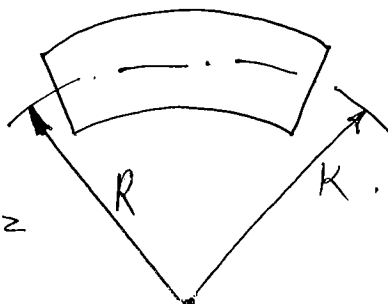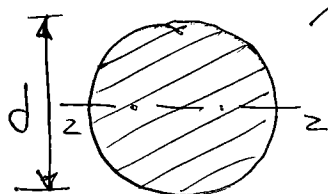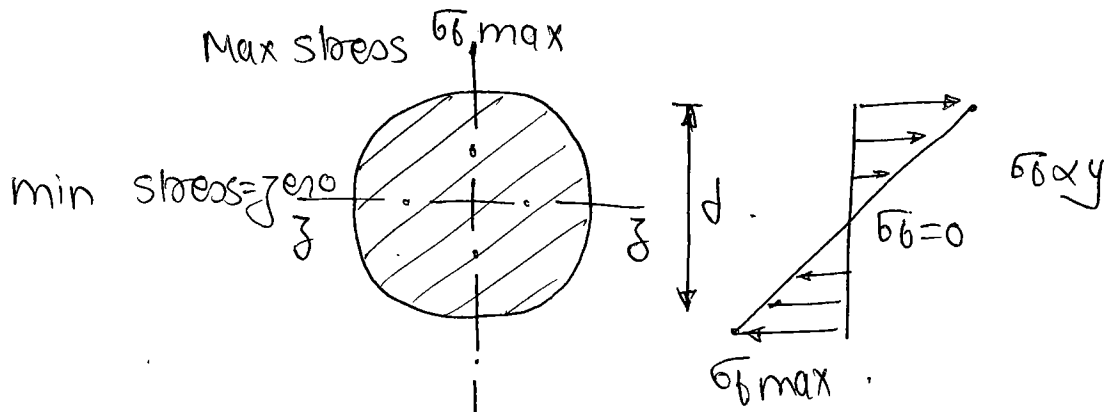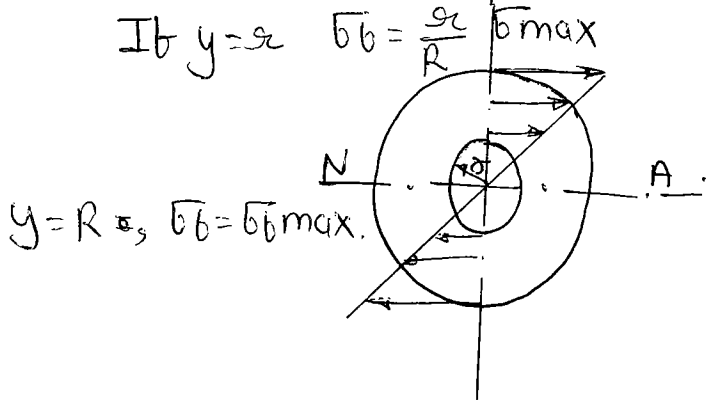
Along the N-A. Hence σ_b min is zero.

Torsion Equation.

$z \propto R$

$$\boxed{\dfrac{T}{J} = \dfrac{z}{R}} = \dfrac{G\theta}{\ell}$$

T

T

R
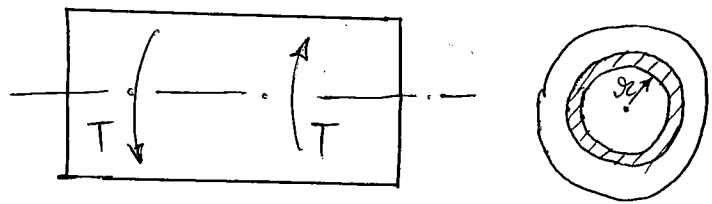
T = Torsion

J = Polar moment of Inertia.

$$= I_{xx} + I_{yy} = \dfrac{\pi}{32}d^4$$

$$= \dfrac{\pi}{32}(D^4 - d^4)$$

z = Shear stress at radius R
G = Modulus of rigidity
θ = Angle of twist.
ℓ = length of member under torsion

For solid.   For hollow.

$$\tau_{min} = \frac{\Re}{R} \tau_{max}$$
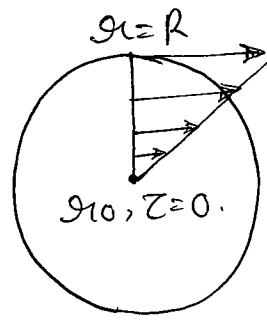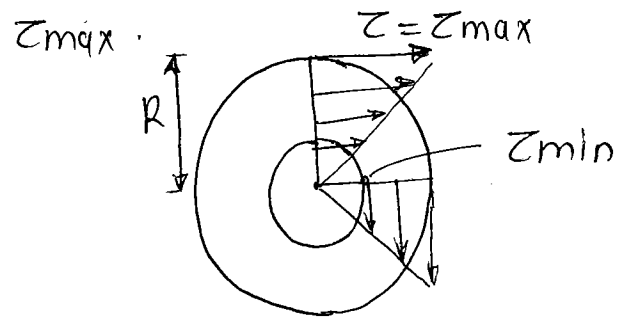
Von mises eq^h for Axial and torsion loading.
$\qquad$ (Bending)
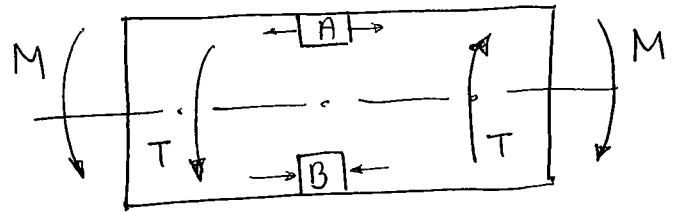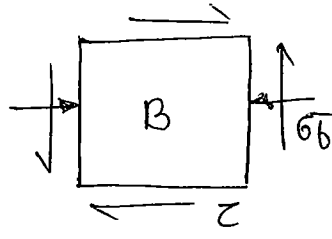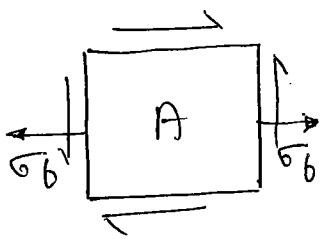
$$\left[\sigma_1{}^2 + \sigma_2{}^2 - \sigma_1\sigma_2\right]^{1/2} = \frac{Syt}{F\cdot S} = \sigma \text{ equivalent}$$

for A.

$$\sigma x = \sigma$$
$$\sigma y = 0.$$

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}.$$

$$\sigma_1 = \frac{\sigma}{2} + R$$
$$\sigma_2 = \frac{\sigma}{2} - R.$$

$\qquad$ Put in eq^h

$$\left[\left(\frac{\sigma}{2} + R\right)^2 + \left(\frac{\sigma}{2} - R\right)^2 - \left(\frac{\sigma}{2} + R\right)\left(\frac{\sigma}{2} - R\right)\right]^{1/2} = \sigma \text{ equivalent}.$$

$$\left[\left(\frac{\sigma}{2}\right)^2 + 3R^2\right]^{1/2} = \sigma eq$$

$$\left[\left(\tfrac{\sigma}{2}\right)^2 + 3\left\{\sqrt{\left(\tfrac{\sigma}{2}\right)^2 + \tau^2}\right\}^2 \right]^{1/2} = \sigma_{eq}$$

$$\left[\left(\tfrac{\sigma}{2}\right)^2 + 3\left(\tfrac{\sigma}{2}\right)^2 + 3\tau^2 \right]^{1/2} = \sigma_{eq}.$$

$$\left[4\left(\tfrac{\sigma}{2}\right)^2 + 3\tau^2 \right]^{1/2} = \sigma_{eq}.$$

$$2\tfrac{\sigma}{2} + \tau(1+3)^{1/2} = \sigma_{eq}.$$

$$\boxed{\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2}.}$$

Q: The homogeneous state of stress for a metal part undergoing plastic deformation is

$$T = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ 10 & 5 & 0 \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ 5 & 20 & 0 \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \\ 0 & 0 & -10 \end{bmatrix}$$

where the stress component value are in MPa. using Von moises yield criterism.

a] 9·50   b] 16·07   c] 28·52   d] 19·41

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}.$$

$$= \frac{10+20}{2} + \sqrt{\left(\frac{10-20}{2}\right)^2 + 5^2}$$

$$= 22 \cdot 071.$$

$$\sigma_2 = \frac{\sigma_y + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_y - \sigma_z}{2}\right)^2 + \tau_{yz}^2}$$

$$= \frac{20 - 10}{2} + \sqrt{15^2 + 5^2}$$

$$= 7 \cdot 93.$$

$$\sigma_3 = -10 \, mpa.$$

$$\frac{S_{yt}}{FS} = \sigma_{eq} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_2 + \sigma_3\sigma_1)}$$

$$= 27.83.$$

$$Z = 0.557 \, \sigma_{eq}.$$

$$= 0.557 \times 27.83$$

$$= 16.057.$$

⑥ . Mohr's Theory (Coloumb – Mohr's theory)
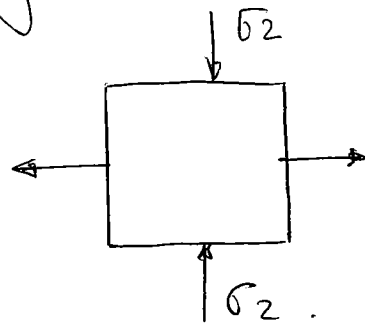
For Duchle $S_{yt} = S_{yc}$ (isotropic – Assumption)

$$\sigma_1 - \sigma_2 = \frac{S_{yt}}{FS}.$$

$$\frac{\sigma_1}{S_{yt}} - \frac{\sigma_2}{S_{yt}} = \frac{1}{FS}$$

For Brittle Material ($S_{yt} \neq S_{yc}$)

$$\frac{\sigma_1}{S_{yt}} - \frac{\sigma_2}{S_{yc}} = \frac{1}{F.S}.$$

$$\frac{\sigma_{1so}}{S_{ut}} - \frac{\sigma_2}{S_{ut}} = \frac{1}{F \cdot S}$$

$$\sigma_2 = 0$$
$$\sigma_1 = S_{ut}$$

$$\frac{-\sigma_1}{S_{ut}} - \frac{\sigma_2}{S_{uc}} = \frac{1}{F \cdot S}$$

$$\sigma_2 = 0$$
$$S_{ut} = -\sigma_1$$

Pure shear

Failure envelope

Safe Zone

Suc

$S_{uc} > S_{ut}$
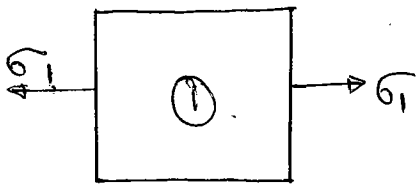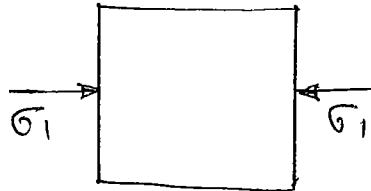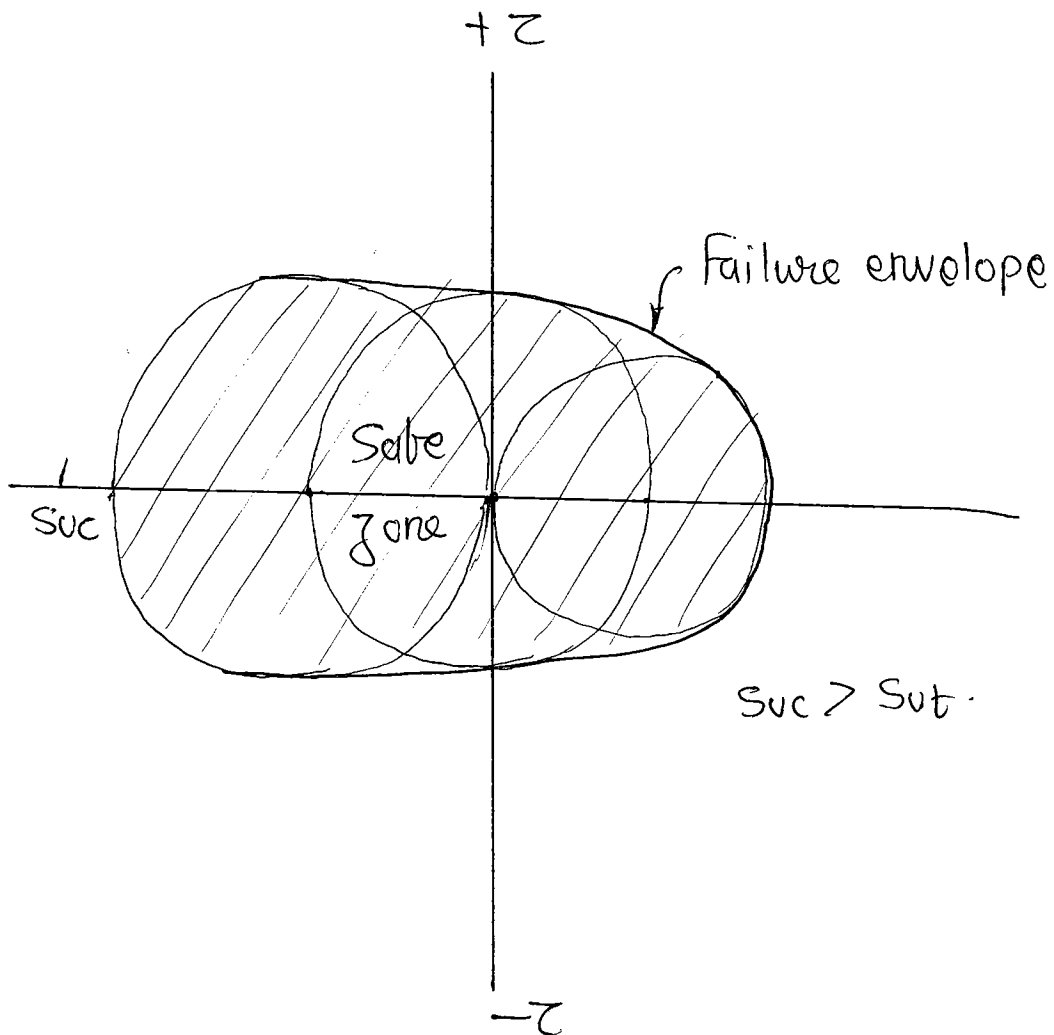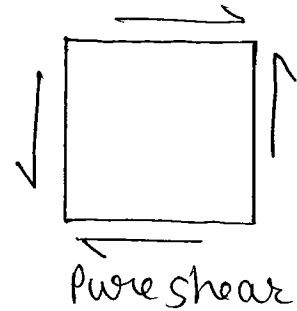
Design for Brittle material

Q A steel pin subjected to a shear force 25 kN and the direct compression of 41 kN. The permissible compressive stresses in pin material is 50 MPa, then the suitable dia of pin according to distortion energy theory is

a] 75$mm   b] 38 mm   c] 20mm   d] 45mm.

$$T = \frac{\pi}{16} f_s d^3$$

$f_s = 25 \text{ KN}$

$F_t = 41 \text{ KN}$

$\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2}$

$$50 = \sqrt{\left(\frac{41}{A}\right)^2 + 3\left(\frac{25}{A}\right)^2} = 59.63$$

$$= \sqrt{\frac{41^2 + 3\times25^2}{A^2}}$$

$$A = \frac{2\sqrt{889}}{50}$$

$$A = 1192.64$$

$$\frac{\pi}{4} d^2 = 1192.64$$

$$\boxed{d = 38 \text{ mm}}$$

Q A load P 45 KN is applied to a crank shaft of dia. 90mm at a distance of 200mm from bearing. The material is 30C4 with $S_{yt} = 315$ MPa. Factor of safety according to Guest's theory is

a] 1   b] 2   c] 3   d] 4.

$$\frac{\sigma_1 - \sigma_2}{2} = \frac{S_{yt}}{2 FS}$$

(torsion) $T = P \times 150$
$= 45 \times 150$
$= 6750 \text{ KN-mm}$

(Bending) $M = P \times 200$
$= 45 \times 200$
$= 9000 \text{ KN-mm}$

load p is applied ⊥ᵉʳ to plane of paper.

$$T = \frac{\pi}{16} f_s d^3$$

0.047
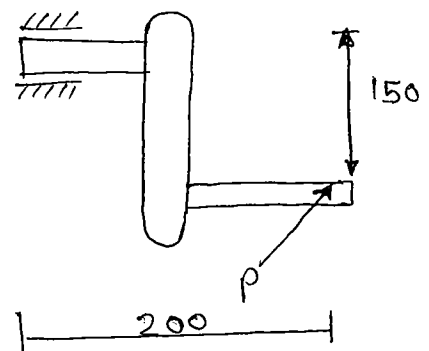
$$\sigma_b = \frac{32\,M}{\pi d^3}$$

$$= \frac{32 \times 9000}{\pi \times 90^3}$$

$$= 0.125 \ KN/mm^2$$

$$= 125.75 \ N/mm^2$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{125.75}{2}\right)^2 + (47.15)^2}$$

$$= 78.59$$

$$\tau_{max} = \frac{S_{yt}}{2 \times F \cdot S}$$

$$78.59 \times 2 \times FS = 315$$

$$\boxed{FS = 2}$$

Q A component subjected to Fluxtuating stresses

Desing for Fatigue.

A component is subjected to a fluctuating stresses fails at a stress much below their static strength. This phenomenon is called as Fatigue.

wohler - Fatigue.

Residual energy.
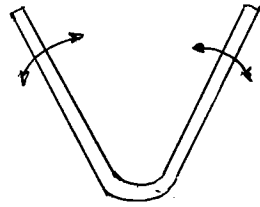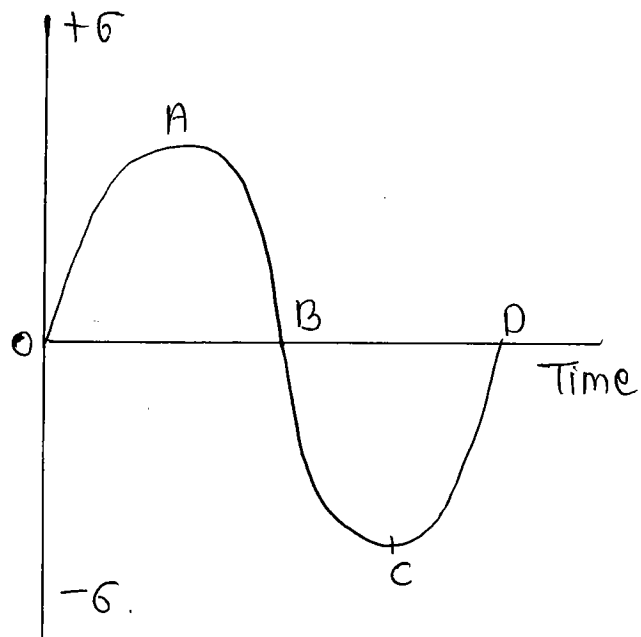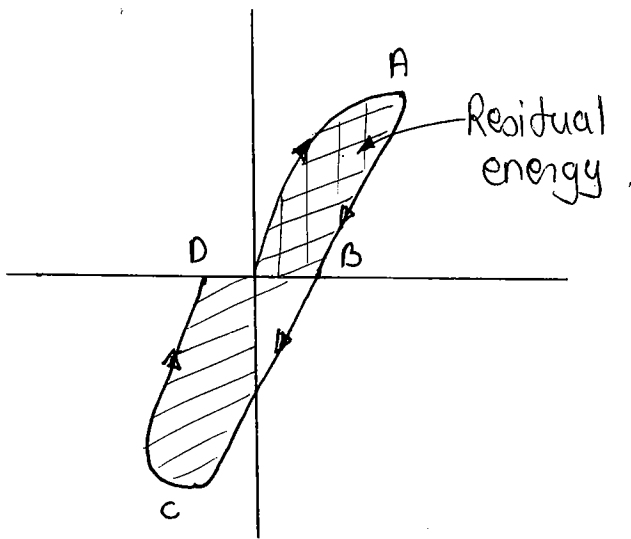
Ex:- 100 KN- strength.
10 KN - Residual ↑ then strength become 90KN↓

Terminology.

i]

Fluxtuahing load.

mean stress, $\sigma m = \dfrac{\sigma max + \sigma min}{2}$

Stress amplitude $\sigma a = \dfrac{\sigma max - \sigma min}{2}$

Stress Ratio, $R = \dfrac{\sigma min}{\sigma max}$.

2]

Repeated stress ($\sigma m = \sigma a$)

$\sigma m = \dfrac{\sigma max + \sigma min}{2} = \dfrac{\sigma + 0}{2} = \sigma/2$.

$\sigma a = \dfrac{\sigma max - \sigma min}{2} = \dfrac{\sigma + 0}{2} = \sigma/2$

$R = \dfrac{\sigma min}{\sigma max} = 0$.

3]

Completely Reversible

$\sigma m = \dfrac{\sigma max + \sigma min}{2} = \dfrac{\sigma + (-\sigma)}{2} = 0$

$\sigma a = \dfrac{\sigma max - \sigma min}{2} = \dfrac{\sigma - (-\sigma)}{2} = \dfrac{2\sigma}{2} = \sigma$.

$R = \dfrac{\sigma min}{\sigma max} = \dfrac{-\sigma}{\sigma} = -1$.

a] Rail Road car axle.

b] IC Engine

Pressure in IC Engine

c]

Gears

D]

Fluctuating flag.

h

E.

wheel.
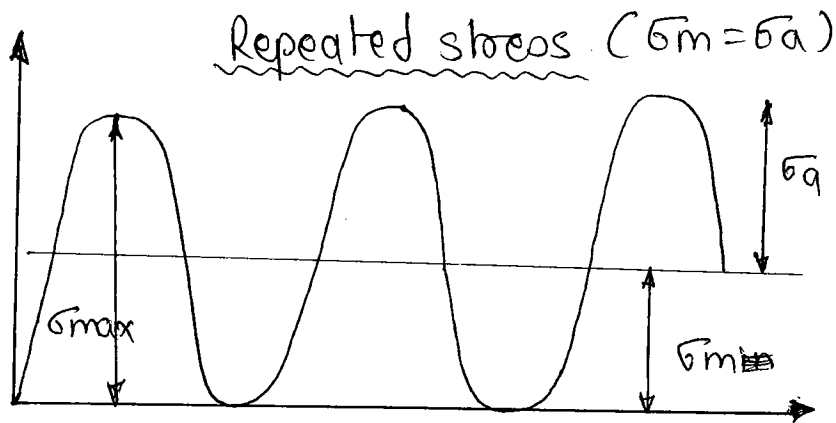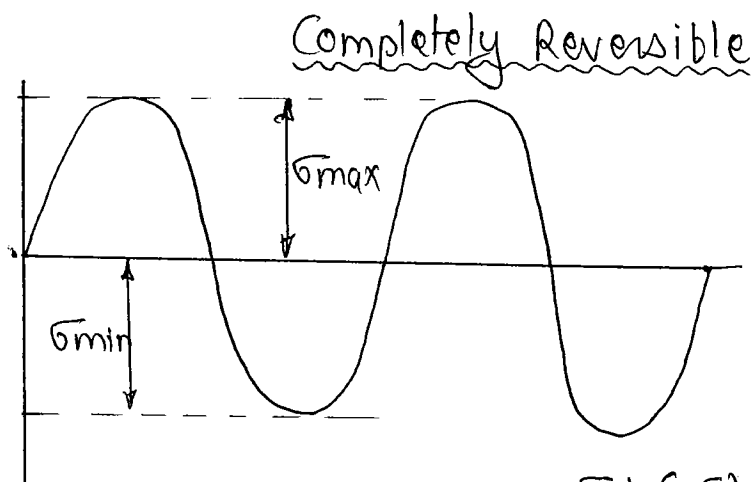
Axle → stationary or rotating member and is loaded in bending.

a]

$$\sigma_b = \frac{32\, M_{ax}}{\pi d^3}.$$

B.M

m max.

$\frac{w}{2} \times a$ $\frac{w}{2} \times a$

Pure Bending

$\frac{w}{2}$ S.F

$\frac{w}{2}$

comp.

$+\sigma_b$

B] IC Engine.

Pr.

cycle continue hence Repeated load.

85mm

1725 rpm

Motor

F_i $i = 1, 2, \cdots$ (variable)

7·6mm

$R = 250mm$

revolution counter.

F_i

Mmax.

$\sigma_b = \dfrac{32M}{\pi d^3} = \sigma_a$.

σ_a

$\sigma_m = 0$

load		speed rpm
σ_{a1} =	F_1	N_1
σ_{a2} =	F_2	N_2
σ_{a3} =	F_3	N_3
:		:

(S-N curve)

Stress amplitude / log (stress)

σ_b static load · Sut

low cycle Fatigue   High cycle Fatigue

Sf

Se'

Finite life   Infinite life.

10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 No of revolution

log (speed) N
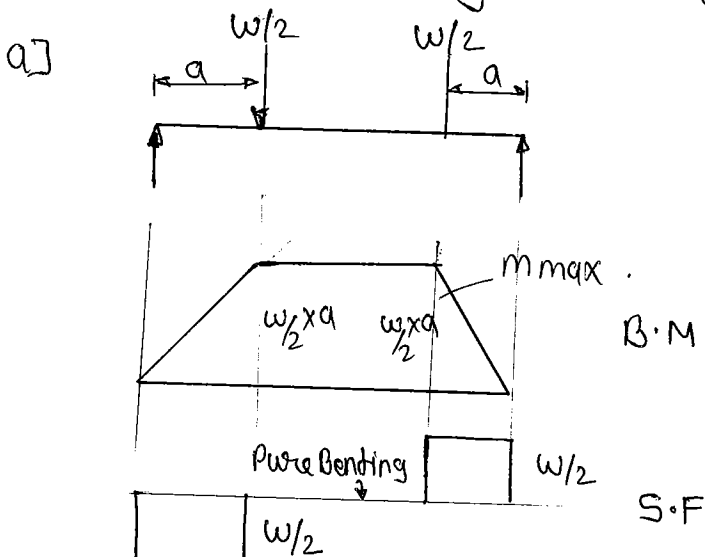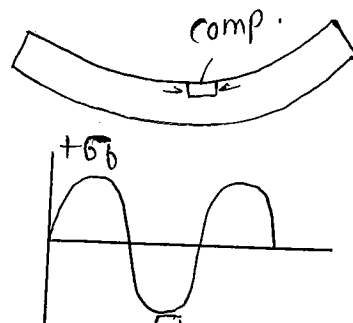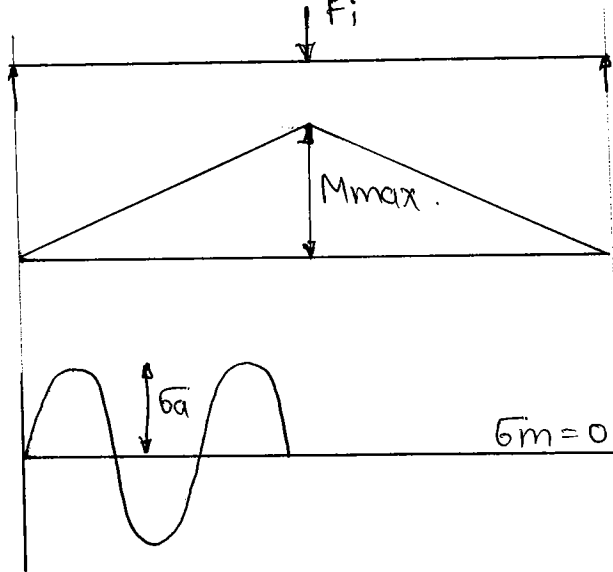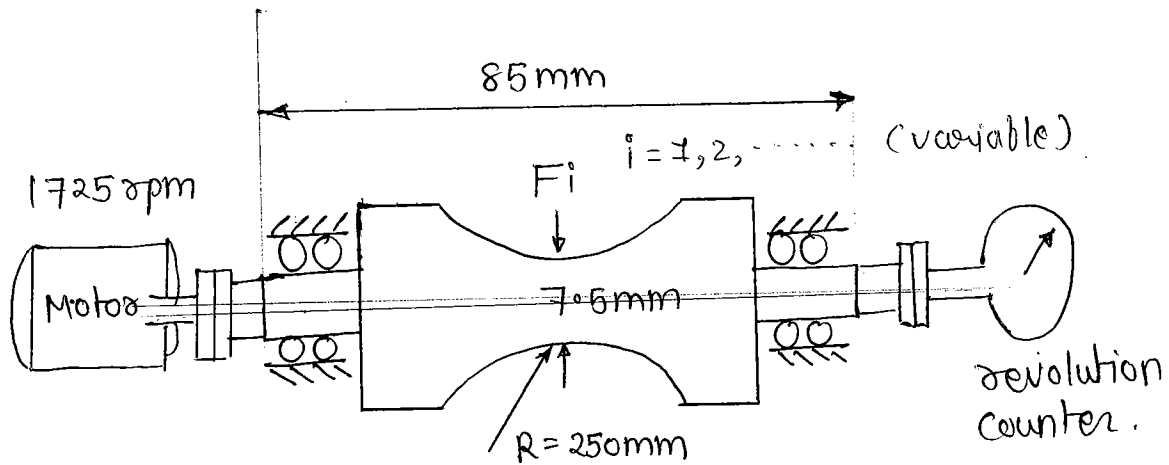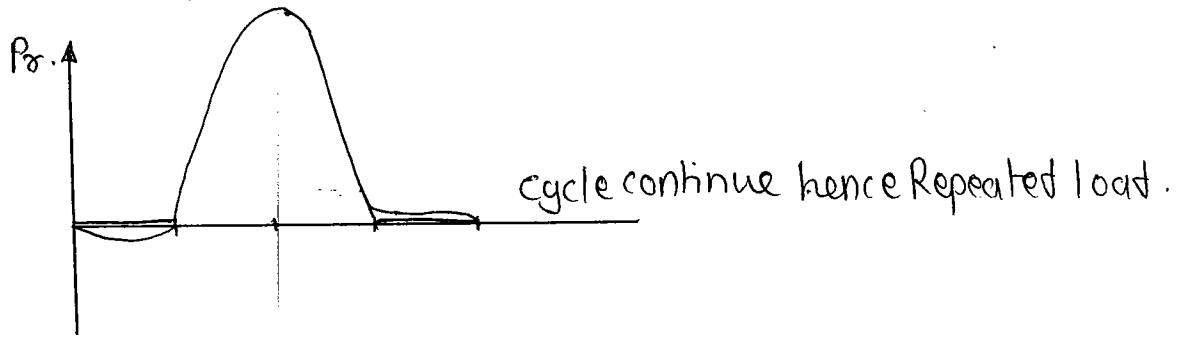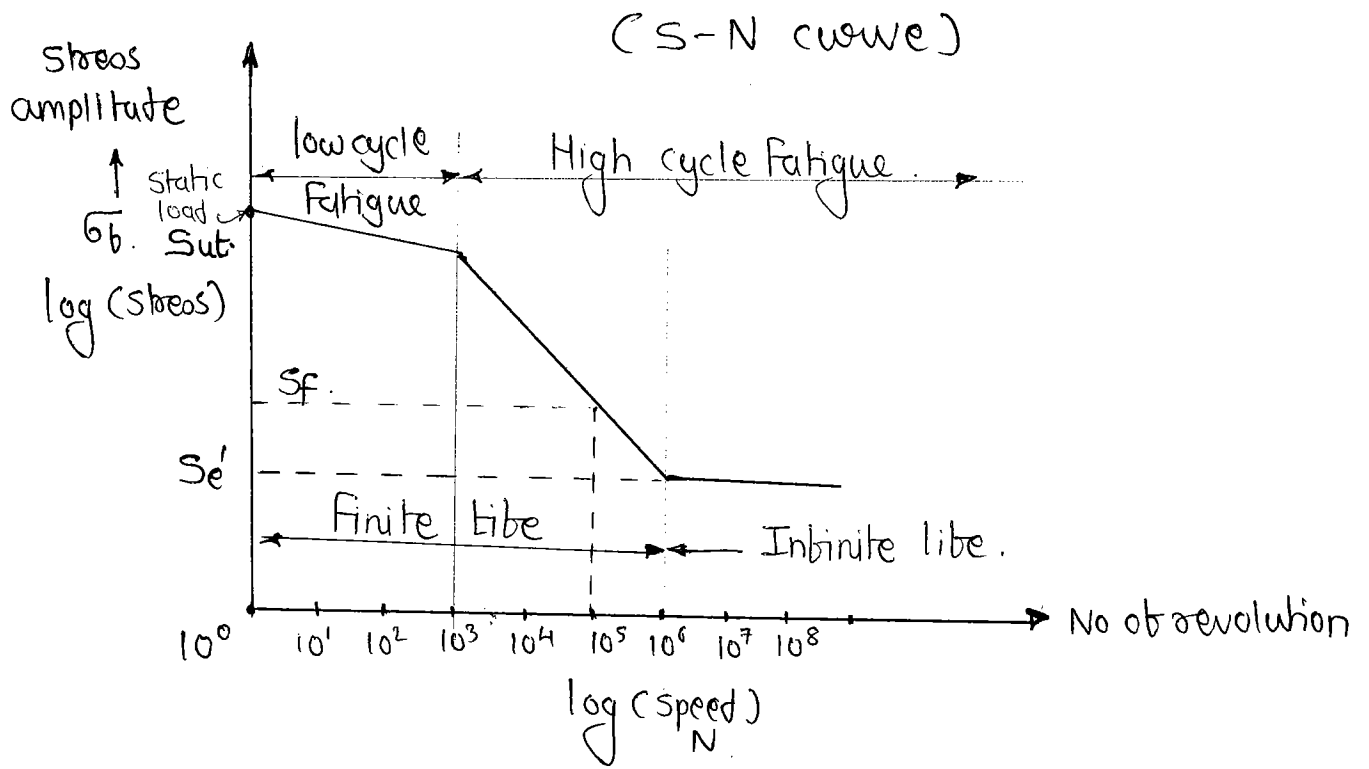
Endurance strength (Se')

It is the maximum value of a completely reversible bending stress which a standard polished specimen can withstand for a a minimum life of one million revolution.

Stress of finite life expressed as Fatigue strength
Stress of infinite life expressed as endurance limit.

low cycle

Sut

Non Ferrous melels Al3, cu etc

Se'

knee

Finite life   Infinite life.

10^3 10^5 10^6 10^7 10^8
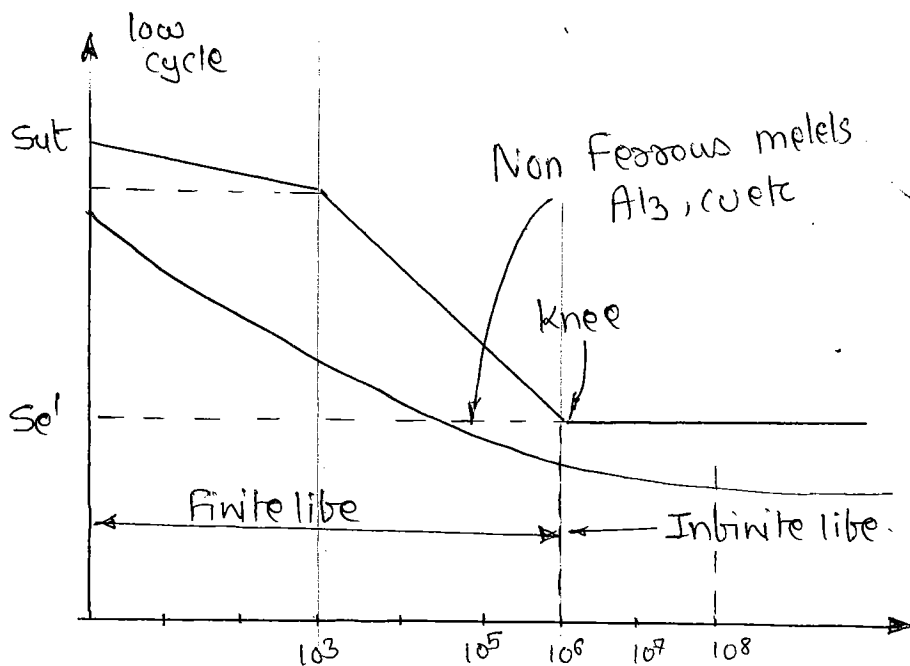
10^6 rev — 9.6 hr.

10^8 rev — 9.6 × 100

= $\dfrac{960 \text{ hr}}{24}$ = 40.2 days

= 1½ months.

Empirical relation

For steel — $Se' = 0.5$ Sut   if Sut < 1400 MPa

= 700 MPa   if Sut ≥ 1400Pa.

For C.I

$Se' = 0.4$ Sut

For non Ferrous materials.

$Se' = 0.3$ Sut

Factors affecting the endurance strength of component

Corrected endurance strength

$Se = Ka\, Kb\, kc\, ka\, ke\, kf \cdots\cdots Se'$

I) **Surface Finish factor (ka)**

Ka = 1 for polished specimen.
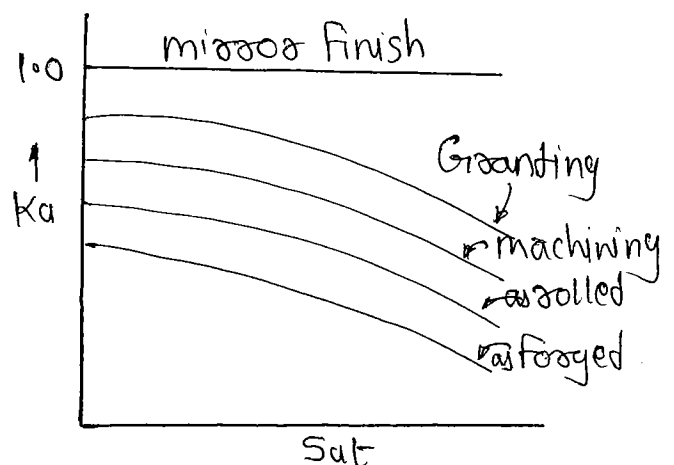
2] K_b = Size Factor

$K_b = 1 \quad d \leq 8mm$

$= 1.189 \, d^{-0.097}$
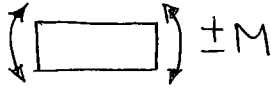
As dia of component increases the value of size factor reduces

of $K_b \downarrow$

3] K_c = Reliability Factor

$= 1 \qquad 50\%$ reliability

$= 0.897 \qquad 90\% \quad "$

4] K_e = load factor.

$= 1.$ Completely reversed bending. $\left\{ \boxed{} \right\} \pm M$

$\pm F \qquad \qquad \pm F$

$= 0.7 \text{ to } 0.8$ (If two values limit is given & select list one)

Ex:- Connecting rod
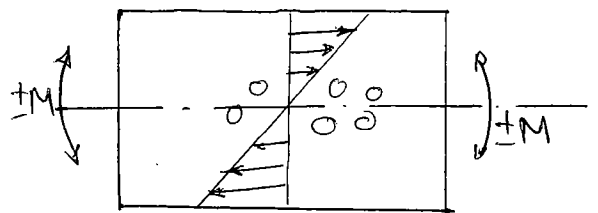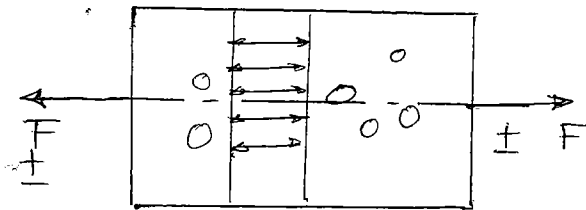
5] K_g = Temperature factor

$= 1 \quad T < 300°C$

$= 0.5 \quad T > 300°C$

K_d = Modifying stress Concentration Factor

$$= \frac{1}{K_f}.$$

K_f = Actual stress concentration Factor

Stress Concentration

Localised high stress near the discontinuity is called Stress concentration
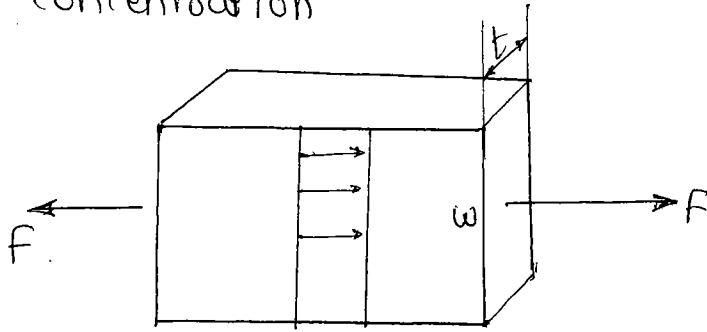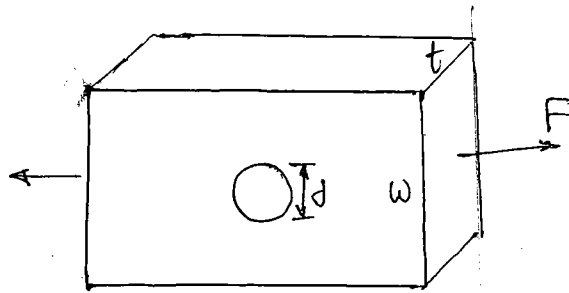
$\sigma\, uniform = F$
$$= \frac{F}{w \times t}$$

$$\sigma_0 = \frac{F}{(w-d)t}$$
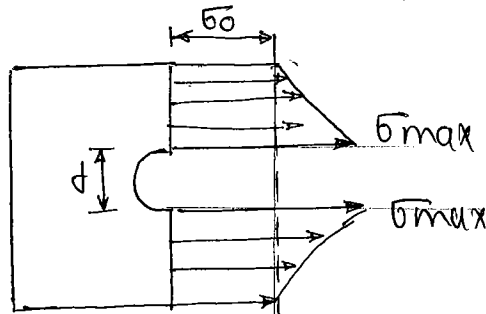
Theoretical stress concentration Factor,

$$K_t = \frac{\text{Maximum stress near the discontinuity}}{\text{Nominal stress obtained by min area of c/s.}}$$

$$K_t = \frac{\sigma max}{\sigma_0}.$$

$\sigma_0 = \sigma uniform$ $(d << w)$

Infinite plate

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right) \qquad (\sigma_0 = \sigma_\theta)$$

i] If $a = 0$.

$$\sigma_{max} = \sigma$$

ii] If $b = 0$

$$\sigma_{max} = \infty$$
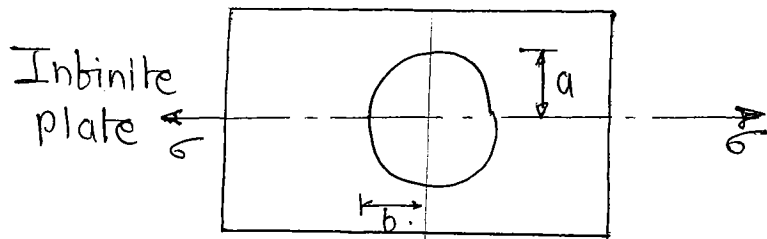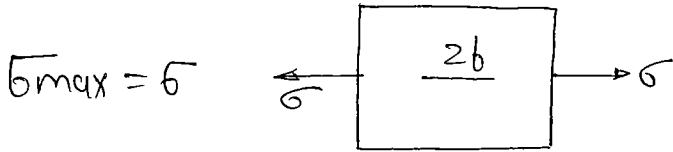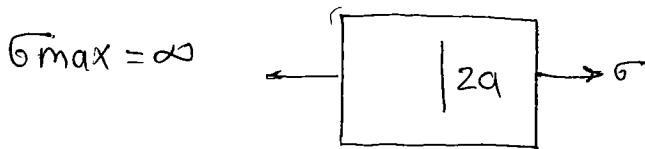
iii] If $a = b$. A circular hole

$$\sigma_{max} = 3\sigma.$$
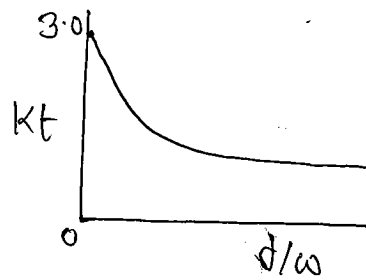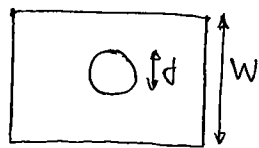
Methods of determining, K_t.

1] Mathematical analysis
      Theory of elasticity

2] Experimental method
  - a] Photo-elasticity .
    b] Brittle Coating
    c] Strain gauge method .
    d] Grid method
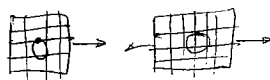
3] Computer based

Notch Sensitivity Factor (q)

The degree to which the effect of theoratical stress concentration is actually reached

$$q = \frac{\text{Increse of actual stress over nominal stress}}{\text{Increse theoratical stress over nominal stress}}$$

$$K_f = \frac{\sigma_{max}\,actual}{\sigma_0}$$
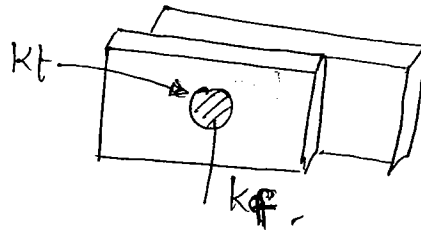
$$\sigma_{max}\,actual = K_f\,\sigma_0$$

$$\text{Increase} = K_f\,\sigma_0 - \sigma_0$$

$$K_t = \frac{\sigma_{max}}{\sigma_0}$$

$$\sigma_{max} = K_t\,\sigma_0$$

$$\text{Increased} = K_t\,\sigma_0 - \sigma_0$$

$$q = \frac{K_f\,\sigma_0 - \sigma_0}{K_t\,\sigma_0 - \sigma_0} = \frac{(K_f - 1)\sigma_0}{(K_t - 1)\sigma_0}$$
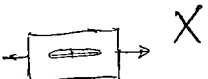
$$= \frac{K_f - 1}{K_t - 1}$$

$$\boxed{K_f = 1 + q(K_t - 1)} \qquad q \leq 1$$

If $q = 1$

$$\boxed{K_f = K_t}$$

1) Effect of K_t on type of Material

Type	Static load	Fluctuating load
Ductile Material	X	✓ K_t
Brittle Material	✓ K_t	✓ K_t

Ductile material.

$S_{yt} = 300\,MPa$.

at 50 MPa

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right).$$

at 100 MPa, 150.

$150 = \sigma$

100

As $\sigma\uparrow$ $\sigma_{max}\downarrow$
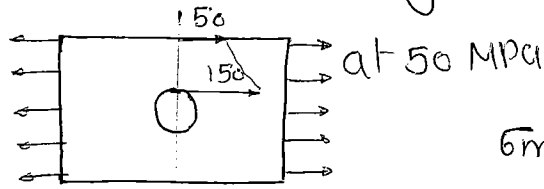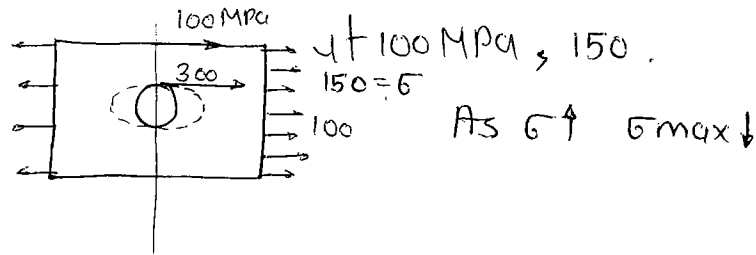
Effect of Mean stress on endurance limit.
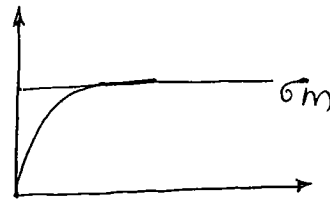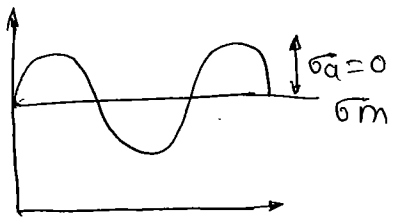
$\sigma_a = 0$ static load.

we have considered completely reverse condition

Air

static load

Torsion

$\pm M$

static/mean   Completely reversed

$\pm M$

σ

stress

No of cycle

$\sigma_m = 0$

Fluxituating

σ_a

σ_m

Ex:- ①

Normal (Bending)
a load
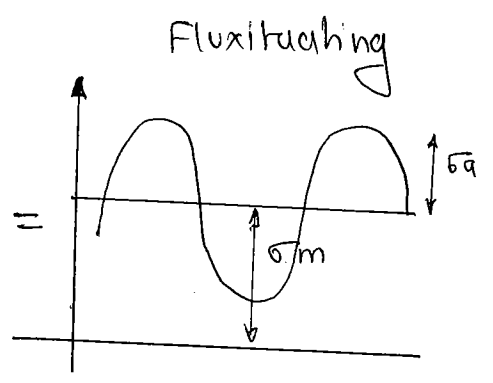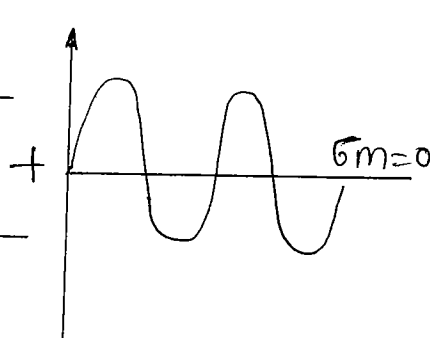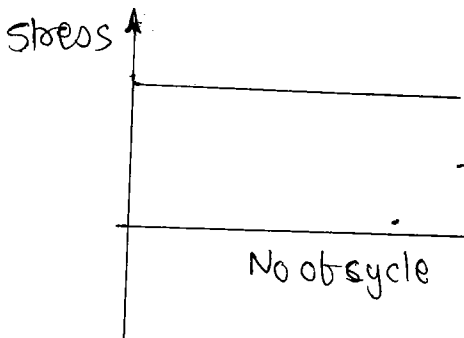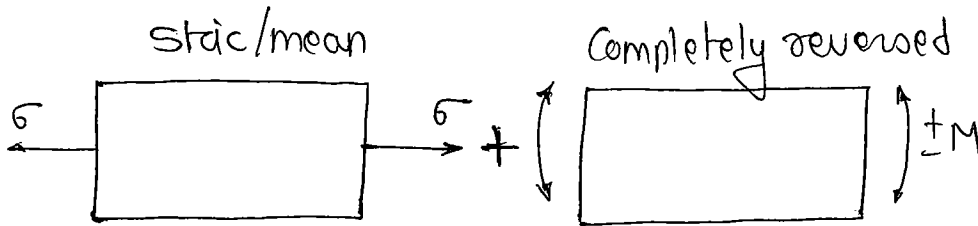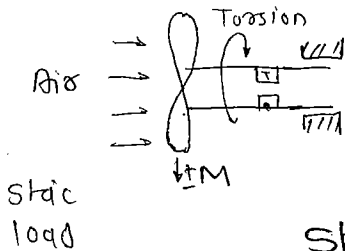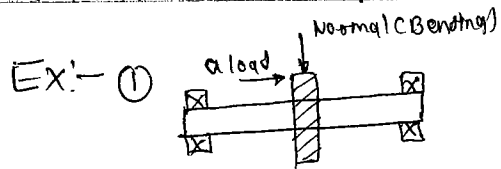
②. IC Engine connecting rod

③ Preloaded ~~load~~ boalt of ~~IC engine~~ steam engine

④ Cycle spokes.
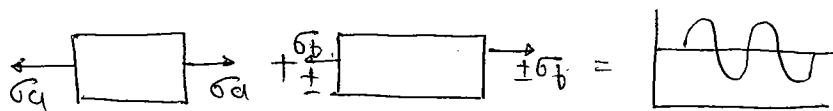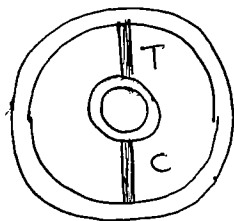
$\sigma_{max} = \sigma_a + \sigma_b$

$\sigma_{min} = \sigma_a - \sigma_b$

Effect of mean stress on Endurance strength.

Completely reversed.

Goaber's line.

Soderberg's line.

Goodmen's line.

σ_a

S_e

O σ_{yt} S_{ut} . (static load) σ_m

② (0, Se)

Goodmen's line

(Sut, 0) ①
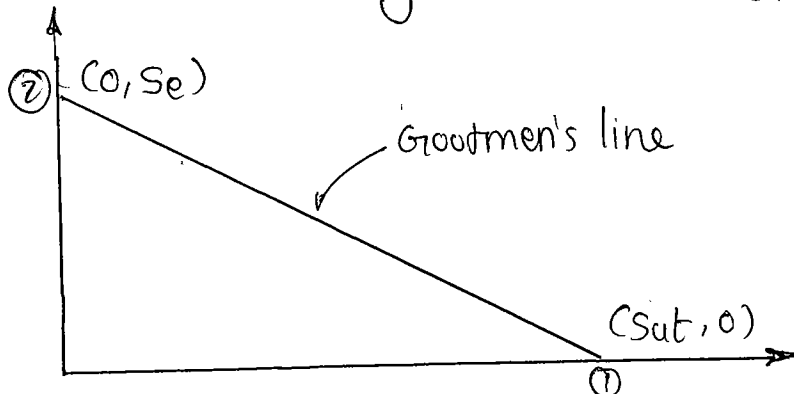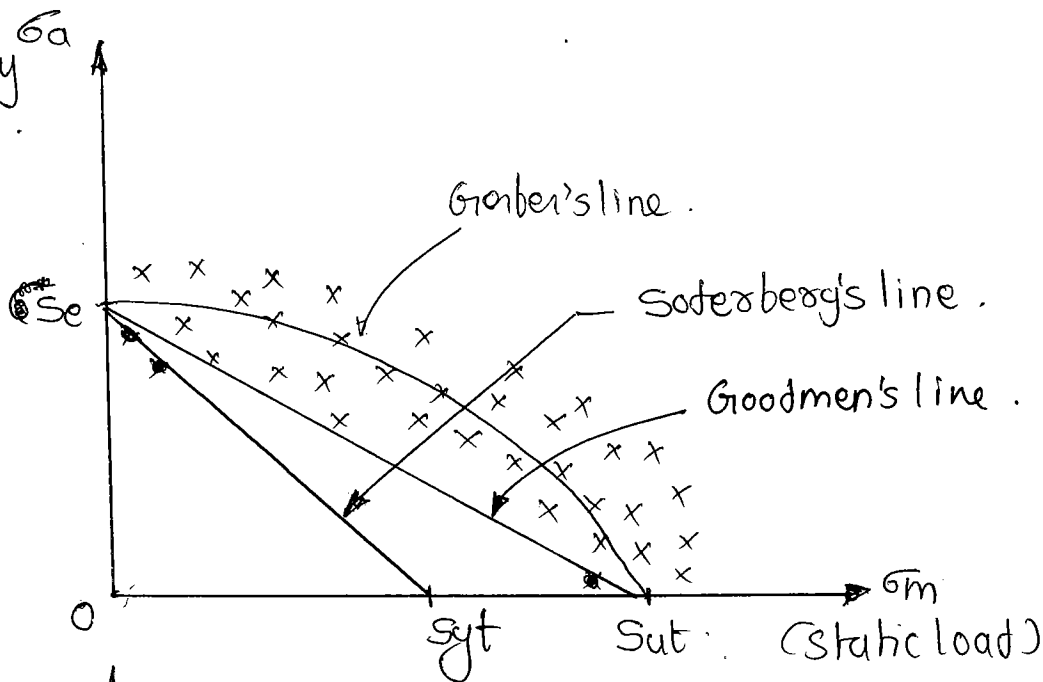
$y - y_1 = \dfrac{y_1 - y_2}{x_1 - x_2} (x - x_1)$

$\sigma_a - 0 = \dfrac{0 - Se}{Sut - 0} (\sigma_m - Sut)$

$$\frac{\sigma_a}{\sigma_{me}} = -\frac{\sigma_m}{S_{ut}} + \frac{S_{ut}}{S_{ut}}$$

$$\frac{\sigma_a}{\dfrac{Se}{F\cdot S}} + \frac{\sigma_m}{\dfrac{S_{ut}}{F\cdot S}} = 1.$$

$$\boxed{\frac{\sigma_a}{Se} + \frac{\sigma_m}{S_{ut}} = \frac{1}{F\cdot S}}$$

2] Soderberg's equations.

$$\boxed{\frac{\sigma_a}{Se} + \frac{\sigma_m}{S_{yt}} = \frac{1}{F\cdot S}}$$

3] Gerber's equation.

$$\frac{\sigma_a}{Se} + \left(\frac{\sigma_m}{S_{ut}}\right)^2 = 1.$$

$$\boxed{\frac{\sigma_a}{Se/F\cdot S} + \left(\frac{\sigma_m}{S_{ut}/F\cdot S}\right)^2 = 1}$$

Q :- A steel plate shown in fig is subjected to completely reversed load of 16 KN. The plate is made of steel $(S_{ut} = 440\ MPa)$, $q = 0.8$, $ka = 0.67$, $kb = 0.85$, $kc = 0.9$, $kd = 0.897$, $kt = 2.35$ and $F\cdot S = 1.5$

a) Then the plate thickness for infinite life is

a] 6mm   b] 12mm   c] 20mm   d] 30mm

b] Optimum thickness is obtaind by

a] Gerber's Method
b) Goodman eqⁿ
c] Soderber's eqⁿ
d] Both Goodman & Soderbergs method

±16 KN   50   10 ∅   t   ±16 KN

$$\frac{\sigma a}{Se} + \frac{\sigma m}{Sut}^{0} = \frac{1}{Fs} \qquad \text{Goodmen}$$

$$\frac{\sigma a}{Se} + \frac{\sigma m}{Syt}^{0} = \frac{1}{Fs} \qquad \text{Sodenberge.}$$

Ans $]$
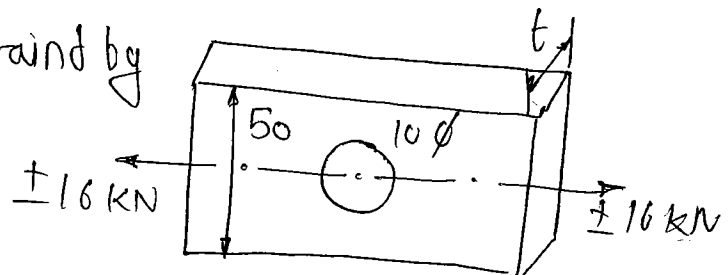
$$\frac{\sigma a}{Se} = \frac{1}{F \cdot S}$$

$F = \pm 16 \, kw$

$Sut = 440 \, MPa$

$q = 0.8$

$ka = 0.67$

$kb = 0.85$

$kc = 0.9$

$kd = 0.897$

$t = ?$

$Fs = 1.5$

$Kt = 2.35 \div $ If given then

$$Se = ka \, kb \, Kc \, ke \cdots Se'$$
$$= 0.67 \times 0.85 \times 0.9 \times 0.897 \times \frac{1}{2.08} \times 220$$
$$= 48.62 \, MPa.$$

$$Se' = 0.5 \, Sut$$
$$= \underline{440 \times 0.5} \quad \text{if} < 1400 \, MPa.$$
$$= 220 \, MPa$$

ke — Modifing stress concentration buctor.

$$ke = \frac{1}{kf} = \frac{1}{1 + q(Kt - 1)}$$
$$= \frac{1}{1 + 0.8(2.35 - 1)} = \frac{1}{2.08}.$$

$$\sigma a = \frac{16 \times 10^3}{(w - d)t} = \frac{16 \times 10^3}{(50 - 10)t} =$$

$$\frac{\sigma a}{Se} = \frac{1}{F \cdot S}$$

$$\frac{16 \times 10^3}{40 \times t} = \frac{48.62}{1.5}$$

$$\boxed{t = 12 \, mm}$$

Compressive strength of material.
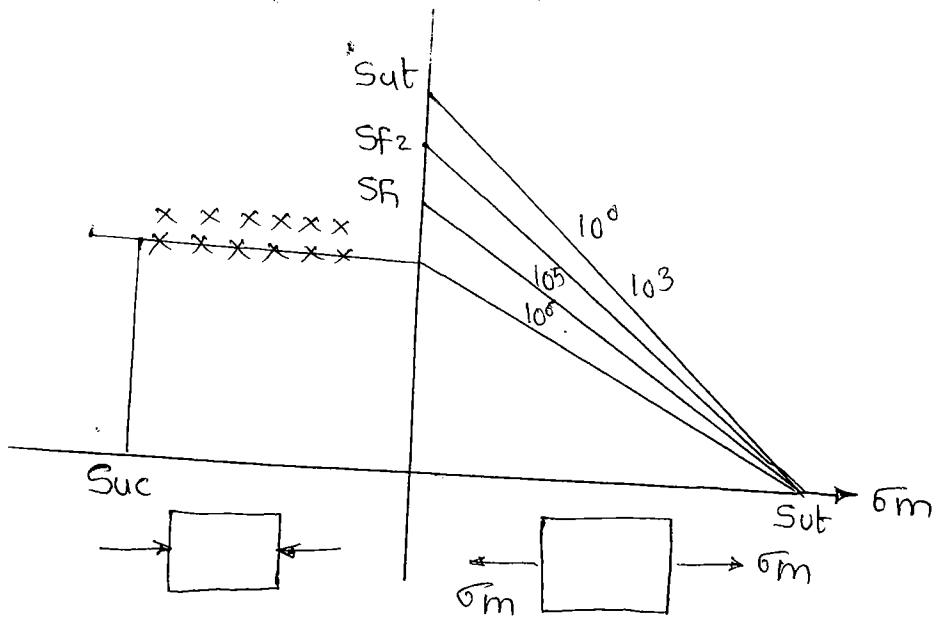
To improve Fatigue strength

1] Cold rolling
2] Burnishing
3] Shot pining.
4] Case hardning
5] Coining.

⎫
⎬ Residual Stress or
⎭ Preloading

Shot pining

Nozzle

Becoz. of intention the
   strength is increased
in case of sand cushing
Surface scratches produce
which retures surface
fatigue strength.
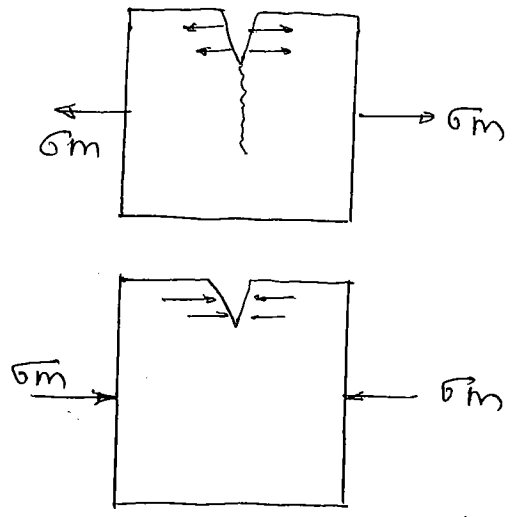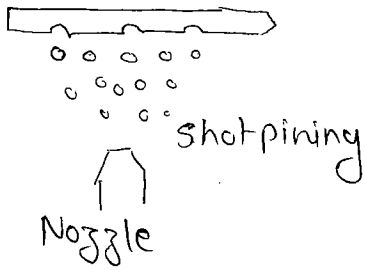
Decrese the effects of
Surface knotchs
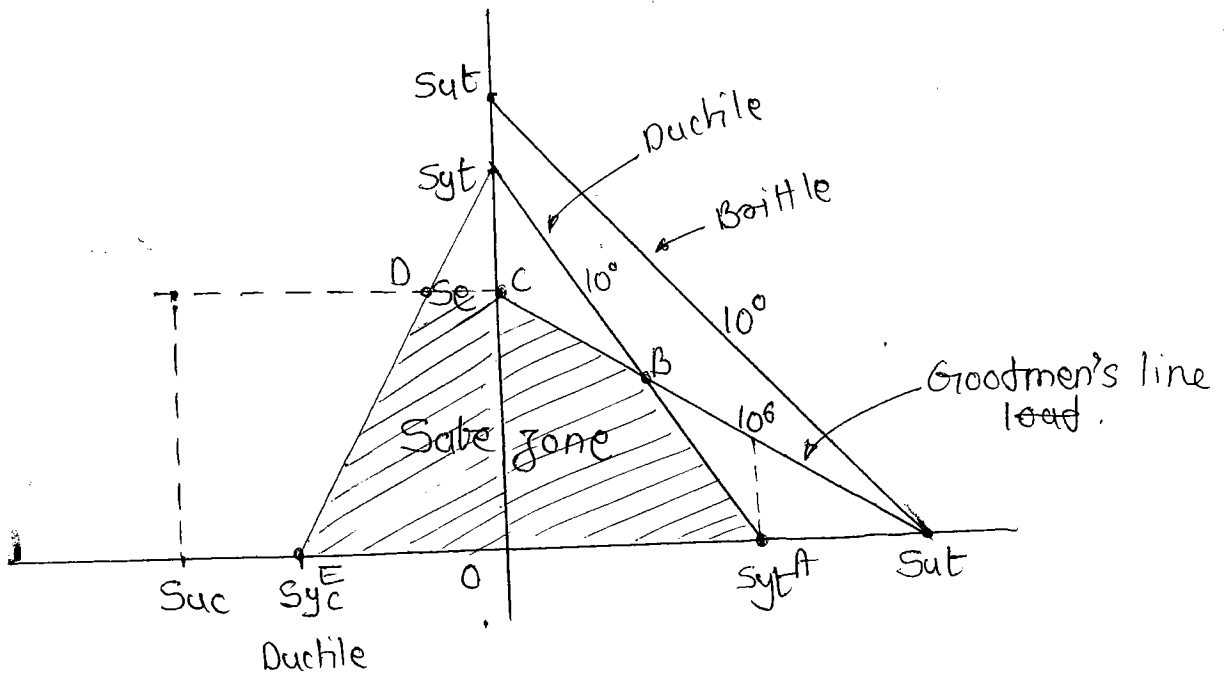
Assuming material will fail at syt line hence give name 10⁰

Design for finite life
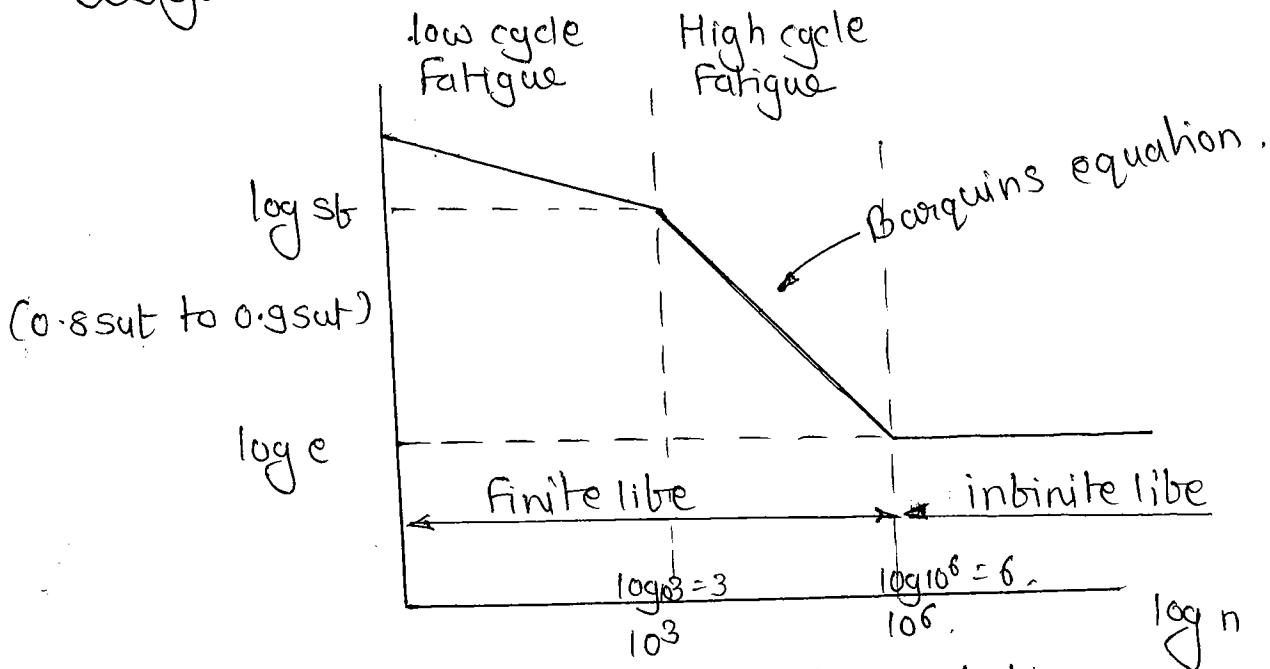
Design for finite life and high cycle fatigue

Sf corresponding to 10³ rev is 0.8 sut to 0.9 sut.

Basquin's Equations.

$$A = S_f L^B.$$

A and B are constants.
S_f - fatigue strength
L - Life in revolution.

$$A = S_e (10^6)^B \quad -①$$
$$A = S_f (10^3)^B \quad -②.$$

Find the value of A & B then, using above ① & ② eq^n.

Cumulative fatigue.
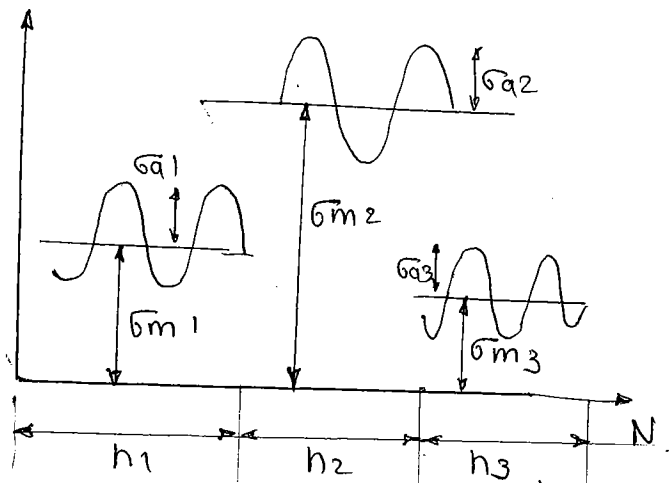
Ex:- Pulley, Bearings

Cumulative fatigue — Miner's equation

Let, a component is subjected to

Stress		no of cycle.
σ_1	for	n_1	no of cycle.
σ_2	"	n_2	"
σ_3	"	n_3	"
σ_4	"	n_4	"

It the component is subjected to only stress σ_1 then,
lets its life $= L_1$ revolution

Life completed in one $= \dfrac{1}{L_1} \leftarrow$ total life
revolution

Life completed during n_1 revolution $= \dfrac{n_1}{L_1}$

Total life consumed $= \dfrac{n_1}{L_1} + \dfrac{n_2}{L_2} + \dfrac{n_3}{n_3} + \cdots \cdots + \dfrac{n_x}{L_x} = 1$.

Let, $n_1 = \alpha_1 L$

$\quad\quad n_2 = \alpha_2 L$

$L \to$ total life

$\alpha_i \to$ part of the total life at G_i

$\dfrac{\alpha_1}{L_1} + \dfrac{\alpha_2}{L_2} + \cdots \cdots + \dfrac{\alpha_x}{L_x} = \dfrac{1}{L}$.

Q19% A machine part is made of steel with $Sut = 630\,MPa$, $Se = 0.22\,Sut$, Fatigue strength for 10^3 cycles is $0.9\,Sut$ The life of part is 25000 cycles The load for 50% of time is $\pm 225\,MPa$ and 30% of time is $\pm 145\,MPa$.

The lives of part at 225 MPa and 145 MPa respectively are (in revolution)

a] 9.29×10^4, 8.01×10^5 c] 8×10^4, 4×10^5

b] 9.7×10^4, 6×10^5 d] 3×10^5, 5×10^5.

Ans $Sut = 630\,MPa$ $L = 25000$

$Se = 0.22\,Sut$ $Sf = 10^3$ cycles is $0.9\,Sut$

$$A = Sf\,L^B.$$

$Sf_1 = \pm 225\,MPa$ $\alpha_1 = 0.5$

$Sf_2 = \pm 145\,MPa$. $\alpha_2 = 0.3$.

$Se = 0.22 \times 630 = 138.6 \cdots\cdots 10^6\,rev$

$Sf = 0.9 \times 630 = 567 \cdots\cdots 10^3\,rev$.

$$A = 138.6 \times 10^{6\,B}$$
$$A = 567 \times 10^{3\,B}.$$

$138.6 \times 10^{6\,B} = 567 \times 10^{3\,B}$, $10^{6B - 3B} = \dfrac{5\,63 \times 10^3}{138.6 \times 10^6}$

$$B = 0.203$$
$$A = 2304$$

$$230 H = Sf L^{0.203}$$

$$230 H = 225 L_1^{0.203}$$

$$L_1 = 9.32 \times 10^4$$

$$L_2 = 8.03 \times 10^5$$

The load during the remaining time.

a) 250 MPa   b) 375 MPa   c) 395 MPa   d) 415 MPa.

$$\frac{\alpha_1}{L_1} + \frac{\alpha_2}{L_2} + \frac{\alpha_3}{L_3} = \frac{1}{L}$$

$$\frac{0.5}{9.32 \times 10^4} + \frac{0.3}{8.03 \times 10^5} + \frac{0.2}{L_3} = \frac{1}{25600}$$

$$L_3 = \cancel{8756.15} \; 5837.46 \text{ cycle.}$$

$$230 H = Sf \, L^{0.203}$$

$$230 H = Sf \times 5837.46^{0.203}$$

$$\boxed{Sf = 396 \text{ MPa}}$$

29] The life of a machine component under heavy test load was 10 hrs. The life under combine loading was 18 hours of which at heavy load and 8.2 hours was at the normal load. What is the expected life under normal operating load

a) 54.5 hr   b) 6 hrs   c) 410 hr   d) 490 hr.

$$\frac{n_1}{L_1} + \frac{n_2}{L_2} = 1.$$

$$\frac{9.8}{10} + \frac{8.2}{L_2} = 1$$

$$\boxed{L_2 = 410}$$

OR

$$\alpha_1 = \frac{9.8}{18}, \quad \alpha_2 = \frac{8.2}{18}$$

$$\frac{\alpha_1}{L_1} + \frac{\alpha_2}{L_2} = \frac{1}{L} \qquad \frac{0.511}{10} + \frac{0.455}{L_2} = \frac{1}{18}$$

Welding

Welding is a process of joining metals by the application of heat with and without the application of pressure.

Advantages

1] Highest Joint efficiency
2] Lighter in weight
3] Easy to weld any part of the Structure at any point.
4] Easy to make alteration

Disadvantages

1] Due to non uniform heating and cooling thermal stresses will be introduced it result in destortion of component.

Joints $\left\{\begin{array}{l}\text{Butt} \\ \text{Fillet}\end{array}\right.$

Rebo Reinforcement
(kt = 1·2)
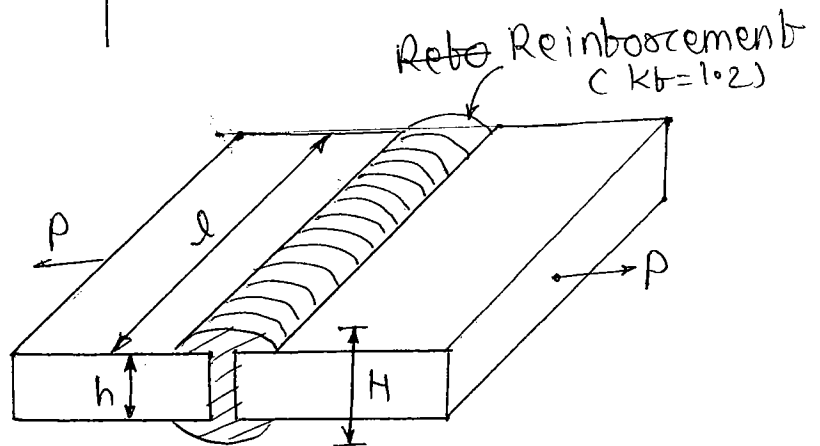
$$\sigma_t = \frac{P}{h \times L}$$
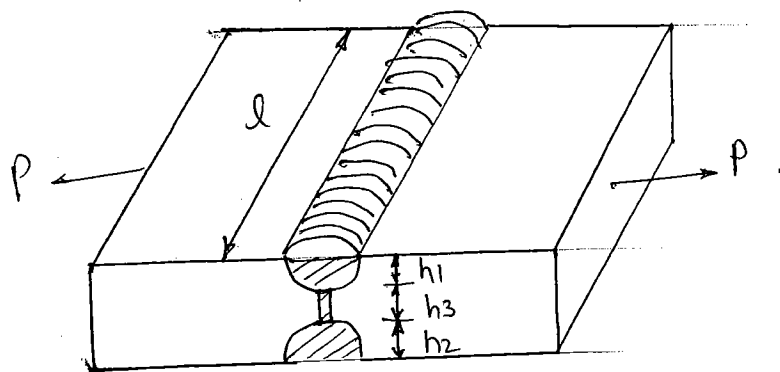
$$\sigma_t = \frac{P}{(h_1 + h_2) \, l}$$

Fillet weld.
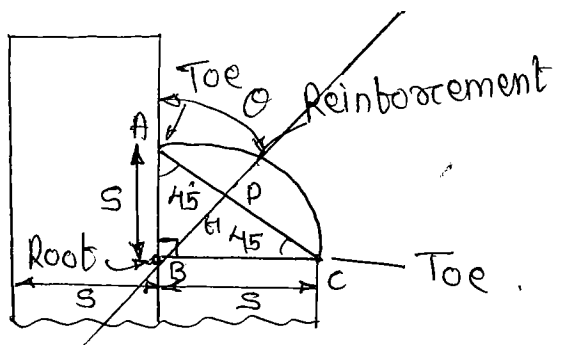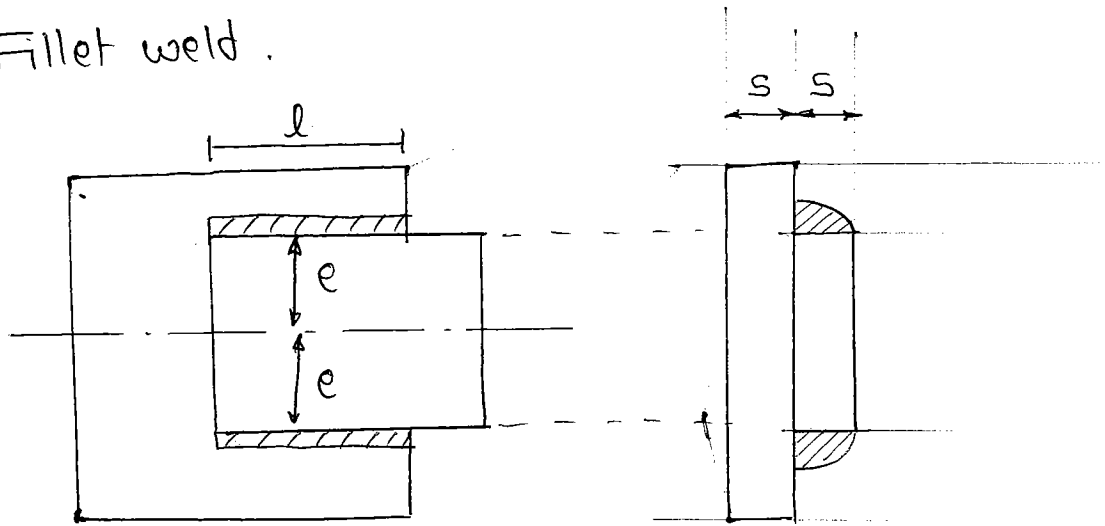
Toe and Root are the weakest point because of high stress concentration

$$\frac{t_1}{\sin 45} = \frac{S}{\sin(180-(45+\theta))}$$

$$t_1 = \frac{S \sin 45}{\sin(45+\theta)}$$

$$= \frac{S \sin 45}{\sin 45 \cos\theta + \cos 45 \sin\theta}$$

$$= \frac{S}{\cos\theta + \sin\theta}$$
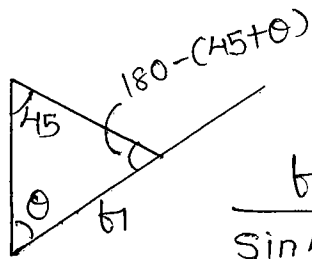
$$t_1 = \frac{S}{\sin\theta + \cos\theta}$$

Shear stress on the plane at an angle θ

$$\tau_\theta = \frac{P}{t \times l}$$

$$\tau_\theta = \frac{P(\sin\theta + \cos\theta)}{S \times l}$$

Max^m shear stress

$$\frac{\partial \tau}{\partial \theta} = 0$$

$$\frac{P}{Sl} \neq 0.$$

$$\frac{\partial}{\partial \theta}(\sin\theta + \cos\theta) = 0$$

$$\cos\theta - \sin\theta = 0$$

$$\cos\theta = \sin\theta = 45°.$$

$$t = \frac{S}{\sin\theta + \cos\theta} \qquad \theta = 45.$$

$$\boxed{t = 0.707\,S}$$ — throat thickness.

$$\tau_{45°} = \tau_{max} = \frac{P(\sin 45 + \cos 45)}{Sl}.$$

$$\tau_{max} = \frac{\sqrt{2}\,P}{Sl}$$

$$\boxed{P = \frac{1}{\sqrt{2}} Sl\,\tau_{max}}$$

Load carrying capacity of single parallel fillet weld

$$P = 0.707\,Sl\left(\frac{S_{sy}}{F \cdot S}\right) -$$

$$P = 1.404\,Sl\left(\frac{S_{sy}}{F \cdot S}\right) -\text{ Double fillet weld.}$$

$$\tau_{90} = \frac{P}{Sl} \qquad \frac{\tau_{90}}{\tau_{45}} = 0.707$$

$$\tau_0 = \frac{P}{Sl}$$

Transverse fillet weld.

$$\tau_{max} = \tau_{67.5}$$

shear
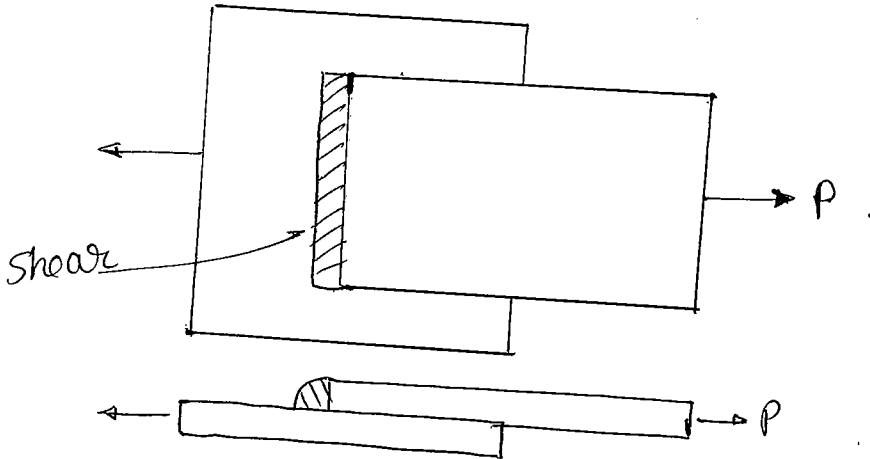
$$\tau_{max} = \tau_{67.5}$$

$$P = 0.707 \, Sl \, \tau_{max} \quad - \quad \text{Transverse fillet weld}$$

$$\frac{\tau_{67.5}}{\tau_{45}} < 1.$$

$$H =$$

$$H = P \sin\theta$$
$$N = P \cos\theta$$
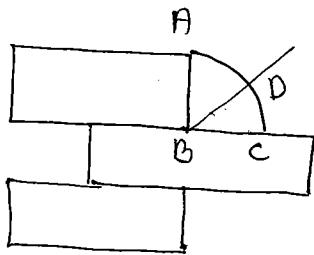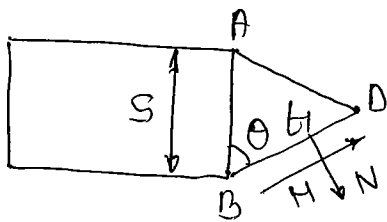
$$\tau = \frac{H}{t_1 \, l}$$
$$= \frac{P \sin\theta \, (\sin\theta + \cos\theta)}{Sl}.$$

$$\frac{\partial \tau}{\partial \theta} = 0$$

$$2\theta = 135°.$$

Axially loaded unsymmetrical welds.
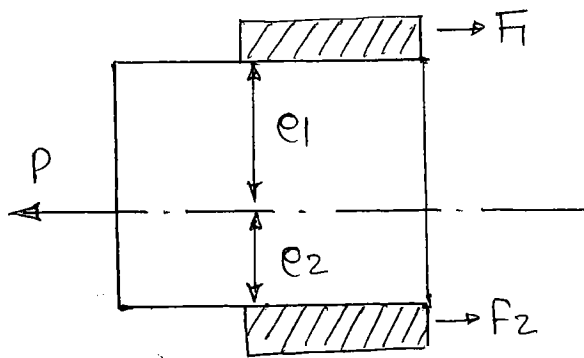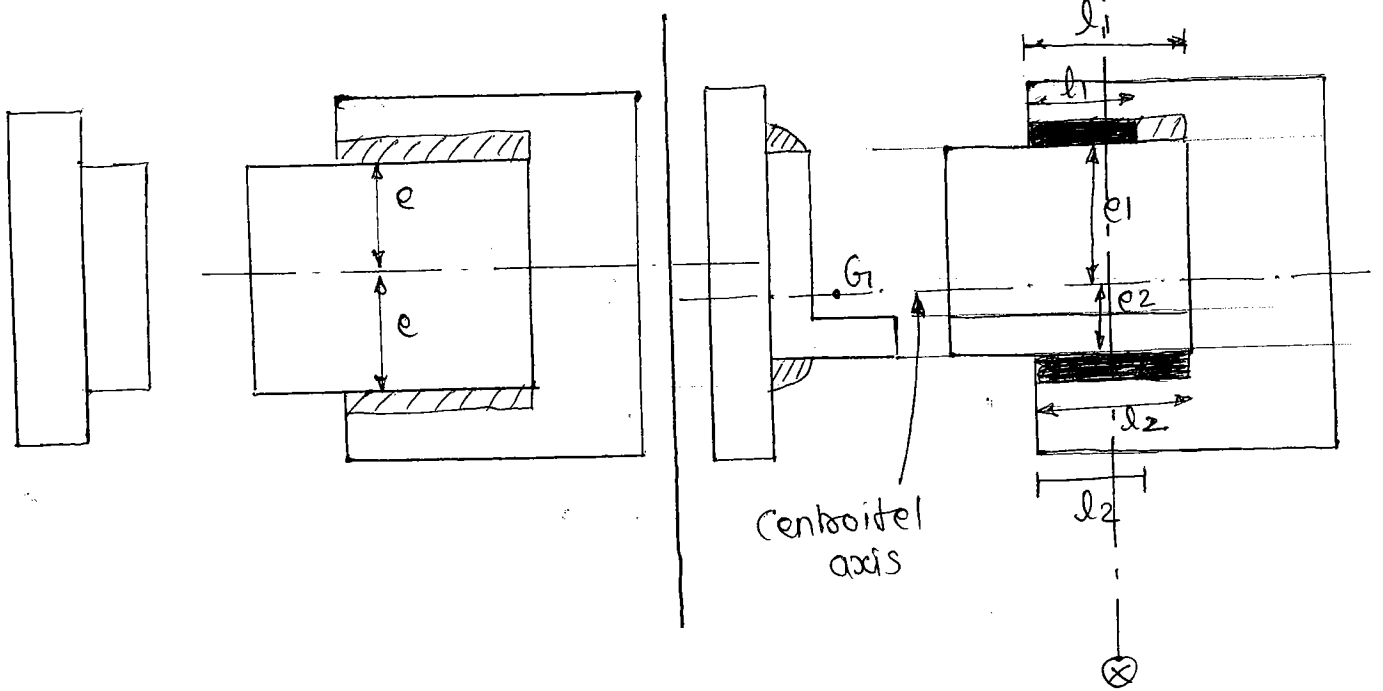
Centroidel
axis

$$P = F_1 + F_2$$

$$\sum M_{centroidal} = 0.$$
$$\qquad axis$$

$$F_1 \times e_1 - F_2 \times e_2 = 0.$$

$$F_1 e_1 = F_2 e_2$$

$$\frac{F_1}{e_2} = \frac{F_2}{e_2} = \frac{F_1 + F_2}{e_1 + e_2} = \frac{P}{e_1 + e_2}$$

$$F_1 = \frac{e_2}{e_1 + e_2} \times P.$$

$$F_2 = \frac{e_1}{e_1 + e_2} P_2$$

Ex:-

$$P = 0.707 \, Sl \left(\frac{Ssy}{F \cdot S} \right)$$

$$l = l_1 + l_2$$

$$F_1 = 0.707 \, S \times l_1 \left(\frac{Ssy}{F \cdot S} \right)$$

$$F_2 = 0.707 \, S \times l_2 \left(\frac{Ssy}{fs} \right)$$

$$P = F_1 + F_2$$

$$l = l_1 + l_2$$

$$P \propto l$$
$$F_1 \propto l_1$$
$$F_2 \propto l_2$$

$$F_1 = \frac{e_2}{e_1 + e_2} \, P.$$

$$l_1 = \frac{e_2}{e_1 + e_2} \times l$$

$$l_2 = \frac{e_1}{e_1 + e_2} \times l.$$
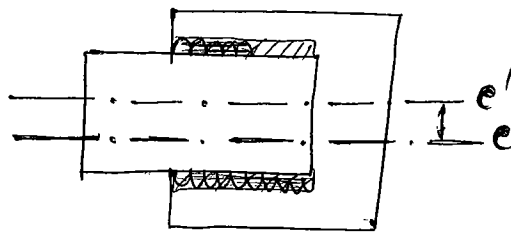
Bending movement will take place because of change in centroidle axis.

Eccentrically loaded Welted Joint.
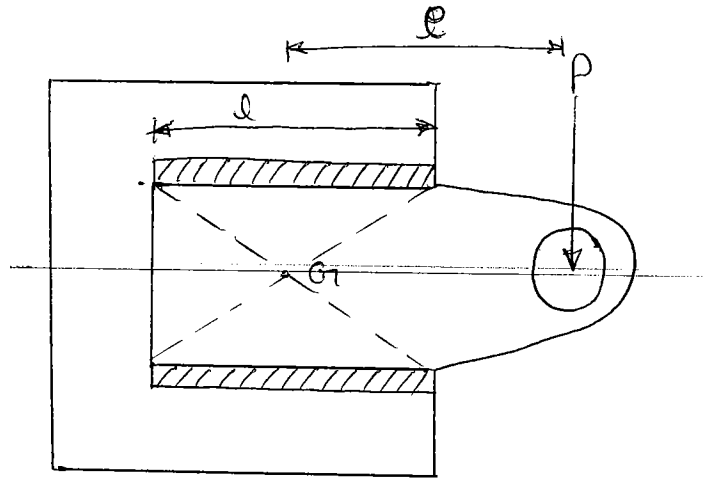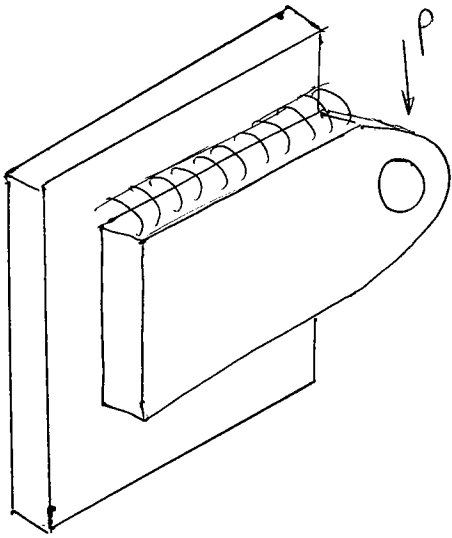
eccentric load cause a Movement Stress it may be bending or torsion according to the application of load
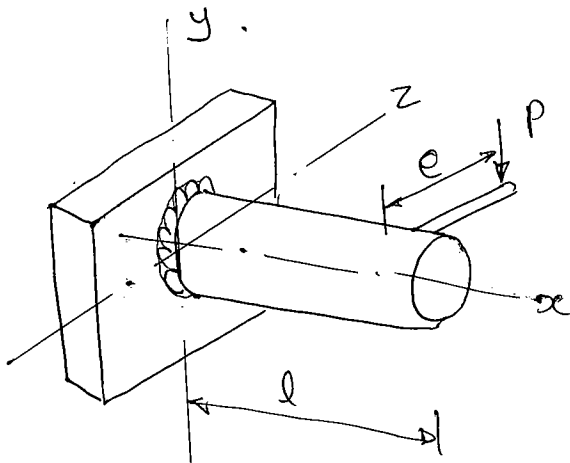
$$M_x = P \times e = T.$$
$$M_y = 0$$
$$M_z = P \times l$$

$$M_x \hat{i} + M_y \hat{j} + M_z \hat{k}$$
$$T + 0 + B$$

Any member trying to move member about its own axis is called Torsion.

If $l = 0$. there is only torque.

$t << l$
$t << e$
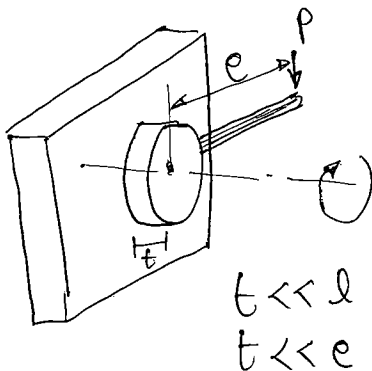
Hence it causes a torsional movement.

$$\tau_1 = \frac{P}{2t \times \ell}$$

$$+$$

$$\tau_2 = \frac{T}{J} \times r$$

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\tau = \frac{T}{J} \times r$$

$$\tau \propto r.$$
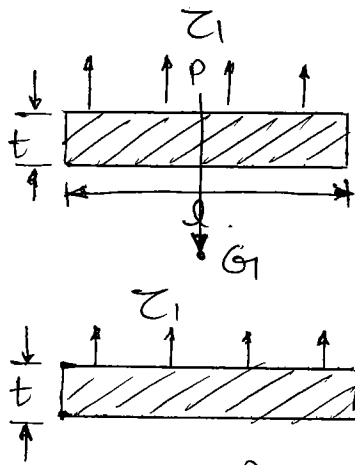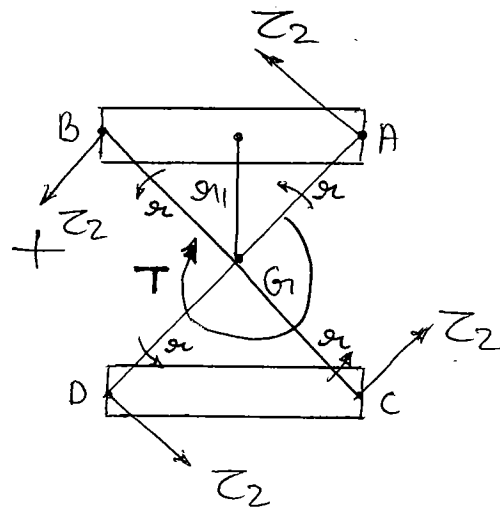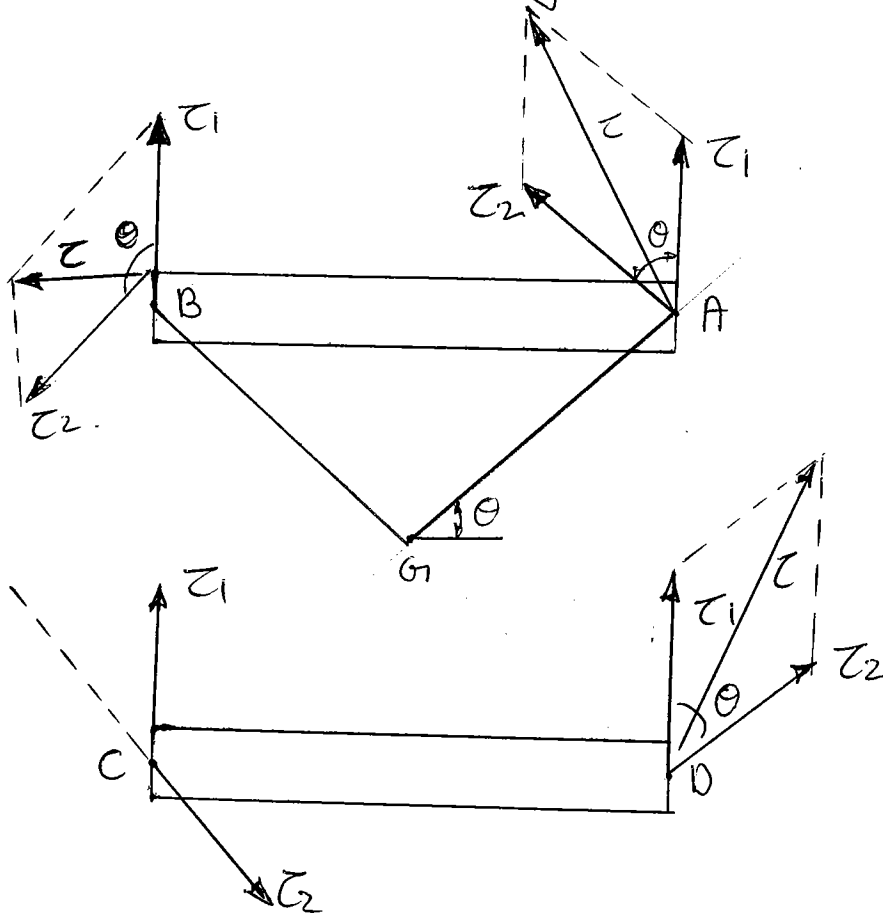
$$\tau = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1 \tau_2 \cos\theta}$$

$$\tau = \phi(\tau_1, \tau_2, \theta)$$

$$\theta\downarrow \quad \cos\theta\uparrow$$

Hence object will first fail from point A & D

Polar movement of Inertia.
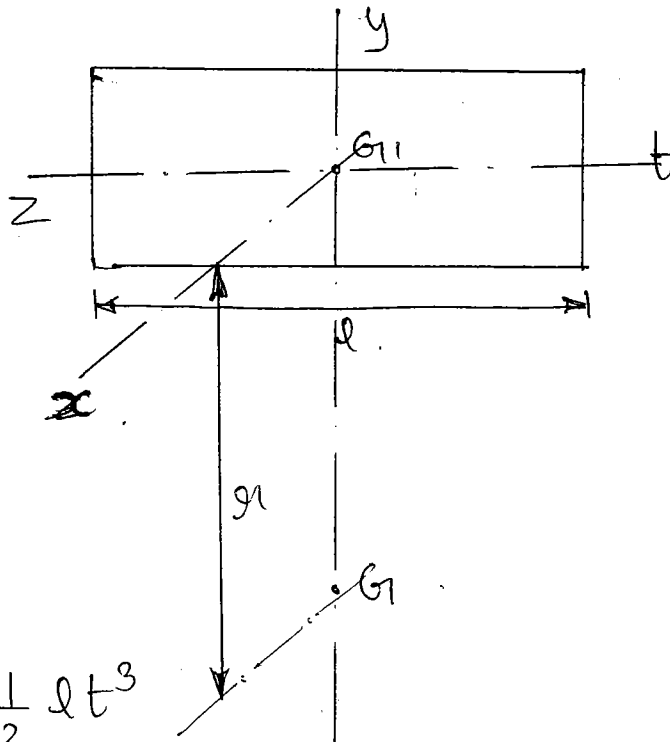
$$I_z = \frac{1}{12} \ell t^3$$

$$I_y = \frac{1}{12} t \ell^3$$

$$I_x = J_{G_1} = I_z + I_y$$

$$= \frac{1}{12} \ell t^3 + \frac{1}{12} t \ell^3 \qquad t <<< \ell .$$

$$\boxed{I_{G_1} = \frac{1}{12} t \ell^3 .}$$

$$I_G = I_{G_1} + A r^2$$

$$= \frac{1}{12} t \ell^3 + t \times \ell \cdot r^2 .$$

$$\boxed{I_G = A \left[\frac{\ell^2}{12} + r^2 \right]}$$

For both side

$$\boxed{I_G = 2A \left[\frac{\ell^2}{12} + r^2 \right]}$$

Note:- It two thickness value is given then take
the lowest value for weld design .

1] The permissible shear stress in a fillet used is 100 MPa. The fillet has equal leg length of 15mm each. The allowable shear load on the weldment per mm is

a] 2.25 KN b] 1.15 KN c] 1.06 KN d] 0.15 KN

$S = Size = leg$

$S = size = leg$
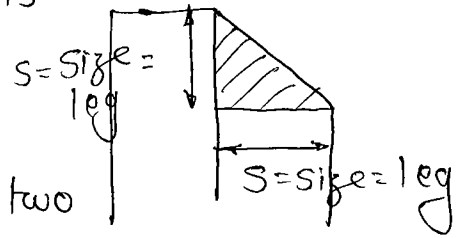
Size of weld = least thickness of two plates

$$P = 0.707 \, Sl \left(\frac{Ssy}{Fs} \right)$$

$$= 0.707 \times 15 \times 1 \times 100$$

$$= 1.06 \; KN$$

2] The allowable load per linear centimeter of the fillet weld for the 10mm size, assume shear stress intensity to be 80 MPa is

a] 2.8 KN   b] 5.6 KN   c] 8 KN   d] 11.3 KN.

$$P = 0.707 \, Sl \left(\frac{Ssy}{Fs} \right)$$

$$= 0.707 \times 10 \times 10 \times 80$$

$$= 5.6 \; KN.$$

Estimate the shear stress due to Torsion (T)

$$Z = \frac{T}{J} \propto r$$

$t = 0.7078$

θ — shear stress

Z

D_M

T, z

$$I_Z = \pi t r^3$$

$$I_y = \pi t r^3$$

$$I_x = \int_{\theta=0}^{\theta=2\pi} y^2 \, dA$$

$y = r\sin\theta$

$dA = r\, d\theta\, t$

$$= \int_{\theta=0}^{\theta=2\pi} r^2 \sin\theta \times r\, d\theta \times t$$
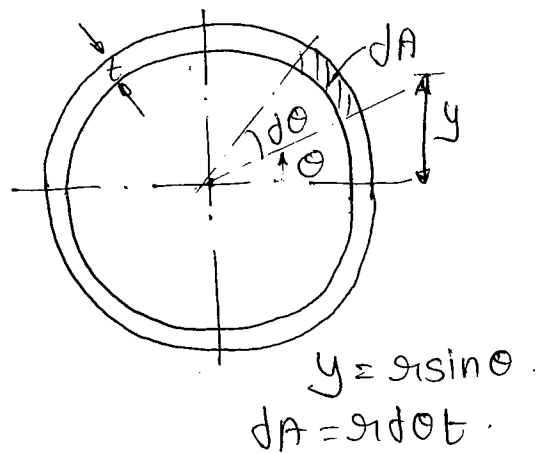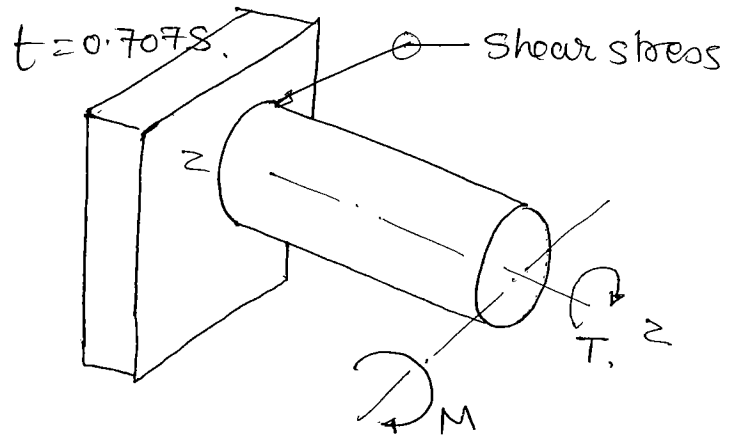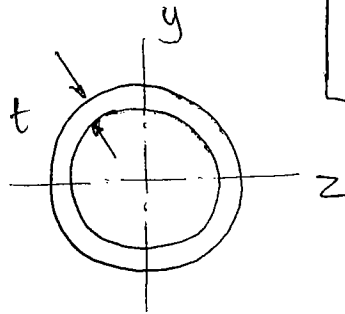
$$= r^3 t \int_{\theta=0}^{\theta=2\pi} \left(\frac{1-\cos2\theta}{2}\right) dt$$

$$= \frac{r^3 t}{2} \int 1$$

$$= r^3 t \pi$$

$$= \pi t r^3$$

$$I_y = \pi t r^3$$

$$J = I_x = I_z + I_y = 2\pi t r^3$$

$$Z = \frac{T}{J} \times r$$

$$= \frac{T}{2\pi t r^3} \times r = \frac{T}{2\pi t r^2} \qquad \boxed{\tau = \frac{T}{2\pi t r^2}}$$

Bending stress (σ_b) due to B·M

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{M}{I} \times y$$

$$= \frac{M}{\pi t r^3} \times r$$

$$\boxed{\sigma_b = \frac{M}{\pi t r^2}}$$

Ratio beth the Bending and Shear stress due to M & T.

$$\frac{\sigma_b}{\tau} = \frac{M/\pi t r^2}{T/2\pi t r^2}$$

$$\frac{\sigma_b}{\tau} = \frac{M}{T/2}$$

$$\boxed{\frac{\sigma_b}{\tau} = \frac{2M}{T}}$$

Q:- The moment on the circular billet weld whose axis coinside with x axis is given by.

$$M = \overset{Mx}{40\hat{j}} + \overset{M_z}{20\hat{k}}$$ The ratio of bending & Shear stress is

a) 2   b) 4   c) 3   d) 1

$$\frac{\sigma_b}{\tau} = \frac{2 \times 20}{40} = 1.$$

$$\frac{\sigma_b}{\tau} = \frac{2M}{T}.$$

Q The torque T, that can be applied, if the permissible shear stress is 140 MPa, is

a] 2332.5 N-m   b] 1452.3 N-m   c] 1252 N-m

d] None.

$$\tau = \frac{T_q}{2\pi t r^2} = 2$$

$$T = \tau \, 2\pi t r^2 \quad 2 = \xrightarrow{0.707 \times 6}$$

$$= 140 \times 2\pi \times t \times 25^2$$
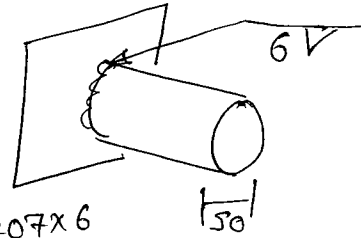
$$= 2332.5 \, N-m .$$

Q25 - The fillet weld weld of length 100mm each are used to support flat 'b' on the bracket 'a' Flat 'b' carries only tensile load. The leg length of both fillet weld is 5mm. The properties of weld metal are

$S_{ut} = 480$ MPa, $S_{yt} = 410$ MPa, $S_{sy} = 230$ MPa and reduction in area 40%. Find the load P.

a] 230 KN   b] 162.6 KN   c] 480 KN   d] 80.8 KN

$$t = 0.707 S = 0.707 \times 5$$

$$X \, P = 2 \times l \times t \times S_{yt} \times 1.4 \qquad P = 0.707 \times S \times l \left(\frac{S_y}{FS}\right) \times 2$$

$$= \qquad\qquad\qquad\qquad\qquad = 0.707 \times 5 \times 100 \, (230) \times 2$$

$$= 162.61 \times 10^3$$

$$= 162.61 \, MPa .$$
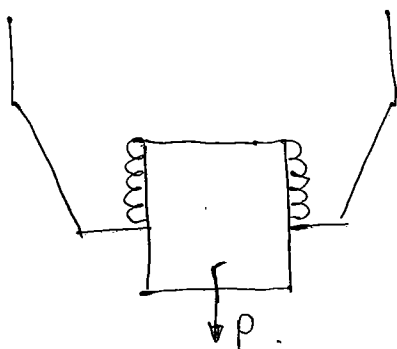
Q: A fillet weld joint is subjected to transverse loading 'F' as shown in fig. Both legs of the fillets are of 10 mm size and the weld depth is 30mm. If the allowable shear stress of the weld metal is 94 mpa. Considering the minimum throat area of the weld, the maximum allowable transverse load in KN is
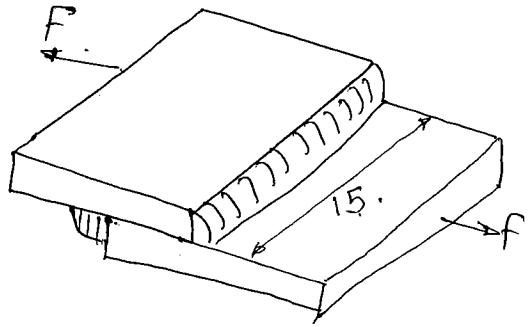
A] 14.44
b] 17.92
c] 19.93
d] 22.16

$$P = l \times 0.707 \times S \times \tau.$$
$$= 0.707 \times 10 \times 30 \times 94$$
$$P = 19.93 \times 10^3 \text{ N}.$$
$$= 19.93 \text{ KN}.$$

Q) 26 & 27 An angle plate is welded as shown in fig. The allowable stress for weld is 75 N/mm²

26] The length of weld at top in mm is

a] 103.7   b] 96.3   © 90   d] 110

27] The force carried by top weld is

a] 55 KN   b] 60 KN   © 65 KN   d] 50 KN

$$l_1 = \frac{e_2}{e_1 + e_2} l.$$
$$l_1 = \frac{55}{155 + 45} \text{ that } (l_1 + l_2)$$
$$l_1 = \frac{5.11}{40} l.$$
$$l_2 = \frac{145}{200} (l)$$
$$l_2 = \frac{29}{40} l.$$
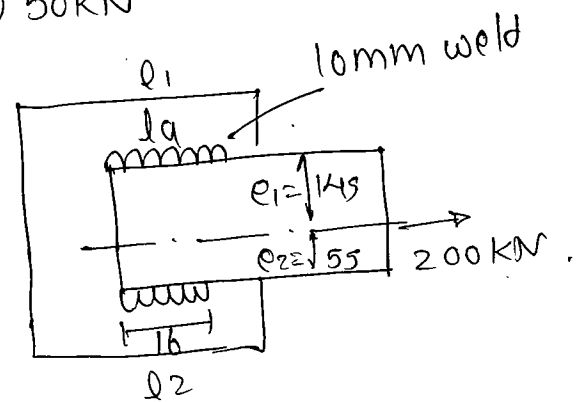
$$F_1 = \frac{e_2}{e_1 + e_2} P.$$
$$= \frac{55}{200} \times 200$$
$$= 55 KN.$$

$$F_1 = 0.707 \, S l_1 \left(\frac{S_{sy}}{F_S}\right)$$
$$55 = 0.707 \times 10.l_1 (75)$$
$$= 0.103 \, m$$
$$l_1 = 103.72$$

48]

Q 30. In the welded joint shown in fig. If the weld B has thicker fillets than at A then the load carrying capacity P of the joint will be.

a] Increses
b] Decreses.
d] unaffected
d] exactly got dob double
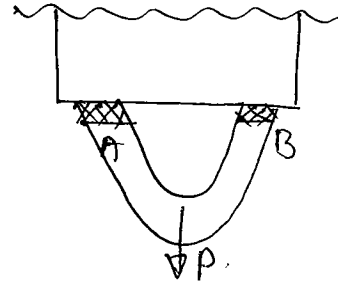
Pg 53

Q 10] Compound fillet welding joint is loaded by P as shown. The maximum shearing as stress accure at a point

a] A and B
b] B & D
c] only A
d] only c
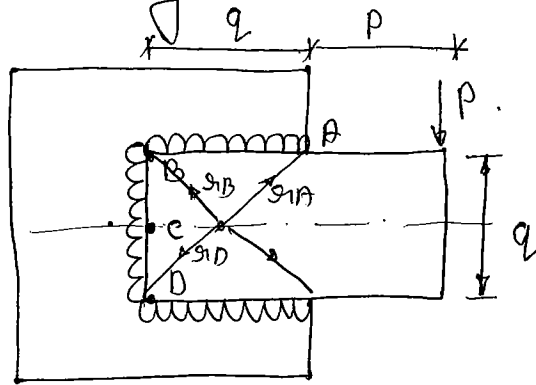
12) A spherical pressure vessel made of 10mm steel plate hemispheres butt welded together and the tank is to be 15m diameter. The tensile strength of weld metal is 85 MPa. The allowable internal pressure inside the tank is

a] 0.227 MPa
b] 0.452 MPa
c] 0.161 MPa
d] 0.752 MPa.

$$\sigma_h = \frac{PD}{4t}$$

$$85 = \frac{P \times 15 \times 10^3}{4 \times 10}$$

$$\boxed{P = 0.226 \ MPa}$$

Screwed Fasteners.

Designation

Md x P

- Metric
- Nominal dia
- Pitch

Screw Forms

① BSW

55°

② BA

47 ½

③. ANS

60

④ Isometric.

Clearance

H/8

H/4

d_c d
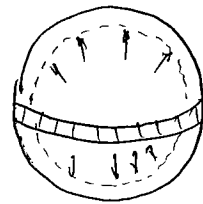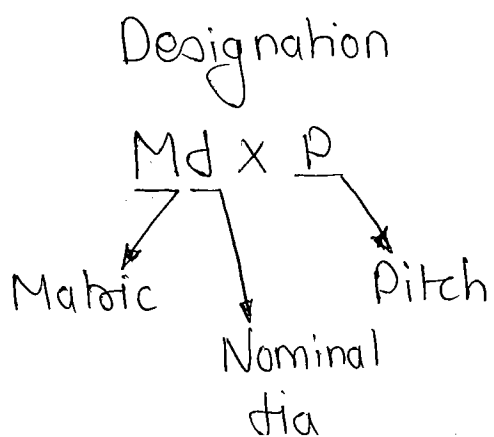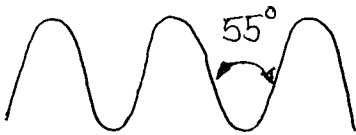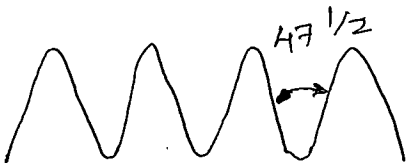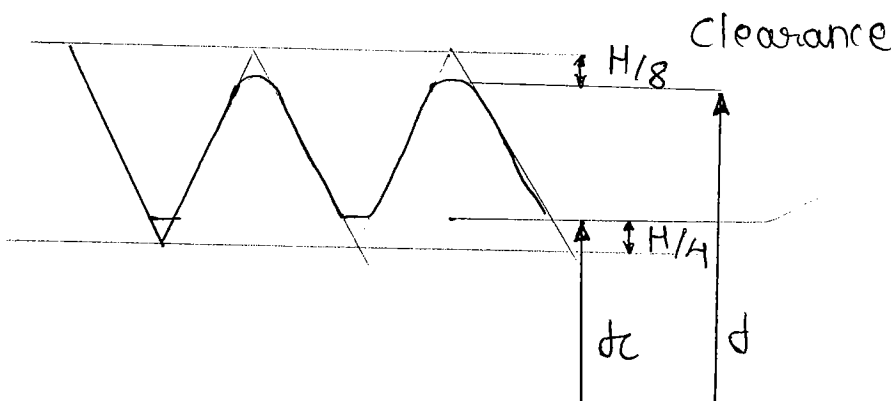
⑤ Square thread angle = 0°

P

P/2

⑥.

P

2g°

ACME Thread.

P/2

⑦

P

45°

3/4 P
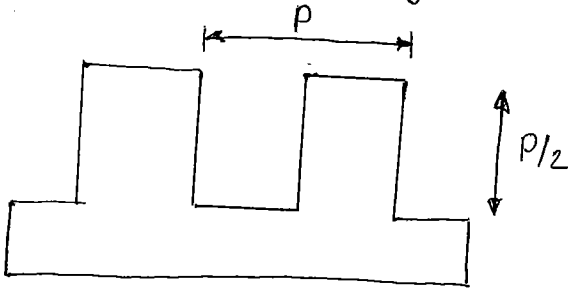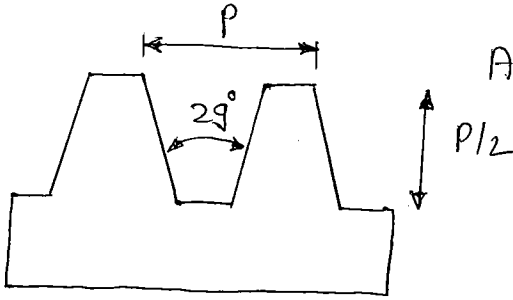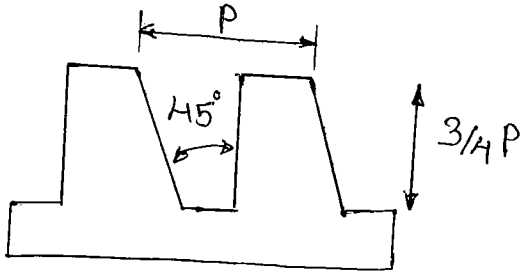
Bolted Joint   Power Transmission.

Fcosβ Power force

F

2β

β

F sinβ
Bursting
Force

Screw Axis.
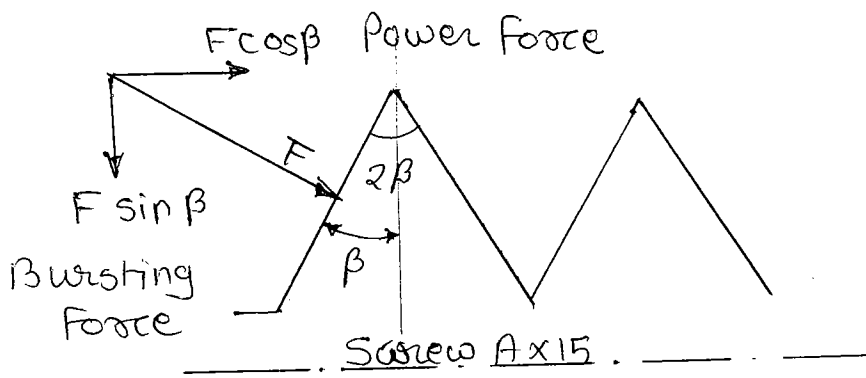
Locking force/Bursting
Force
= F sin β

β↑ F sinβ↓↑

Power Force
= F cos β

β↓ Power↑

for Max power β = 0. (square thread)

Stresses in screw.

2d

d

dc

Shank   Thread.

0.8d

Represent
the nut
side.

Nut

T

d c

Bolt

SCREW

T
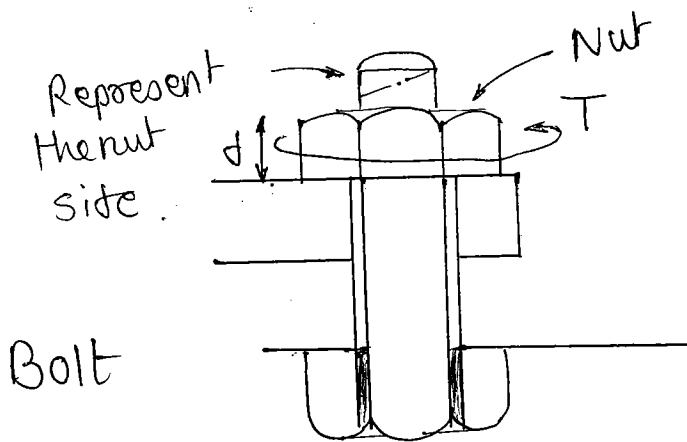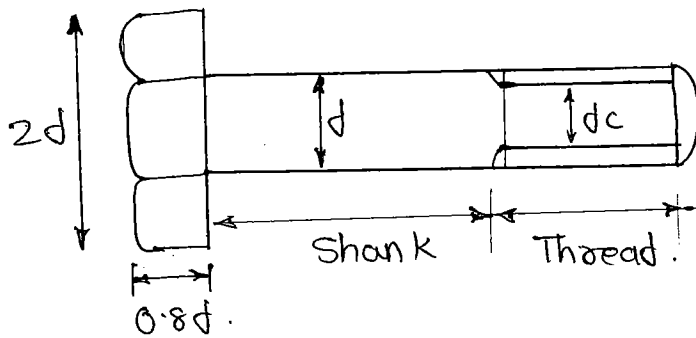
→ Bolt is alway having a nuts & torque given to nut
→ Screw is made boo taped hole. It don't have nut
  and torque given to head

Stresses in Screw

Tightning load

I.C Engine

Shear

Wall Brackets.

tension(try to separate the pipe)

tensile

Eye Bolt

tensile.

Power vessel
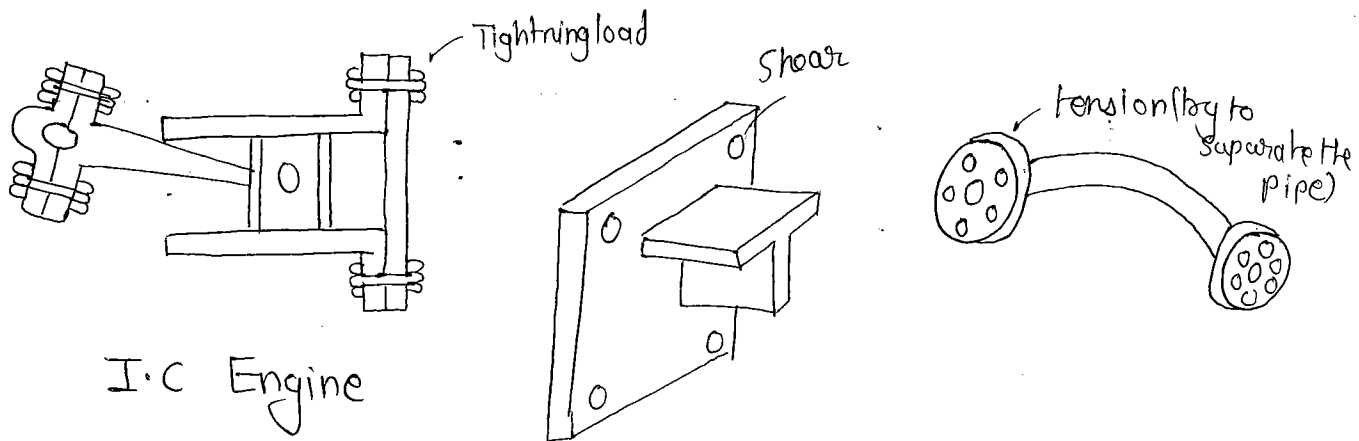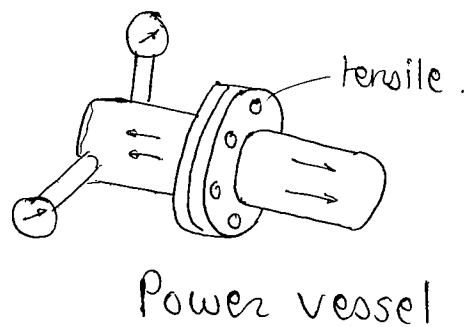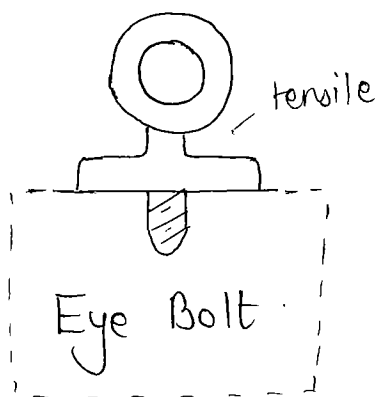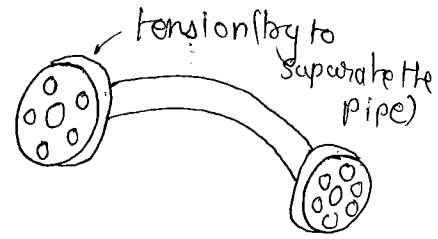
We are designing the ~~load~~ Bolt for tensile force.

① Preload (To tight the bolts)

② External force (load to which bolt is subjected)

Stresses in Screw

① Stresses due to preload / Initial tightining

② Stresses due to external load

③ Stresses due to combine preload and external load.

Stresses due to pre load.

Bolt   Connected members .   Head   Gasket   Flange
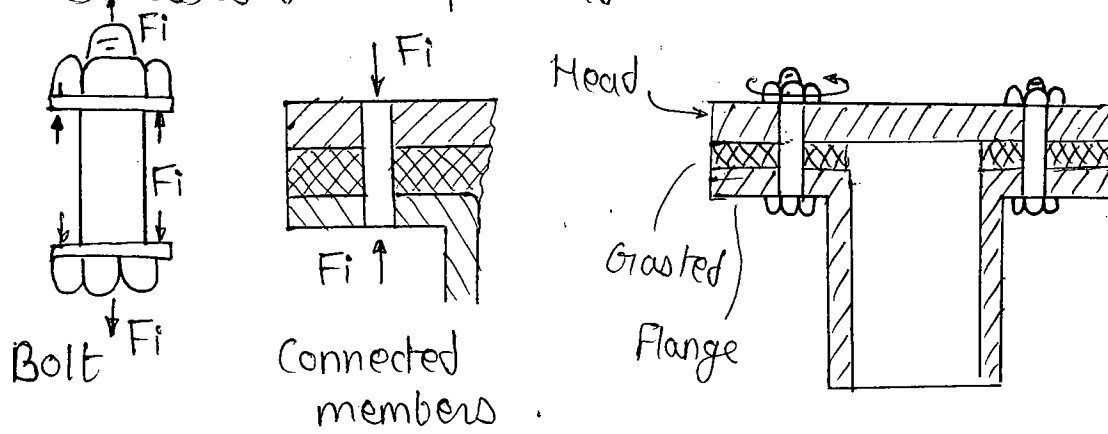
$F_i = 2840d$ N   For leak proof joint based on the Proof strength (without permanent deformation)

$F_i = 1420d$ N   For ordinary joint
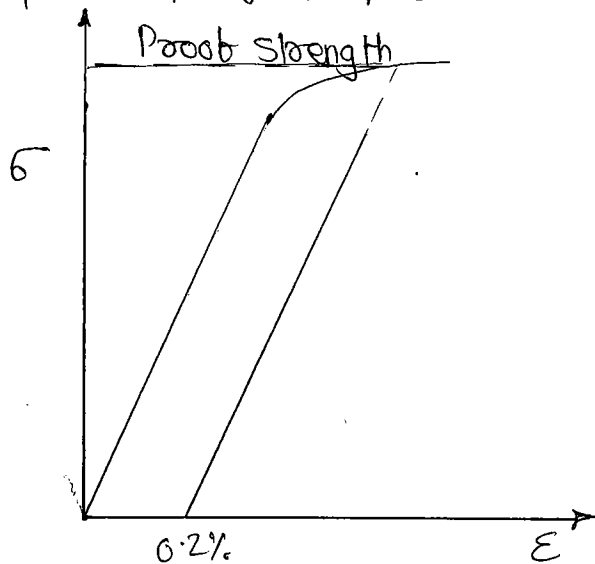
Initial load is expressed in term of dia of bolt or in term of load applied

$$F_i = 2840d \quad or \quad F_i = 1.5 P .$$

Stresses due to Preload

① Tensile stresses.

$$\sigma_t = \frac{F_i}{\frac{\pi}{4} d_c^2}$$

②. Torsional shear stress

$$\tau = \frac{16T}{\pi d_c^3}$$

③. Shear stress accross thread.

nut made from a soft material

Bolt made up of hard material
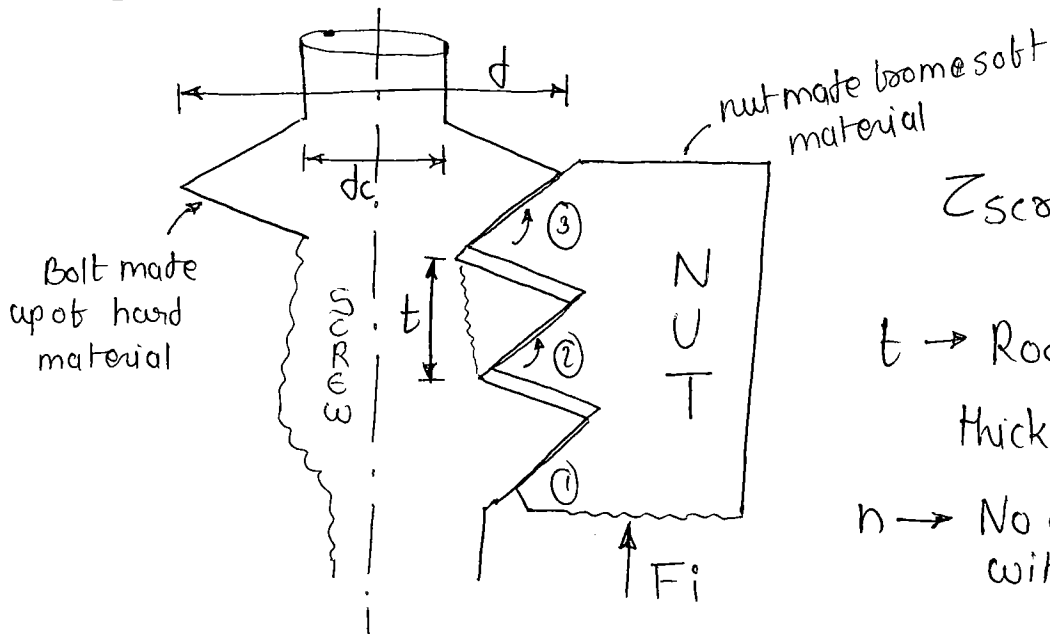
$$\tau_{screw} = \frac{F_i}{\pi d_c \times t \times n}$$

$t \rightarrow$ Root thickness or thickness of thread

$n \rightarrow$ No of thread in contact with nut.

$$\tau_{nut} = \frac{F_i}{\pi d t \times n}$$

$d \rightarrow$ external dia
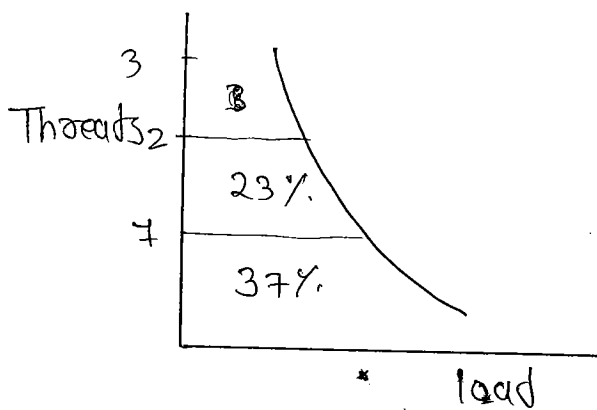
Increasing the size of nut will not give advantage because last thread take zero load

Bearing preossure - When two threads are in contact they transmits the load called as bearing
pressure or crushing stress

Ⓐ Crushing stress

$$\sigma_c = \frac{Fi}{\frac{\pi}{4}(d^2 - d_c^2)\, n}$$
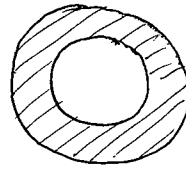
⑤ Bending stress
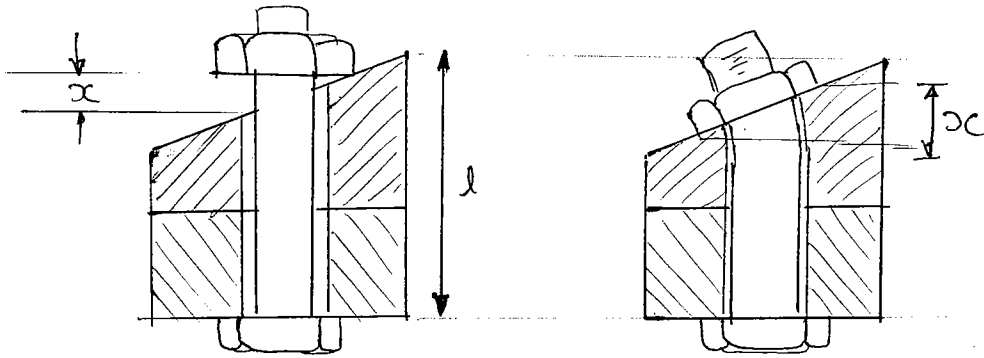
$$\sigma_b = \frac{x \times E}{2\ell\ell}$$

x – distance bet^h the external corner of nut
ℓ – holding length
E – Youngs Modulus of elasticity.

Stresses due to external load.

① Tensile stress

$$\sigma_t = \frac{P}{\frac{\pi}{4}d_c^2}$$

P – External load.

Eye Bolt.

② Shear stress.

$$Z = \frac{P}{\frac{\pi}{4}d^2}$$

$P_s \longleftarrow$ $\longrightarrow P_s$
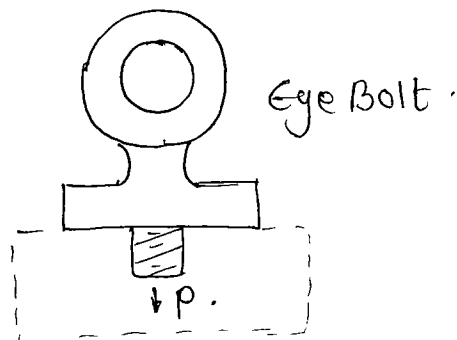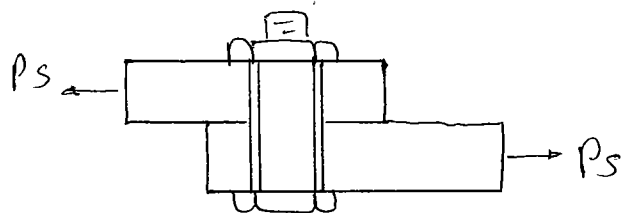
Note - Due to shear & tension combinedly acting $\sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2}$

① Guest's Theory.

$$\tau_{max} = \left[\sqrt{\left(\frac{P_t}{2}\right)^2 + P_s^2}\,\right]\frac{1}{A}$$

$$\frac{1}{A}\sqrt{\left(\frac{P_t}{2}\right)^2 + P_s^2}$$
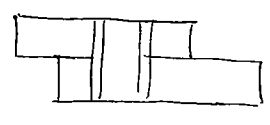
② Distortion energy theory.

$$\sigma_e = \sqrt{\sigma^2 + 3\tau^2}$$
$$= \sqrt{\sigma_t^2 + 3\tau^2}$$

Note:- Take d_c in both shear and tension to avoide confusions.

Stresses due to combined external and preloads

Let, P_r = pressure
D = Bore dia.

External load $P = P_r \times \frac{\pi}{4} D^2$. N.

P = Total load.

$P = P_b + P_m$

$\boxed{F_i = 1.5 P}$

P_b = External load shared by bolt

P_m = Part of external load taken by Connected member.

F_b = Resultant load on bolt
$= P_b + F_i$

F_m = Resultant load on connected member
$= P_m - F_i$

Δ_b = Elongation of bolt due to F_i
Δ_m = Compression of connected member due to F_i
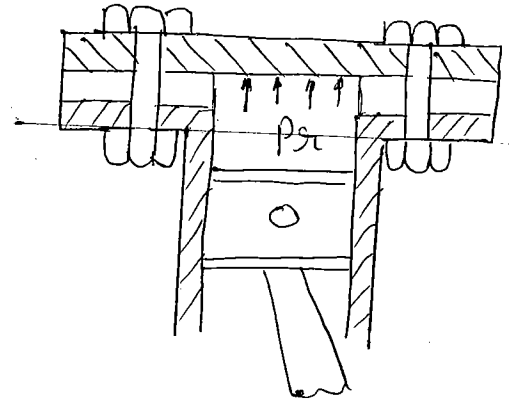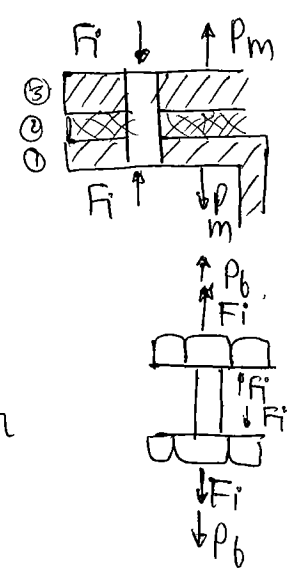
$\delta b = \dfrac{\text{External and Preload}}{\text{Increase in elongation of bolt due to } Pb.}$

$\delta m =$ decrease in compression of conned members due to Pm

For compatability

$$\boxed{\delta b = \delta m}$$

$kb =$ Stiffness of bolt.

$$= \frac{Pb}{\delta b}$$

$km =$ Combined stiffness of connected members (Series)

$$\frac{1}{km} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}.$$

$$km = \frac{Pm}{\delta m}$$

$$\delta b = \delta m$$

$$\frac{Pb}{kb} = \frac{Pm}{km}$$

$$Pm = \frac{km}{kb} \times Pb.$$

$\rightarrow \quad P = Pb + Pm$

$$P = Pb + \frac{km}{kb} \times Pb.$$

$$P = Pb \left[\frac{kb + km}{kb} \right]$$

$$Pb = \left[\frac{kb}{kb + km} \right] P.$$

$$Pm = \left(\frac{km}{kb + km} \right) P.$$

C = stiffness factor.

$$= \frac{kb}{(kb + km)}$$

$$Pb = CP.$$

$$Pm = (1 - c) P.$$

$$\boxed{Fb = Pb + Fi}$$

$$Fm = Pm - Fi$$

$$\boxed{Fm = (1 - c) P - Fi}$$

1] Size of bolt.

$$Fb = \frac{\pi}{4} dc^2 \times \frac{Syt}{F.S}.$$

2] Leakage is prevented.

$$Fm < 0. \quad - \text{Resultant load must be compression}$$

$$(1 - c) P - Fi < 0.$$

$$\boxed{(1 - c) P \leq Fi}.$$

Pno. 34

Q.] The ratio of load required to open the bolt joint to the initial tightening load with stiffness of mating part is equal to four times stiffness of bolt is

a] 0.2   b] 0.8   c] 1.25   d] 2.5.

$$km = 4 kb.$$

$$\frac{P}{Fi}$$

$$(1 - c) P - Fi = 0.$$

$$\frac{P}{Fi} = \frac{1}{1 - c}$$

$$= \frac{1}{1 - 0.2}$$

$$= \frac{1}{0.8}$$

$$\frac{P}{Fi} = 1.25$$

$$c = \frac{kb}{kb + km}$$

$$= \frac{kb}{5 kb} = \frac{1}{5} = 0.2$$

③ A bolted joint connects two members with an axial tightnening for F kN. The bolt has matric threads of 4mm pitch The torque req. to achivie this force F is 1.4 kN-mm what is the value of 'F'?

a] 0·35 kN   b] 0·7 kN   c] 1·05 kN   d] 2·2 kN.

$p = 4mm$.
$T = 1.4 kN-mm$

Work done

$W = Force \times distance = Torque \times \theta$

$F \times 4 \qquad = 1.4 kN-mm \times 2\pi$

$F = 2.2 kN$.

Bolted Joints.

Advantage of Preload.

washer

P_b
F_t
F_i
P_i

Load.

Hence it will cause fatigue Failure

$\delta_b = \delta_m$

$F_a' > F_a$

P_b F_t F_i k_m P_m δ_m k_b δ_b F_a F_a'

Compression Δm Δb ϵ Elongation
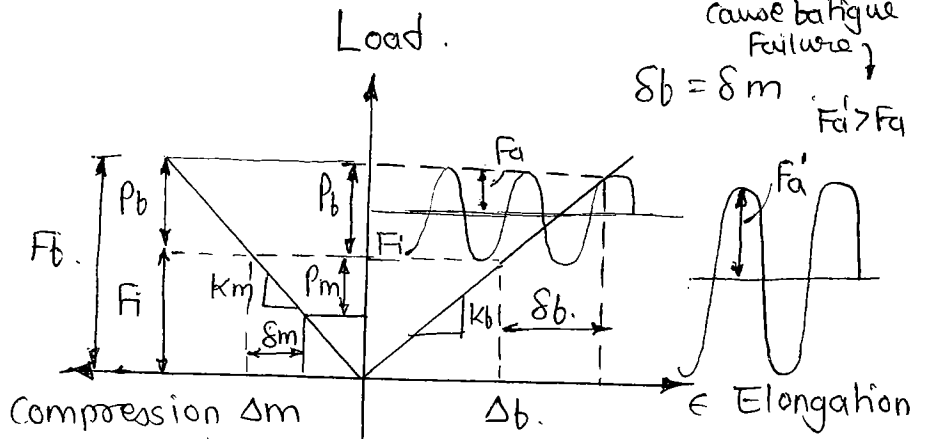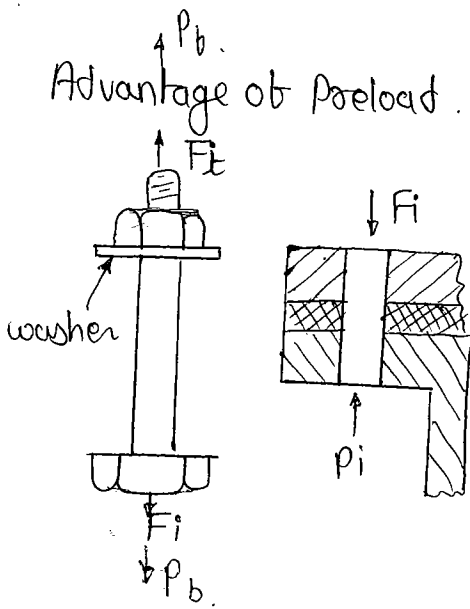
It will produce the locking effect due to less fluctuating load

① To prevent leakage of fluid.

② Increases the fatigue strength of bolt

③ Increases the locking effect.

P_r

D

θ

load amplitude.

No. of cycle.
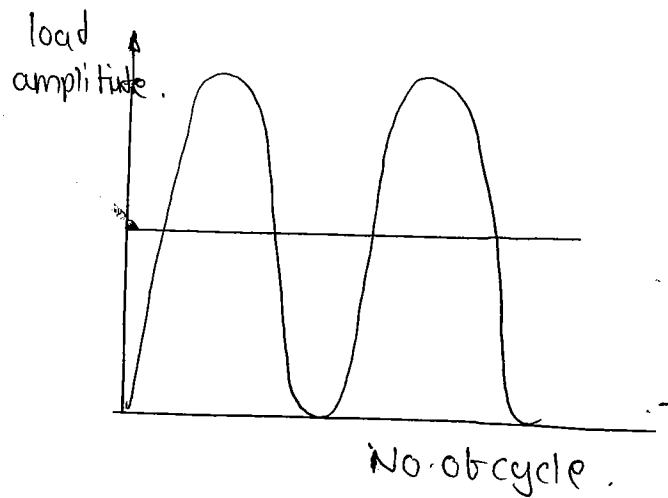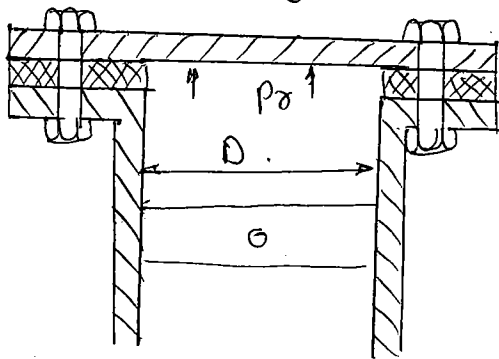
$$P = P_A \times \frac{\pi}{4} D^2$$

On the bolt the P is varying from min to maximum value.

Eccentric load on bolt Joint.

$$P = \frac{\pi}{4} d_c^2 \times n \times \frac{S_{yt}}{F_s}.$$

$$\tan\theta = \frac{a}{b}$$
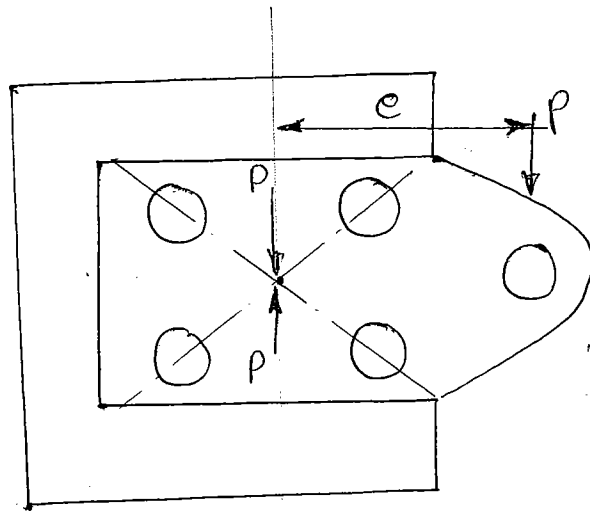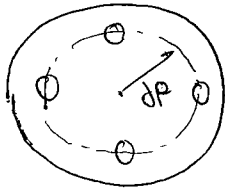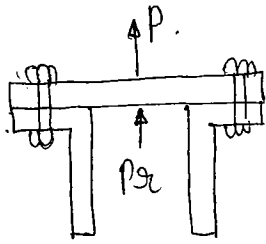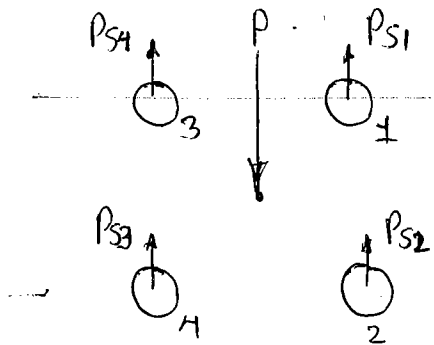
$$P_{S1} = \frac{P}{4}.$$

$$P_{S1} = P_{S2} = P_{S3} = P_{S4} = \frac{P}{4}.$$

Primary shear load
.<u>or</u>
Direct shear load.
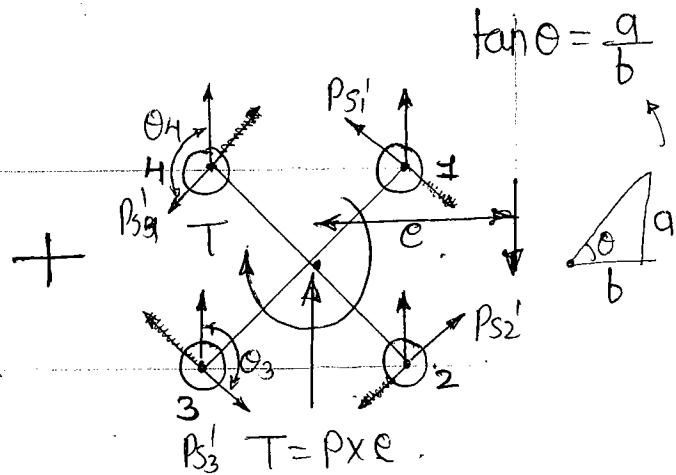
$$T = P \times e.$$

$$\frac{T}{J} = \frac{\mathcal{Z}_2}{r}.$$

$$\mathcal{Z}_2 \propto r$$

Secondary shear load.

Resultant shear loads on the bolt

$$P_S = \sqrt{P_{S_1}^2 + P_{S_1}'^2 + 2 P_S \, P_{S_1}' \cos\theta}$$

$$P_S = \phi \, (P_S, P_S', \theta)$$

$$\theta \downarrow \quad \cos\theta \uparrow \quad P_S \uparrow$$

$$\boxed{(\theta_1 = \theta_2) < (\theta_3 = \theta_4)}$$

Q1. 1) The weakest bolt.   — only one.

2) Total shear force on bolt (2) — $P/3$.

3) Least loaded bolt   — 3

4) Primary shear load on bolt 3 — $P/3$

5] Secondary shear load on bolt ② — 0.
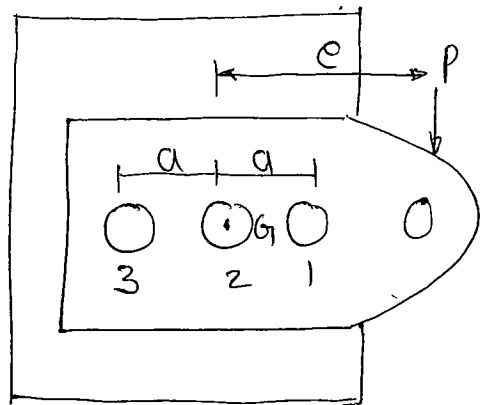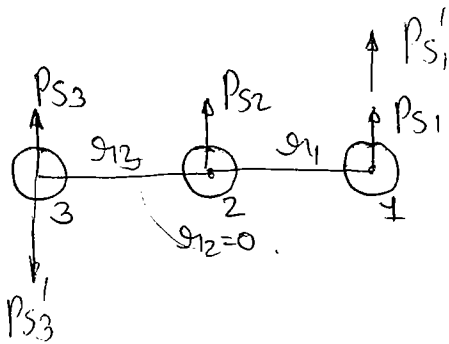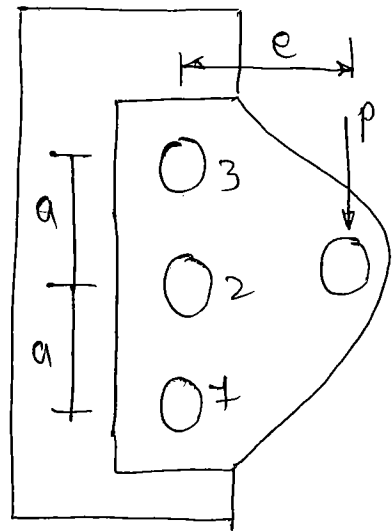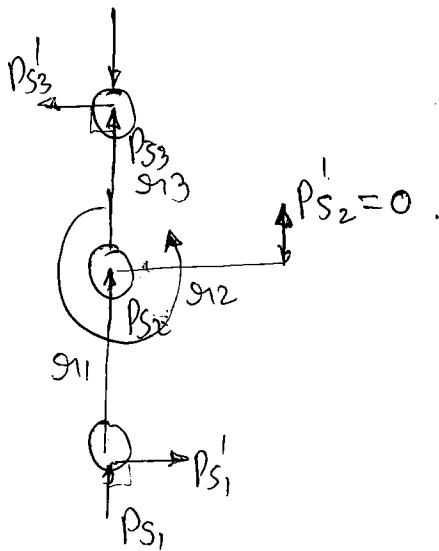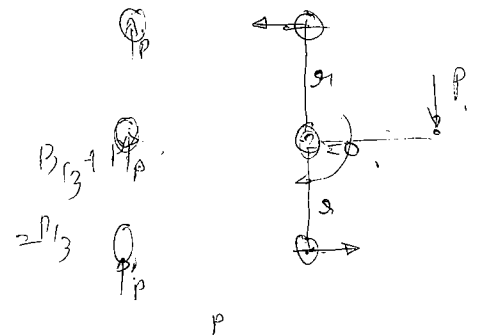
1]

1]   and 3
2] $P/3$.
3]   2.
4] $P/3$
5]   0.

③ A bolt joint is shown in the fig. The maximum shear stress in MPa in bolts A and B respectively are.

3 holes of M10×1.75mm bolts
6mm thick
20, 40, 40, 20
F = 10kN
150
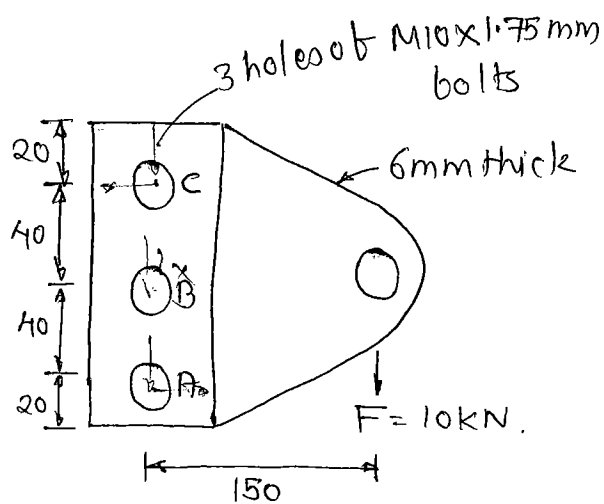
$$\tau_{total} = \frac{10}{\frac{\pi}{4}d^2} + \frac{16\,T}{\pi d^3}$$

$$= \frac{10\times10^3}{3\times\frac{\pi}{4}\times10^2} + \frac{16\times10\times10^3\times150}{\pi\times10^3}$$

$$= \cancel{127\cdot32RN} + 7639\cdot43$$
$$63\cdot66$$

$$\tau_2 = 3333\cdot33 \ N/mm^2.$$

$$\frac{T}{J} = \frac{f_s}{R}$$

$$T = \frac{\pi}{16} f_s d^3.$$

$$f_s = \frac{16\,T}{\pi d^3}.$$

$$= \sqrt{p^2+q^2+2pq\cos\theta}.$$

$$=$$

$$T = f_{s'A}\times \imath_A + f_{sB}'\,\imath_B + f_{sc}'\,\imath_c$$

$$= k\,\imath_A\times\imath_A + k\imath_B\times\imath_B + k\imath_c\times\imath_c$$

$$P\times\ell = k\,(\imath_A^2 + \imath_B^2 + \imath_c^2)$$

$$k = \frac{P\times\ell}{\imath_A^2 + \imath_B^2 + \imath_c^2}$$

$$k = 0.468.$$

$$f_{sA}' = 0.468\times40$$
$$= 18\cdot75 kN.$$

$$P_A = \frac{10}{3} = 3\cdot33$$

$$P_B = \frac{10}{3} = 3\cdot33$$

$$P_C = \frac{10}{3} = 3\cdot33$$

$$P = \sqrt{(P_{sA})^2 + (f_{sA}')^2}$$
$$= \sqrt{(3\cdot33)^2 + (18\cdot75)^2}$$

$$P = 19\cdot04 kN.$$

$$\tau = \frac{19\cdot04}{\frac{\pi}{4}\times10^2}$$

$$\tau = 0.242146 \ kN$$

$$\boxed{\tau = 242\cdot46N.}$$

$$\tau_{SB} = \frac{P_{\tau_{SB}}}{\pi/4 \; 10^2} = \frac{3.33 \times 10^3}{\frac{\pi}{4}(10)^2}$$

$$\tau_{SB} = 42.39 \; N/mm^2$$

a] 242.6 , 42.5   b] 42.5 , 242.6   c] 42.5 , 42.5

d] 242.6 , 248.6 .

— x — x —

Bolts of Uniform strengths.

$U_s < U_t$

Shank   Thread.

$$U = \frac{\sigma^2}{2E}$$

d_h.
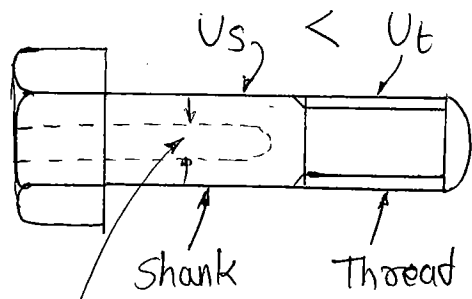
$\sigma_t > \sigma_s$.

$$\sigma_t = \frac{P}{\frac{\pi}{4} d_c^2} \quad , \quad \sigma_s = \frac{P}{\frac{\pi}{4} d^2}.$$

$\sigma_t \uparrow \quad \sigma_s \downarrow$

If the energy stored in shank and bolt is same the bolt is called bolt of uniform strength.

$$\boxed{U_s = U_t}$$

$$\frac{\pi}{4} d_h^2 = \frac{\pi}{4} d^2 - \frac{\pi}{4} d_c^2.$$

$$d_h = \sqrt{d^2 - d_c^2}$$

d_r d_c
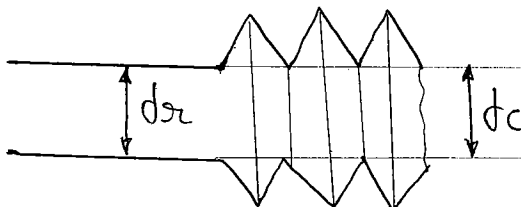
Total strain energy

$$U = \frac{\sigma^2}{2E} \times A \times l .$$

DESIGN OF BEARINGS.

A bearing is a machine element which supports a moving moving member and confines its motion.
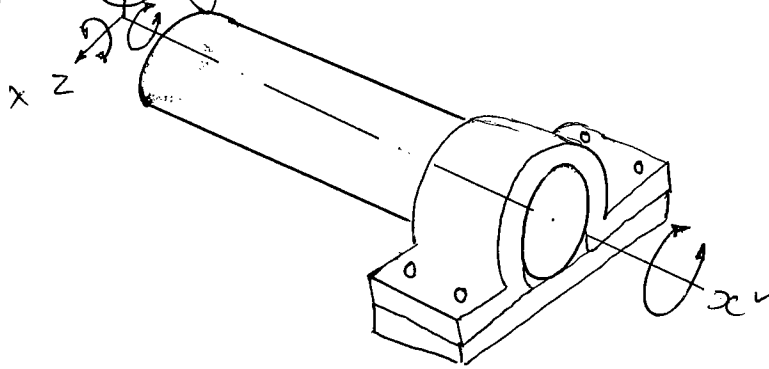
Bearings

1] Sliding Contact bearings
   One surface slides over the other surface

2] Rolling contact Bearing
   Point of or line contact.

Sliding Contact Bearing.

1] Journal Bearing

2] Collar bearings or Thrust Bearings
→ Axial loads.
→ Fx axial load
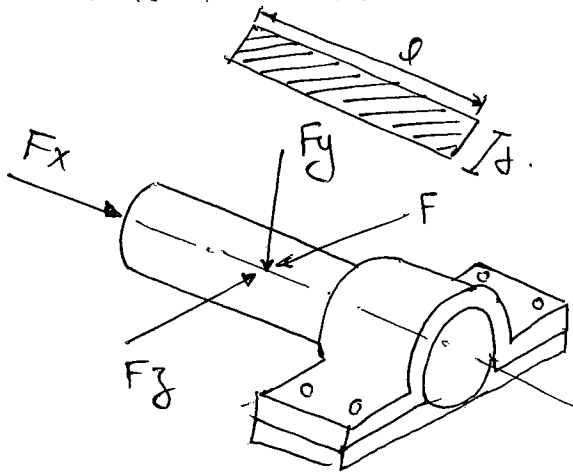
→ Radial loads

→ Fy, F, Fz - Radial loads

1] Journal Bearing

Lubricants
Journal
Bearing

Shaft

Sliding contact type.
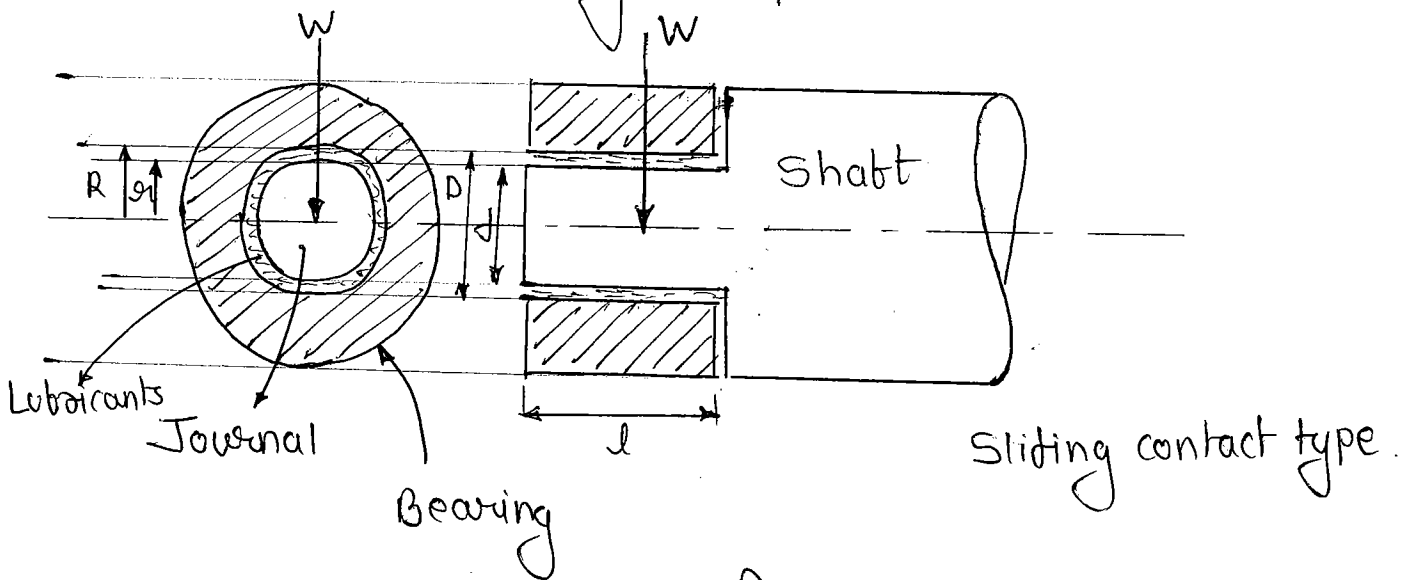
d → dia of Journal
D → dia of Bearing.

Pressure

$$P = \frac{load}{Projected\ area}$$

$$= \frac{W}{l\,d}$$

Note :- load is always ⊥ er to bearing

Clearance used

1] Radial clearance

$$C = R - r$$

2] Diametrical clearance

$$C_d = D - d$$
$$= 2C$$

Types of Journal Bearing

① Full Journal Bearing.

→ Angle of contact 360°

→ Supports load in all radial directions
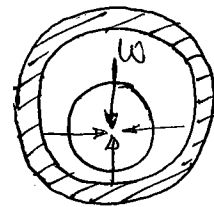
Ex:- IC engines, steam turbine, compressors etc.
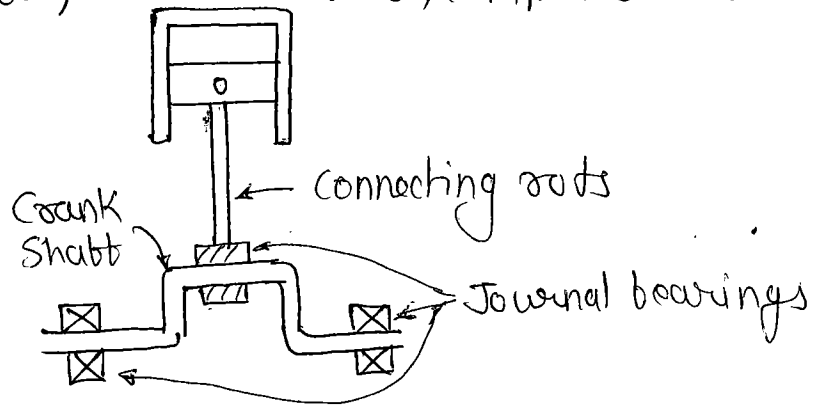
② Partial Journal Bearing.

→ Self cooled bearing.

→ load carry only one direction

→ $\beta < 180°$

Ex:- Rail road car axles.

$\beta = 120$.

Journal
Bearing

Bearing
lubricants
casing
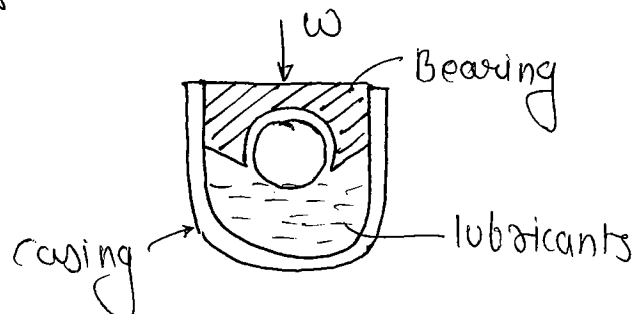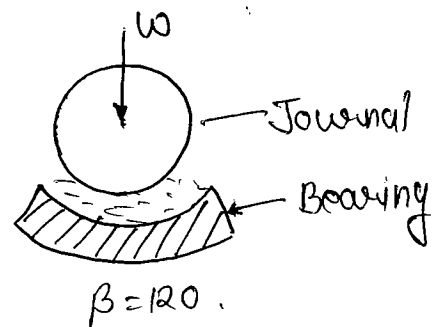
Fitted Journal bearing.

Radius of Journal is always equal to the radius (inner radius) of Bearing.

$$\mathcal{r} = R .$$

Some type time eccentic alinement is taken into account.

$$R > \mathcal{r}$$
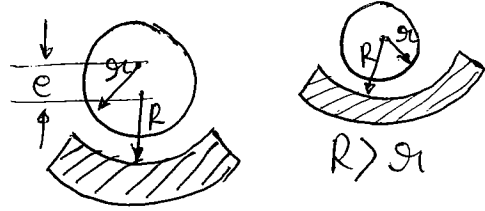
Methods of lubrications

a] Thick Film lubrication

→ Surface finish do not effect the performance hence intipentent of Surface Finish.

→ No contact.
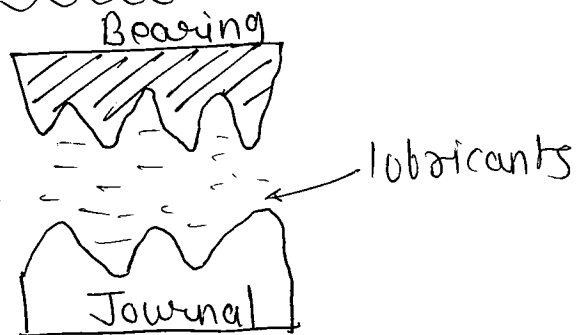
b] Thin Film lubrication

→ Some contact is there
→ High surface finish
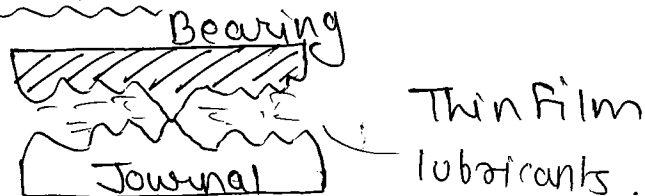
c] Solid lubrication

d] Elasto hydrodynamic & lubrication

used in point contact type of location

$$P = \frac{W}{A}$$

$$A \downarrow \quad P \uparrow$$

Thick Film lubrication.

1] Hydrodynamic lubrication
2] Hydrostatic lubrication

Hydrodynamic lubrication

The load supporting fluid film is created by action of journal

→ No pump is used.

Hydrostatic Bearing

Load supported fluid film is generated by external pressure source.

→ Pump is used to generate the pressure.

Ex:- M/c bead

① working speed is low hence pressure is not generated by bearing

$$\frac{\partial P}{\partial x} = 0.$$

$$\frac{P_2 - P_1}{L}$$

Pressure gradient is zero in x direction

$$Z = \mu \frac{du}{dy}$$

μ = absolute viscosity

$-\frac{\partial P}{\partial x} < 0 \quad -\frac{\partial P}{\partial x} = 0 \quad -\frac{\partial P}{\partial x} > 0$

N-s/m²
Pa-s or
kg/m-s
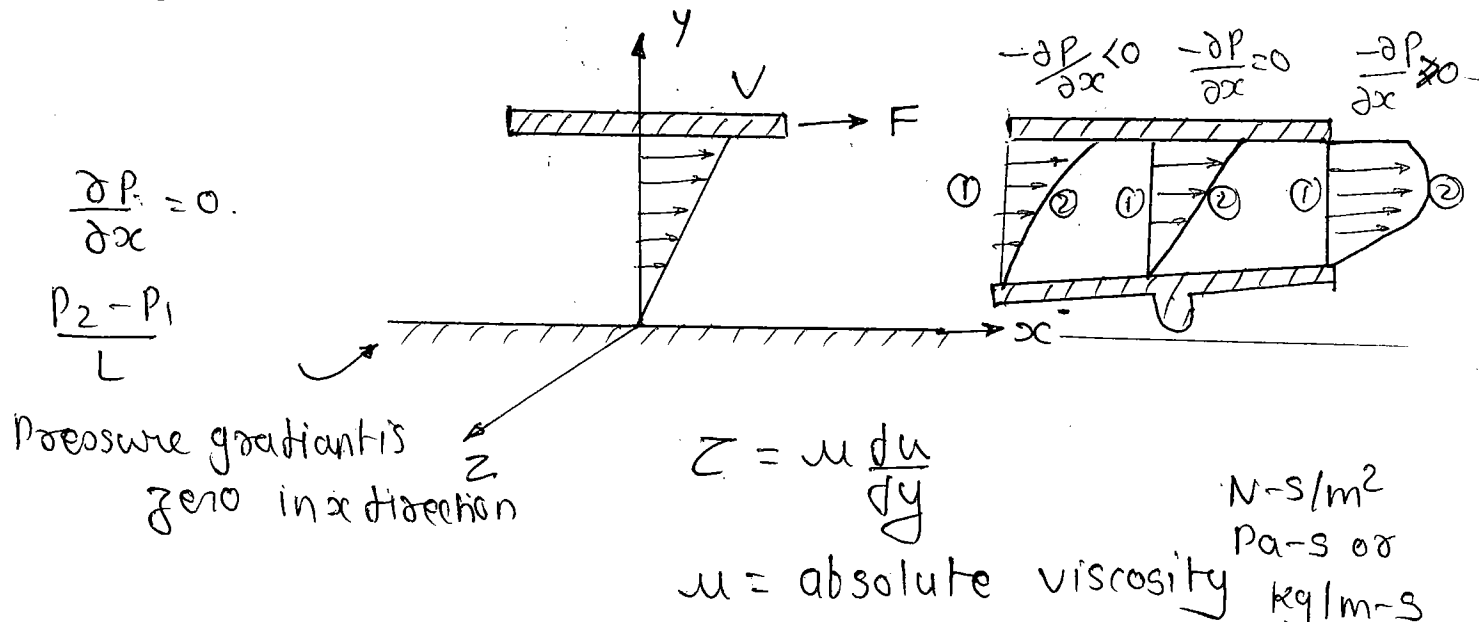
kinematic viscosity

$$\vartheta = \frac{\mu}{\rho} \quad m^2/s \qquad \rho = \text{mass density}.$$

Because of plate velocity and adhesive force take fluid from low pr. to high pressure

$$-\frac{\partial P}{\partial x} < 0 \implies -\frac{\partial P}{\partial x} = 0 \implies -\frac{\partial P}{\partial x} > 0.$$

$$\frac{P_1 - P_2}{L} > 0.$$

→ Conditions for generating pressure

① Wedging action   Converging & diverging forces.

② Proper lubricant

③ Relative velocity.

Rest ω

Metal to Metal Contact

Start ω

$F = \mu \omega$

① Metal to Metal Contact

② Thin film lubrication.

$$V = \frac{\pi D N}{60}$$

N↑ V↑ P↑

③ load is balanced by

ω Running

Suction zone

line of centers

Pressure zone

① No metal to metal contact

② Thick film lubration

③ load is balanced by pressure force
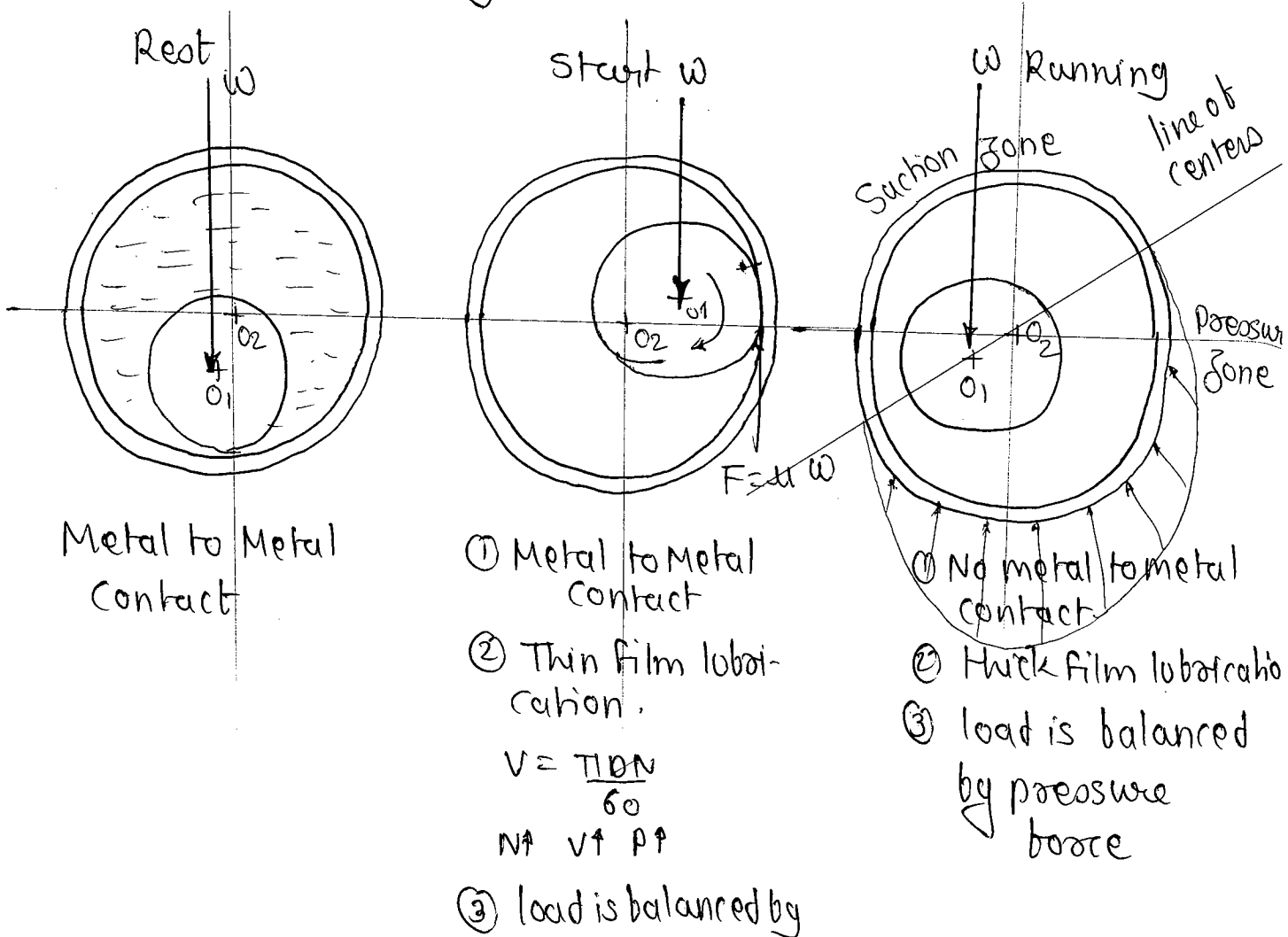
Performance of Journal Bearing.

→ Coff of friction is main to determine the performance
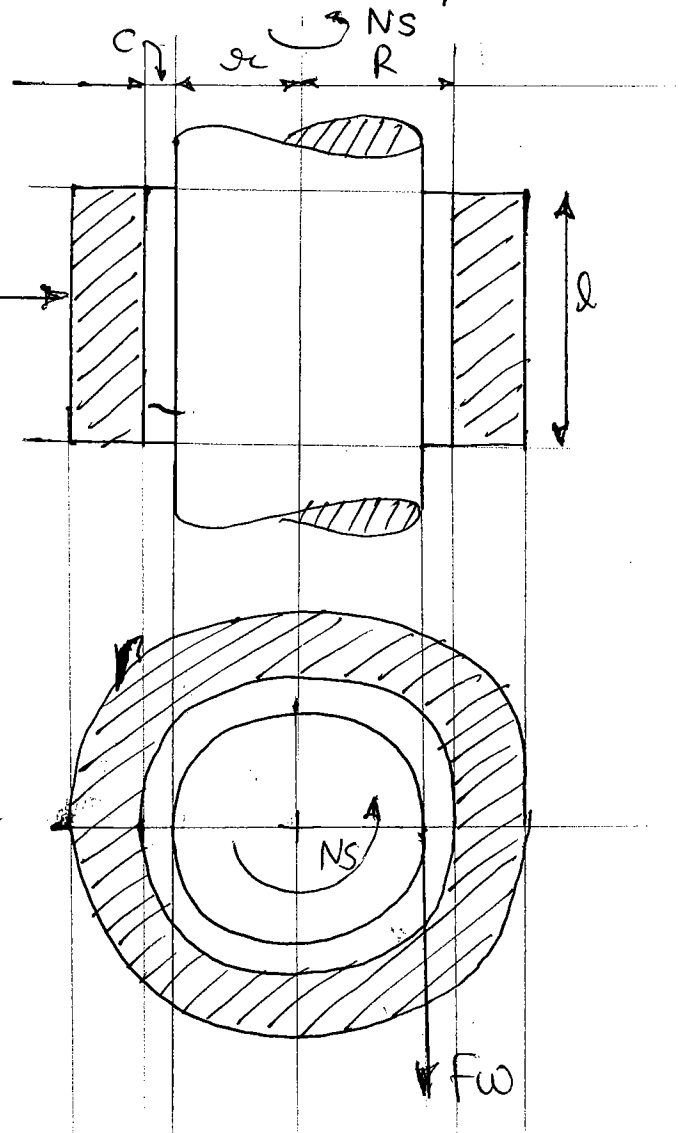
Petroff's equation 1883

Torque $T = $ Force $\times$ Radius

$= $ Stress $\times$ area $\times$

$= Z \times 2\pi r L \times r$

$= \mu \dfrac{du}{dy} \times 2 \times \pi r \ell \times r$

$T = \mu \dfrac{2\pi r Ns}{C} \times 2\pi r \ell \times r$

$T_F = F \times W \times r$

Pressure

$P = \dfrac{W}{\text{Projected area}}$

$P = \dfrac{W}{2r \times \ell}$

$W = P \times 2r\ell$

Friction torque

$T_F = Fw \times r$

$T_F = f \times P \times 2r\ell \times r$

Performace of journal bearing.

$$F_P \times \cancel{2} \cancel{r} \cancel{l} = \mu \times \frac{2\pi \cancel{r} \cancel{l} N_S}{C} \times 2\pi \cancel{r} \cancel{l} \times r$$

$$f = 2\pi^2 \left(\frac{\mu N_S}{P} \right) \left(\frac{r}{C} \right)$$

Assumption in this
   ① Lighter load is applied. $\left. \begin{array}{c} \\ \\ \end{array} \right\}$ Not fit for Bearings
   ② Journal rotates concentrically

$\frac{\mu N_S}{P} \Rightarrow$ Bearing charactrishc Number

$\frac{C}{r} \Rightarrow$ Radial clearance ratio.

2] McKee's Equation.

→ At condition D.

Heat generated ↑
temp ↑
μ ↓
$\left(\frac{\mu NS}{P}\right)$ ↓

→ At condition D'
f↓ Hg↓ T↓ μ↑ $\left(\frac{\mu N3}{P}\right)$↑

→ At condition E
Hg↑ T↑ μ↓ $\left(\frac{\mu N3}{P}\right)$↓ f↑
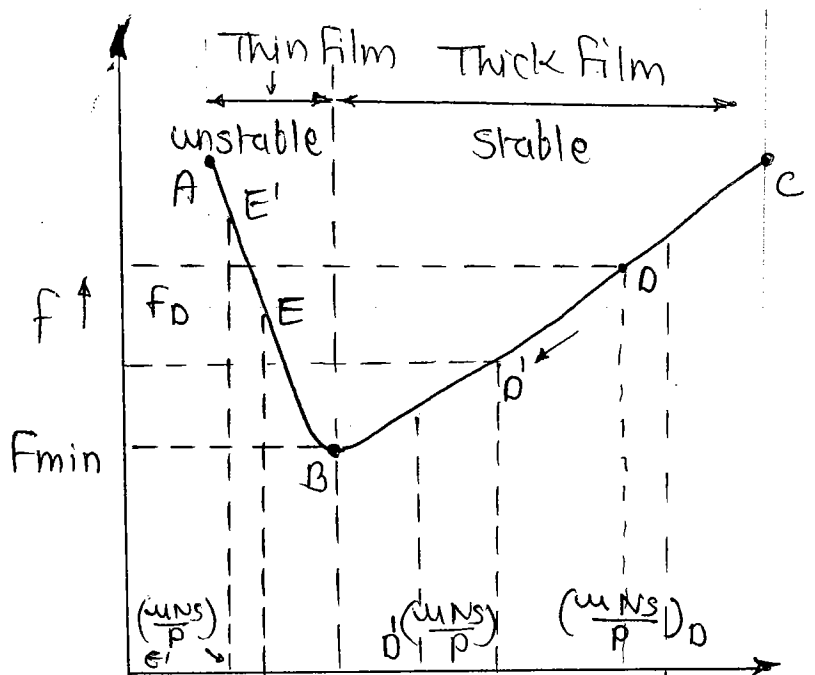
Compound Bearing.

HSB + HDB
Hydrostatic + Hydrodynamic
Bearing   Bearing.

$\frac{\mu NS}{P}$ corresponding to minimum coeff of friction is called bearing modulus, B.

Bearing modulus only deal with Fmin

Operating zone

$$3\beta \lessgtr \left(\frac{\mu NS}{P}\right)_{operating} \lessgtr 15\beta$$

$$f = 0.326 \left(\frac{\mu N}{P} \right) \left(\frac{d}{Cd} \right) + K$$
$$= 0.33$$

N – revolution per min.

μ – N-S/m²

N – rpm

P – N/m²

K – Leakage factor.

K = 0.002

$0.75 < \dfrac{\ell}{d} < 2$.

Friction Torque

$$T = fW \times r$$

Power loss = Heat generated, H_g.

$$= \frac{2\pi N T}{60}$$

$$= fW \cdot V$$

$$V = 2 \frac{\pi d N}{60}.$$

H_d = Heat dissipated

$$= CA (t_b - t_a)$$

C = Heat dissipation cofficient

A = Projected area of beam

t_b = bearing temp.

t_a = Ambient air temp.

Pg no 54.

1] $d = 40mm$.
$W = 3KN$.
$P = 1.3 MPa$.

$$P = \frac{load}{Projected\ area}$$

$$1.3 = \frac{3 \times 10^3}{d \cdot L}$$

$$d \cdot L = \frac{3 \times 10^3}{2 \times 1.3}$$

$$L = \frac{3 \times 10^3}{40 \times 1.3}$$

$$L = 57.69.$$

$$\frac{L}{D} = \frac{57.69}{40} = 1.44. \quad Ans\ b].$$

$\frac{L}{D} > 1$ long bearing

     Design for less friction under thin film lubrication

     Length is more, less leakage.

$\frac{L}{D} > 1$ Short Bearing

     thick film lubrication

     more leakage

$\frac{L}{D} = 1$ Square bearing.
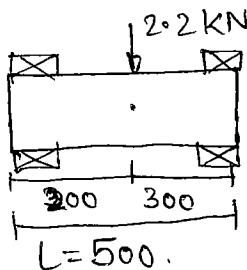
4]. $D = 25mm$

$$P = \frac{load}{Projected\ area}.$$

200   300

$L = 500.$ $\frac{L}{d} = 105$

2·2 KN

300

A

B

500

$$P = \frac{load}{Ld} = \frac{1·32 \times 10^3}{1·5 \times 25 \times 25}$$
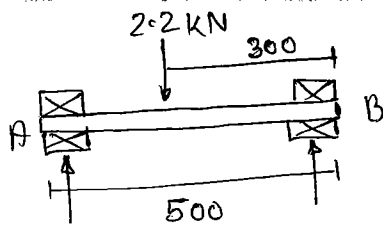
$$= 1·408$$

$$R_A + R_B = 2·2 \, KN$$

$$R_A \times 500 + 2·2 \times 300 = 0$$

$$R_A = 1·32 \, KN$$

$$R_B = 0·88 \, KN$$

$$\frac{L}{D} = 1·5$$

$$L = 1·5 \, p$$

⑤

$d = 75 mm$

$N_1 = 300 \, rpm$

$u_1 = 0·06 \, pa-sec$

$C_1 = 0·2 mm$

$P_1 = 1·4 \, MPa$

$N_2 = 400 \, rpm$

Bearing contact number

$$= \frac{u_1 N_1}{P_1}$$

$$= \frac{0·06 \times 300}{1·4}$$

$$= 12·85$$

To stable the bearing

$$\frac{u_1 N_1}{P_1} = \frac{u_2 N_2}{P_2}$$

$$12·85 = \frac{0·06 \times 400}{P_2}$$

$$\boxed{P_2 = 186 . MPa}$$

④

$d = 150 mm$

$l = 225 mm$

$w = 9 KN$

$N = 1000 \, rp \, m$

$C = 0·075 mm$

$h_d = 90 \, KJ/min$

$u = ?$

$$h_d = CA(T_b - T_a)$$

$$A = h_g = f \omega V . \quad J/s . \quad or \, J/min$$

$$h_g = h_d .$$

$$f \times 9 \, KN \times \pi \times 150 \times 10^{-3} 1000 = 90 \, KJ/min$$

$$f = 0·02$$

$$f = 0.326 \left(\frac{\mu N}{P}\right)\left(\frac{d}{d}\right) + K$$

$$(0.02)\ f = 0.032 \times \left(\frac{\mu \times 1000}{\underset{\underset{m \quad m}{0.150 \times 0.22}}{9 \times 10^3\,N}}\right) \times \frac{150}{2 \times 0.075} + 0.002$$

$$\mu = \underset{0.0157}{\cancel{0.0035}}\ Pa\text{-}s\ .$$

⑨ $d = 50mm$

$\ell = 75mm$

$P = 2\ MPa$.

$N = 500\,rpm$

$C = 11.6\ w/m^2\text{-}K^{\circ}C$

$A = 8 \times \ell d$.

$t_a = 28^\circ C$

$f = 0.0015$

$$Hd = c\hat{A}(t_b - t_a)$$

$$Hg = F\omega \times V$$
$$= 0.0015 \times P \times \cancel{\pi d} \ell d \times \frac{\pi d N}{\underset{60}{\times 10^{-3}}}$$
$$= 0.0015 \times \underset{2}{2} \times 75 \times 50 \times \pi \times 50 \times \frac{500}{60}$$
$$= 883.57 \times 10^3$$

$$Hd = Hg$$
$$\frac{883.57 \times 10^3}{1000} = 11.6 \times 8 \times (75 \times 10^{-3} \times 50 \times 10^{-3}) \times (t_b - 28)$$

$$Hg = 0.0015 \times 2\ \frac{N}{mm^2} \times \frac{50 \times 75}{mm^2} \times \pi \times 50 \times 10^{-3} \times \left(\frac{500}{60}\right)$$
$$= 14.72\ .$$

$$Hg = Hd.$$
$$14.72 = CA\ (t_b - t_a)$$
$$14.72 = 11.6 \times 8 \times 50 \times 10^{-3} \times 75 \times 10^{-3}\ (t_b - 28)$$
$$t_b = 70.31^\circ C$$

$$\boxed{t_b = 343\ .31\,K}$$

Sommerfield Number / Bearing number.

$$S = \left(\frac{\mu N_S}{P}\right)\left(\frac{r^2}{C}\right)$$

$\hookrightarrow$ Radial clearance

Pg 60.

Pno 55] $W = 50 \text{ KN}$,

$\left.\begin{array}{l} L = 150 \text{ mm} \\ d = 150 \text{ mm} \end{array}\right\}$ squre

$C = 0.15 \text{ mm}$

$t_{min} = 0.03 \text{ mm}$

$\mu = 8 \text{ cp.} = 8 \times 10^{-3}$

$S = 0.0446$

$N = ?$

$P = \dfrac{\text{load}}{\text{Projected area}}$

$= \dfrac{50 \times 10^3}{150 \times 10^{-3} \times 150 \times 10^{-3}}$

$= 6666.6 \times 10^3$

$$S = \left(\frac{\mu N_S}{P}\right)\left(\frac{r^2}{C}\right)$$

$$0.0446 = \left(\frac{8 \times 10^{-3} \times N_S}{6666.6 \times 10^3}\right) \times \left(\frac{150}{2 \times 0.15}\right)^2$$

$\cancel{0.0446} = 3 \times 10^{-4} \, N_S$.

$N_S = 49.55$

$N = N_S \times 60$

$N = 2973.33 \text{ rpm}$

Attitude or Eccentricity ratio (ξ) , ξ)

ho - min oil film thickness

$\xi = \dfrac{\text{Eccentricity}}{\text{Radial clearance}} = \dfrac{e}{c}$

$R = e + r + ho$

$\qquad = \dfrac{c - ho}{c}$

$R - r = e + ho$

$\qquad = 1 - \dfrac{ho}{c}$

$c = e + ho$

If $e = 0$, $c = ho$

(Not possible)

ϕ = attitude angle.
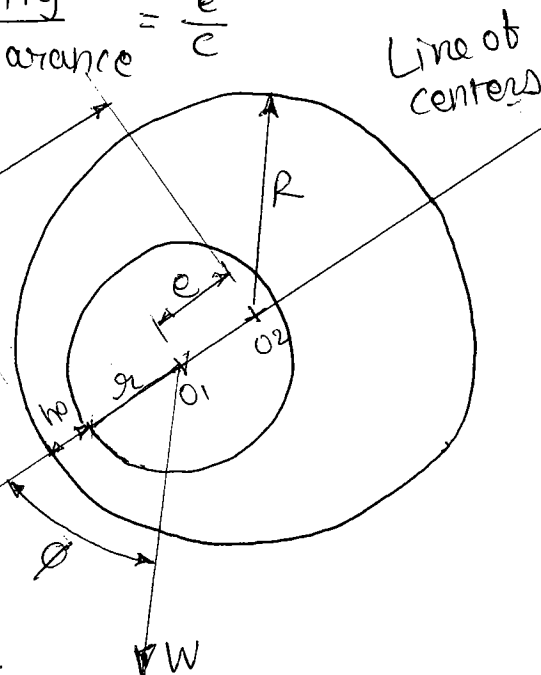
- If the ϕ maintain constant then that type of bearing is stable bearing.
- If the ϕ not maintain constant oil whirl can balance fluid forces created due to vibration.

Line of centers

ϕ = constant
Stable bearing

Oil whirl

unbalanced Fluid forces

Oil whip.

Fluid forces frequency matches with frequency bearing material.

6]

$$\varepsilon = = 0.8$$

$$\frac{ho}{c}$$

$$\varepsilon = 1 - \frac{ho}{c}$$

$$0.8 = 1 - \frac{ho}{c}$$

$$\frac{ho}{c} = 0.2$$

Rolling Contact Bearings.

Rolling contact Bearings (Antifriction bearings)

① **Ball Bearing**

→ Point Contact

② **Roller Bearings**

→ line contact

More load carrying capacity than roller Bearings
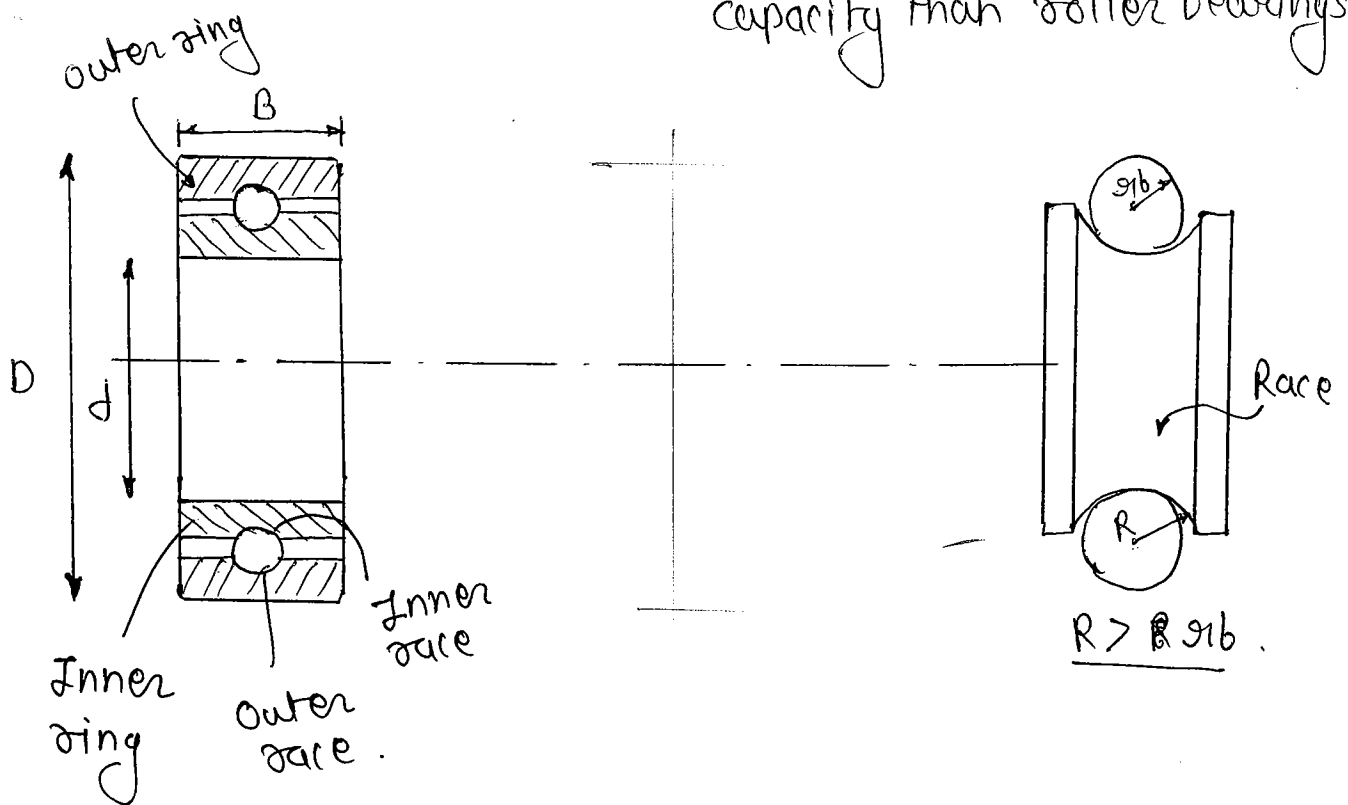
d = Inner dia or dia of shaft

D = Outer dia

B = axial width.

High cromium steel — outer ring

$R > R$ rib.

More rigidity

Interference
bit.
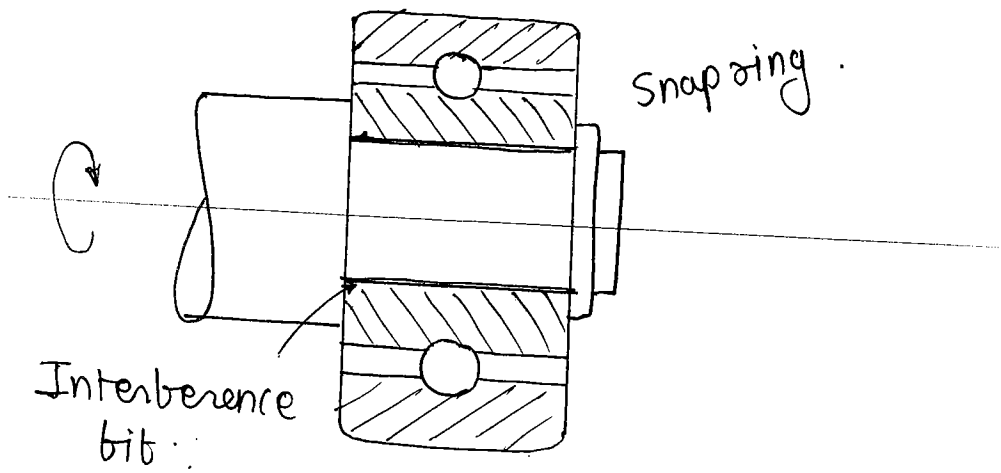
Types of ball bearings.
① Deep groove ball bearings
② Angular contact ball bearings
③ Self aligning " "
④ Thrust ball bearings.
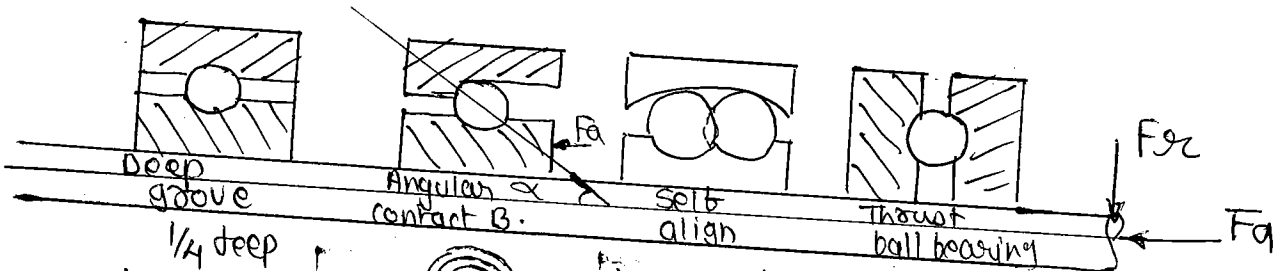
Deep groove	Angular contact B.	Self align	Thrust ball bearing
1/4 deep both side	In this case site is use to insert balls	least coff of friction	only support axial load
Radial & axial load	Design to carry more axil & Raial load.	Radial load and axial load	axial load in both direction
	$\dfrac{Fa}{Fr} > 1$	It will work in angular missalinement	
	hence two bearing is used in pairs side by side.	angular missaligment	

Roller contact bearing

1] Cylindrical Roller bearing
2] Tapered Roller bearing
3] Spherical Roller bearing
4] Needle roller bearing

Cylindrical Roller	Tapered Roller	Spherical Roller	Needle roller

FR
Fa

$$\frac{l}{d} \geqslant 4$$
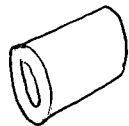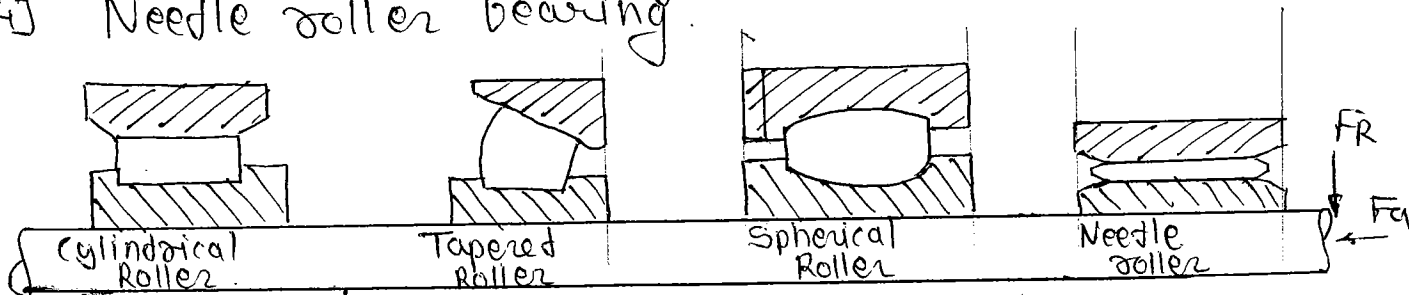
Only medium to heavy radial load

Both radial and axial load

Ex- turbine, automobile axle

Magnitude is heavy.
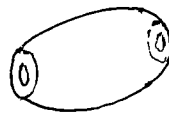
Radial and axial loads

Angular missalignment upto $\alpha = 3°$ it sustain
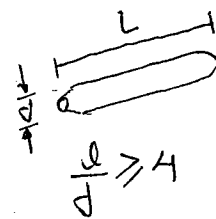
less radial space but more axial space is required

ferbect alignment is req. otherwise it is bent.

Heavy radial load and No axial load can sustain

occillating load can be sustain by this

Ex:- Hevy duty diesel engine

Piston pin bearing can be replaised by this

oil
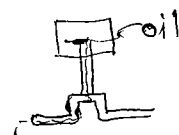
Rolling Contact bearing.

Designation.

1] SKF -

X X X X

Type of Bearing | Load carrying Capacity | X5 Inner dia or Shaft dia.

| 100 | 200 | 300 | 400 |
| Extra light | Light | Medium | heavy |
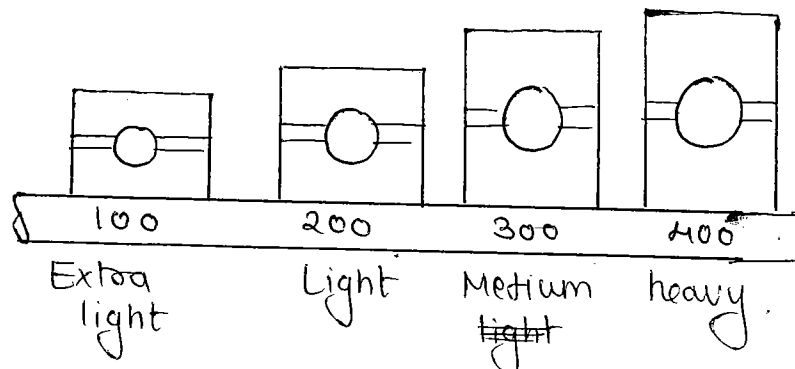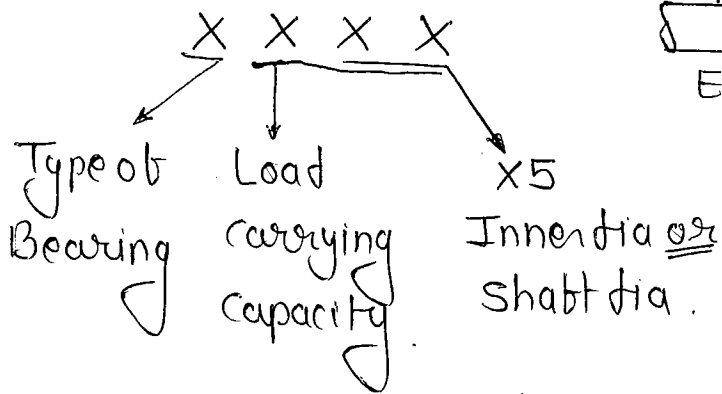
Ex:- 6307

7×5 = 35 mm dia

Medium duty

2] BIS - Antifriction Bearing manufacturing association (AFBMA)

40 B C X X .

Ball Bearing

Deep groove

Width Series

diameter Series.
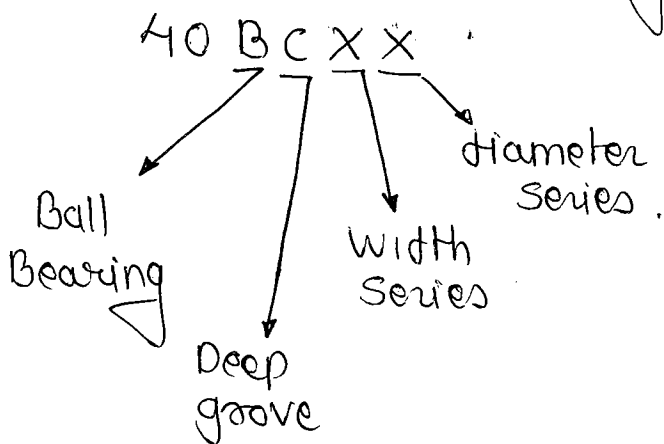
Ex:- ① 6307   07×5

BIS   35 BC 03

② 6410

BIS   50 BC 04

3) SKF
   6 112
   6 212
   6 312
   6 412

BIS   100 series

12×5 BC 01 = 60 BC 01 - light
            = 60 BC 02 - light
              60 BC 03 - Medium
              60 BC 04 - heavy.

Gr by

Designation   d   D   B   r   Co   C   N

d → Inner dia of Bearing

D → Outer dia of Bearing

B → Axil width.

r → Fillet radius

Co → Static load carrying Capacity

C → Rated load . Dynamic load carrying capacity

N → Maximum speed
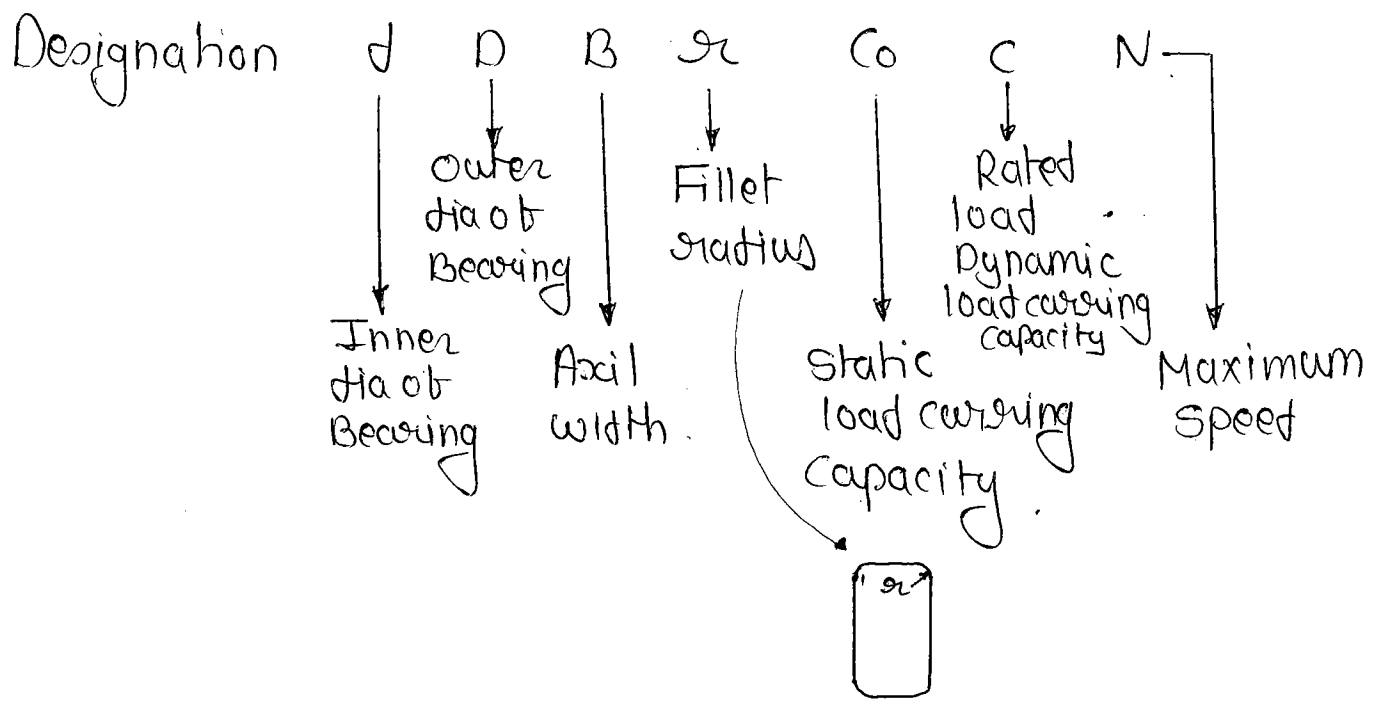
Static load carrying capacity [Co]

Load required to produce permanent deformation in the balls and races at the most heavy stress point of contact equals to 0·0001 D, where D is the dia of ball in mm.

Brinelling failure
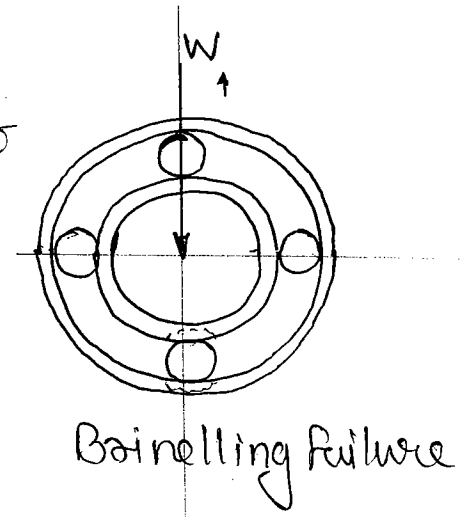
$$Co = \frac{K D^2 Z}{5} \quad \text{— For ball bearing}.$$

K = Constant depends on ball dia.
D = Dia of ball
Z = No of balls.

$$Co = \frac{K D Z L}{5} \quad \text{— For roller bearing}$$

L = length of roller.

Rated Life.

Life → No. of revolution completed before the evidence of Fatigue crack.

Rated life

L_{10} → 90% are in good condition

L_{50} → 50% are in good condition.

↓ avg life.

$L_{50} = 5 \times L_{10}$.

min load

max load

hence Fatigue Failure
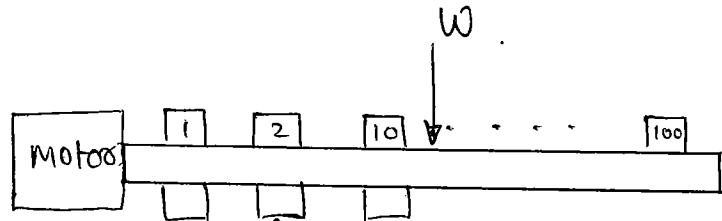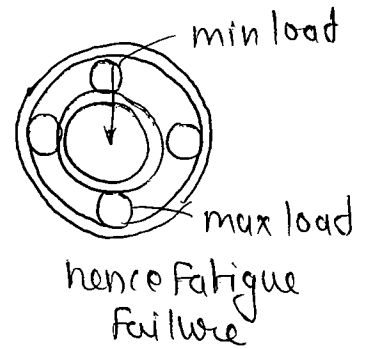
vibration sensing device

1 bearing is Fail then 2 .....
upto 10 bearing is Fail that is called L_{10} life.

The load is applied and calculated the rev. upto 10th bearing Fail. and the the load value is reduce to cover 10^6 rev. and corrosponding load is calculated

Rating Life (L_{10})

This is the life of group of apparently identical Ball or roller bearings, in number of revolution or hours at a given speed that 90% of bearings will complete or exceed before any indication of failure

Basic load Rating (c)

It is that load which a group of apparently identical bearings can withstand for a rating life of one million revolutions.

$$L = \left(\frac{C}{P}\right)^{K}$$

L = Rated libe

C = Basic load Rating

P = equivalent load.

K = 3 For Ball bearing

K = $10/3$ For Roller Bearing

Equivalent load.

$$P = (X V F_r + Y F_a) C_s$$

X = radial load factor.

V = race Rotation Factor

  = 1 For inner race rotation

  = 1.2 For outer race rotation

F_r = radial load.

Y = Axial load factor

F_a = Axial load.

C_s = Service Factor

Libe in million Revolution

$$L = \frac{60 n L_H}{10^6}$$

n = Speed rpm

L_H = Libe in hours.

9]

$\underline{6205}$ → 10·8 KN.

|
200 series .

$\underline{6305}$ → 16·2 KN (rest are less then 10·8 KN)

↓
300 series

Ans d]

10]. 6210 bearing.

3 ←

C = 22·5 KN

L = 27 million rev

P = ?

$$L = \left(\frac{C}{P}\right)^{K}$$

$$27 = \left(\frac{22·5}{P}\right)^{3}$$

P = 7·5 KN.

$$L_{50} = 5 \times L_{10} \quad \text{-Rated life.}$$

$$L = k\left(\frac{c}{p}\right)^K$$

$$P = X V F_{9t} + Y F_a.$$

$$L = \frac{60\, n\, L_H}{10^6}.$$

Pg no 67.

13] $F_{9t} = 2.5$ KN.

$F_a = F_t = 1.5$ KN.

$C_S = 1.5$.

40 hrs/week for 5years

$n = 1000$ rpm

$X = 0.56$

$Y = 1.4$

$$t = \left(\frac{c}{p}\right)^K$$

$$= \left(\frac{1.5}{2.5}\right)^3$$

$$=$$

$$L = \frac{60\, n \times L_H}{10^6} = 60 \times 1000 \times$$

$$n_H = 40 \times \frac{hr}{week} \times 52\ week \times 5years$$

$$= 10400\ hr$$

$$L = \frac{60 \times 1000 \times 10400}{10^6}$$

$$L = 624$$

$$L = \left(\frac{c}{p}\right)^K$$

$$P = (X V F_{9t} + Y F_a) S.$$

$$= (0.56 \times 1 \times 2.5 + 1.5 \times 1.4) 1.5$$

$$= 5.25\ KN.$$

$$624 = \left(\frac{c}{5.25}\right)^3$$

$$C = 44.86$$

(12)

$C = 48.545$ kN.

$LH = 6000$ hrs (Avglibe). $P = (XV F_{91} + Y F_a)G$

$n = 500$ rpm.

$$L = \frac{60 \times 500 \times 6000}{10^6}$$

$L = 180$ million rev.

$$L = \left(\frac{C}{P}\right)^K$$

$$\frac{180}{5} = \left(\frac{48.545}{P}\right)^3 = L_{10}.$$

$P = 14.7$ kN.

Bearing Subjected to a cyclic loads.

Assume a bearing is subjected to load.

P_1 at n_1 rpm   For α_1 % of time.

P_2 at n_2 rpm   For α_2 % of time.

.
:

P_x at n_x rpm   For α_x % of time.

If the bearing is subjected to only load P_1, then its libe is, $L = \left(\frac{C}{P_1}\right)^3 \times 10^6$ revolution

Libe consumed during one revolution, $\dfrac{1}{L_1} = \dfrac{P_1^3}{C^3 \times 10^6}$.

At load

P_1 speed n_1, rpm = α % of time

2000   7 min.

total revolution $N_1 = \alpha_1 n_1$ rev.

N_1 revolution , $\dfrac{N_1}{L_1} = \dfrac{N_1 P_1^3}{C^3 10^6}$.

Life consume for N_2 revolution $= \dfrac{N_2}{L_2} = \dfrac{N_2 P_2^3}{C^3 10^6}$.

$$\dfrac{N_1 P_1^3}{C^3 10^6} + \dfrac{N_2 P_2^3}{C^3 10^6} + \cdots + \dfrac{N_x P_x^3}{C^3 10^6} = \dfrac{N P_e^3}{C^3 10^6}.$$

$$N = N_1 + N_2 + N_3 + \cdots + N_x$$

$$\underset{\text{(equivalent load)}}{P_e} = \left[\dfrac{N_1 P_1^3 + N_2 P_2^3 + \cdots + N_x P_x^3}{N_1 + N_2 + \cdots + N_x} \right]^{1/3}$$

P_e = Equivalent load or Cubic mean load.

Pg no. 67.
Q 19 and 20.

Element no.	% of element time	load (KN)	Speed (rpm)
1	30	5	900
2	40	7	1440
3	30	3	720.

$N_1 = \alpha_1 n_1 = 30 \times 900$

$N_2 = \alpha_2 n_2 = 1440 \times 40$

$N_3 = 720 \times 30$.

1/3

$$P_e = \left[\dfrac{\frac{30 \times 900}{100} \times (5)^3 + \frac{40 \times 1440}{100} \times (7)^3 + \frac{30 \times 720}{100} \times (3)^3}{\frac{30 \times 900}{100} + \frac{40 \times 1440}{100} + \frac{720 \times 30}{100}} \right]$$

$$L_{10} = \left(\frac{C}{Pe}\right)^3 = \left(\frac{16.6}{6.066}\right)^3$$

$$= 20.46 \text{ million rev}$$

Compression

Hydrostatic Bearing
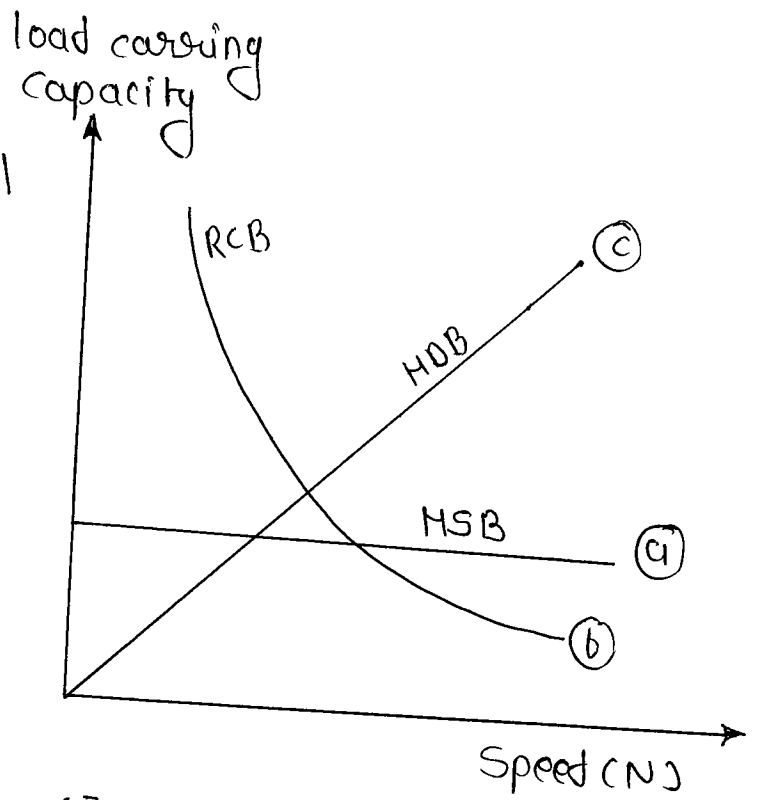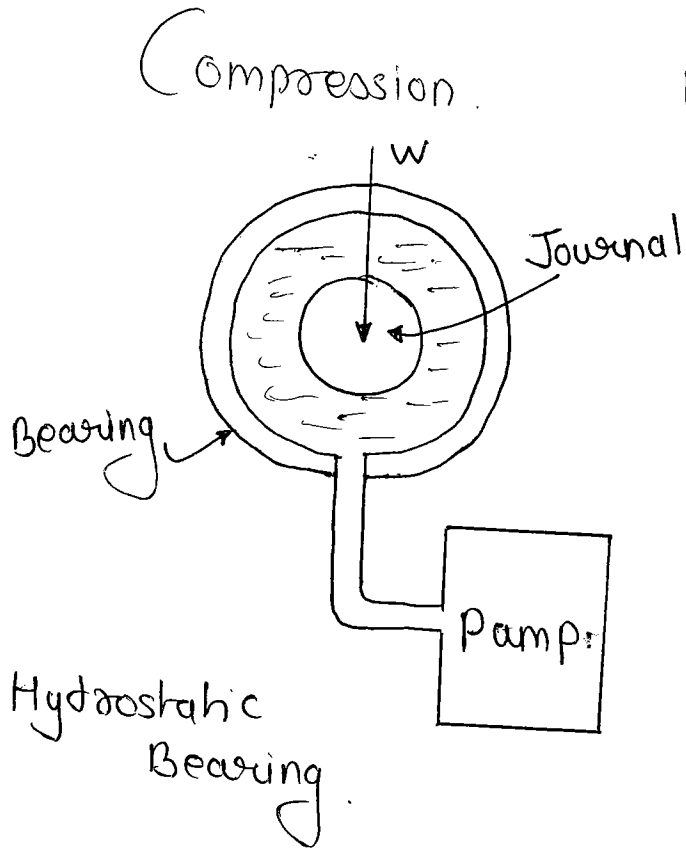
a) Hydrostatic
b) Rolling contact
c) Hydrodynamic.
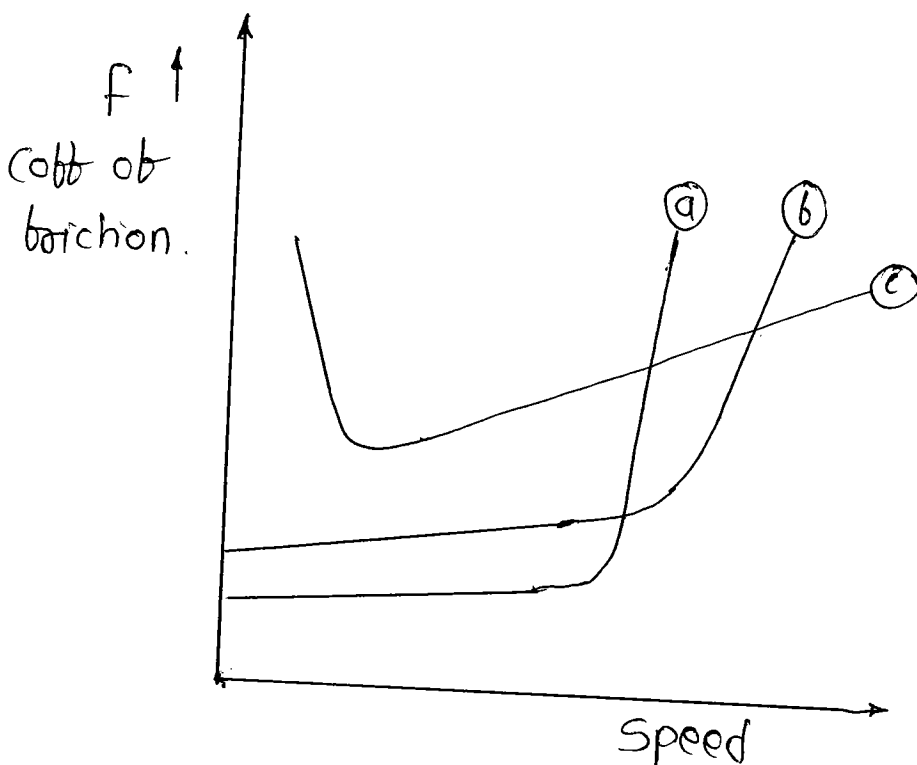
a — Ball
b — Roller
c — Hydrodynamic

3] Starting Torque.

Hydrodynamic bearing (HDB)
- High - because of high starting friction.

Rolling contact bearing (RCB)
- low - because of low starting friction.

4] Space.

→ Hydrodynamic B
More axial space, less radial

→ Rolling. C. B
More radial and less axial.

DESIGN OF BRAKES.

A brake is a device used to control the speed of a moving member by:

1] Providing suitable decceleration
2] Absorbing Energy.

Energies.

1] Kinatic energy of translation, $K \cdot E_T = \frac{1}{2} m(v_1^2 - v_2^2)$
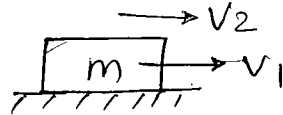
2] K.E of Rotation, $KE_R = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$
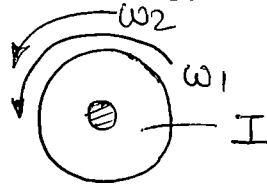
3] Potential Energy, $PE = \omega h$
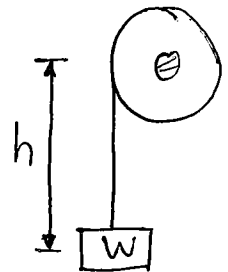
Classifications of Mechanical Brakes.

Brakes.

Radial   Axial.

Radial Force is applied

Axial Force is applied

External   Internal

Ex:- Disc Brake

Ex   Shoe brake band brake

Ex:- Internally expanding Shoe brake

Friction pad

Friction pad

steel disc
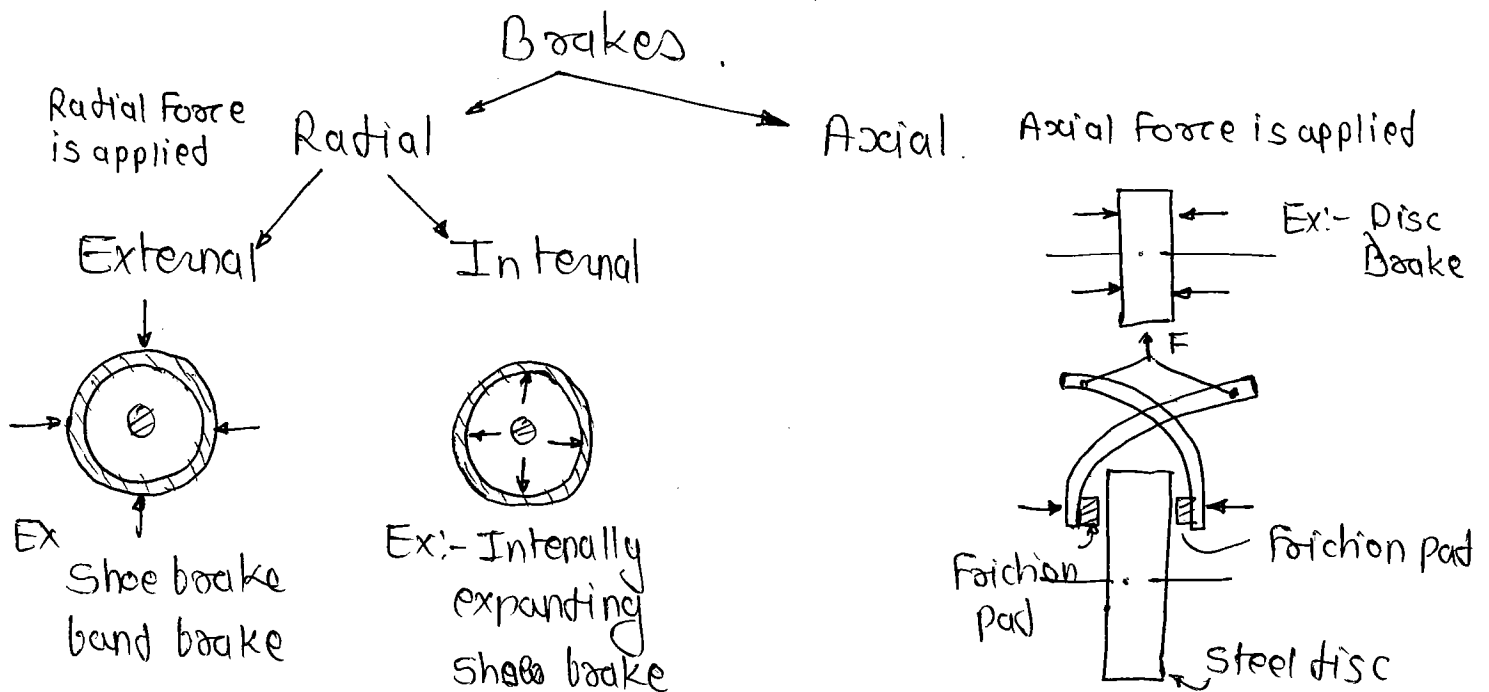
Principle of Shoe Brake.
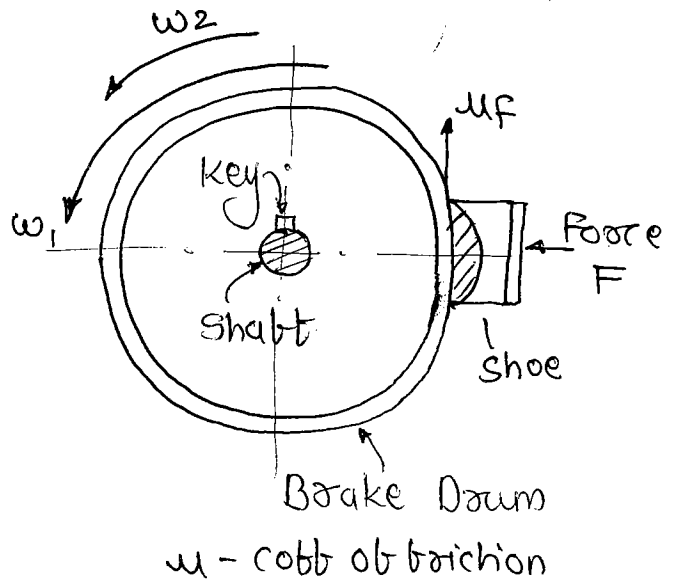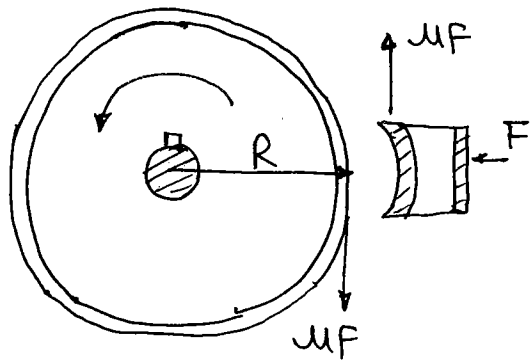
$\longrightarrow$ Friction Torque.

$$T_B = \mu F \times R .$$

$\longrightarrow$ Energy dissipated.

$$\Delta E = T_B \times \theta .$$

① $\quad v = u + at$

$$\omega_2 = \omega_1 + \alpha t .$$

$$\alpha = \frac{\omega_2 - \omega_1}{t} .$$

② $\quad v^2 = u^2 = 2as$

$$\omega_2^2 - \omega_1^2 = 2\alpha\theta .$$

$$\omega_2^2 - \omega_1^2 = 2 \times \frac{\omega_2 - \omega_1}{t} \times \theta$$

$$\theta = \left(\frac{\omega_1 + \omega_2}{2}\right) t .$$

$\longrightarrow$ Heat generated.

$$H_g = T_B \times \left(\frac{\omega_1 + \omega_2}{2}\right) t$$

$$= \mu F \times R \omega \times t$$

$$= \mu F V \times t .$$

$$H_g/sec = \mu F V$$

$$\omega = \frac{\omega_1 + \omega_2}{2}$$

$$V = R\omega$$

$p -$ Pressure.

$\mu -$ coeff of friction

Shoe or Block Brake

Case I.

$$\varepsilon M_{pivot} = 0$$

$$P \times \ell - R_N \times x = 0$$

$$P = \frac{R_N \, x}{\ell}$$

l

P = effort

2θ

μR_N
shoe

Lever

Pivot

R_N

Brake drum

Case II

$$\varepsilon M_{pivot} = 0$$

$$P \times \ell - \underbrace{R_N x}_{AM} + \underbrace{\mu R_N \times a}_{FM} = 0$$
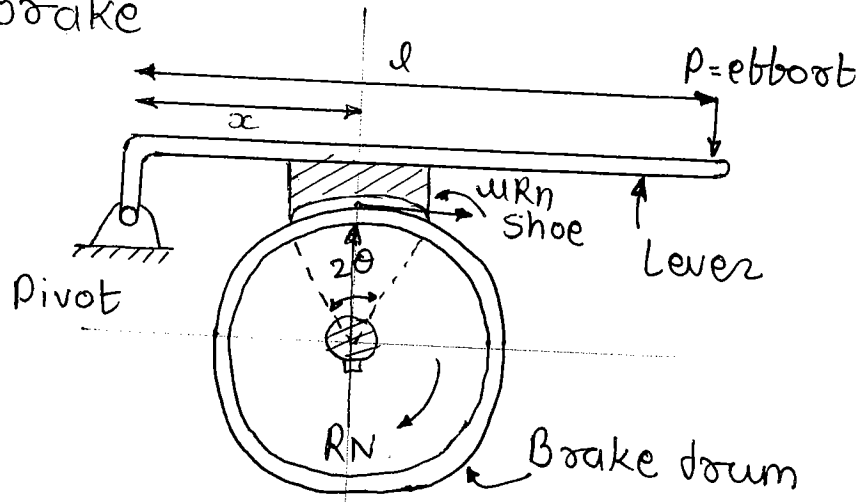
self energising

Movement of $P \times \ell$ and μR_N are in Same direction.

$$P = \frac{R_N \, (x - \mu a)}{\ell}$$

l

x

P

a

R_N

μR_N

If $\mu a > x$

P -ve.
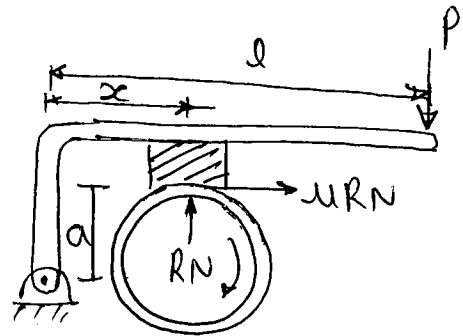
R < 0 self locking Brake.

To avoide self locking

$$x > \mu a$$

P is +ve

AM - Aplied movement
FM - Frichional movement

AM = FM = Same direction
If same magnitude
self locking.

Case III

$$P \times l - R_N x - \mu R_N \times a = 0$$

AM FM
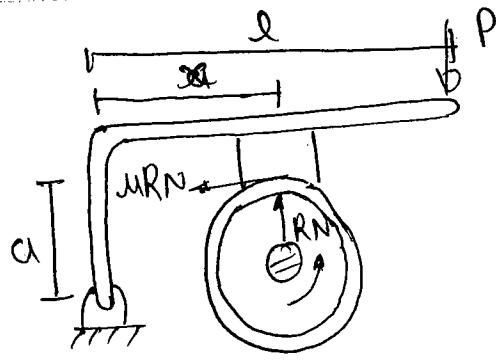
$$P = \frac{R_N(x + \mu a)}{l}$$

→ Not self energising

If $\mu a \geqslant x$ P is +ve

P < 0. – Self locking brake.

Q:– If $\mu = 0.4$ and $x = 500mm$ whatshould be the value of a to get self locking.

$$x = \mu a$$

$$a = \frac{x}{\mu} = \frac{500}{0.4}$$

$$a = 1250mm.$$

$$P = \frac{R_N(x - \mu a)}{l}$$

Short Shoe.

$2\theta \lessgtr 45°$ → Pressure is uniform

Long Shoe

$2\theta \geqslant 45°$ → Pressure in non uniform

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin2\theta}$$ μ' = Virtual coff of friction

If $\theta = 0°$.

$$\mu' = \frac{4\mu \sin0}{2 0° + \sin2 0°}$$ $\sin\theta$ is very small $\sin\theta = \theta$

$$= \frac{4\mu\theta}{2\theta + 2\theta} = \mu$$

$$\boxed{\mu' = \mu}$$

If $\theta = \pi/2$.

$$\mu' = \frac{4\mu \sin\pi/2}{2\times\frac{\pi}{2} + \sin2\times\frac{\pi}{2}} = \frac{4\mu}{\pi}$$

$$\mu' = 1.273\mu$$

$$\boxed{\mu' = 1.3\mu}$$

$$1 \lessgtr \frac{\mu'}{\mu} \lessgtr 1.3.$$

Band Brake

Rope (or) steel band coiled with

Brake drum.
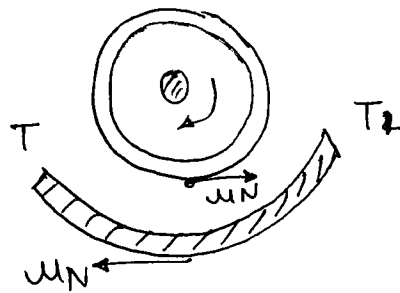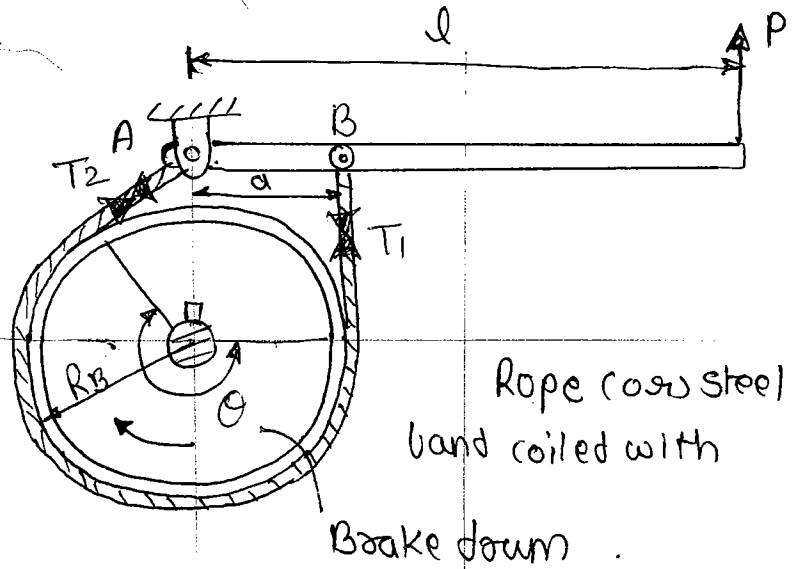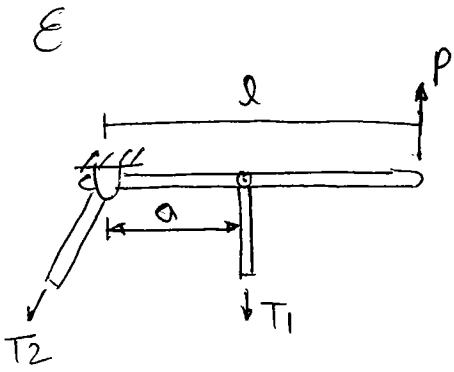
$\sum M_{pivot} = 0.$

case i) $P \times l - T_1 \times a = 0 - ① \ (\curvearrowright)$ <u>clockwise</u>
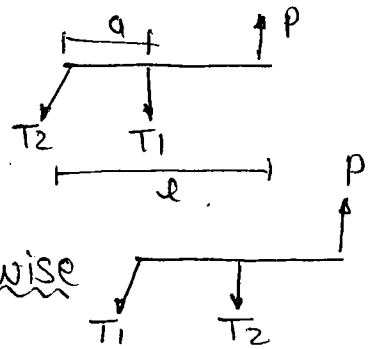
case ii) $P \times l - T_2 \times a = 0 - ② \ (\curvearrowleft)$ <u>Counter clockwise</u>

case ①

$$P = \frac{T_1 \times a}{l}$$

case ②i)

$$P = \frac{T_2 \times a}{l}$$

Not self locking type.

$$\frac{T_1}{T_2} = e^{\mu \theta}$$

(Breaking torque) $T_B = (T_1 - T_2) R_B.$

Simple band Brake.

One end of band is connected to pivot and the other end is connected to lever.

- Non self energising brake
- Not self locking brake.

In case of band brake Pressure is not uniform hence local wear take place in case of shoe brake, ware is uniform hence ~~shoe~~ band brake is not used in automotive.
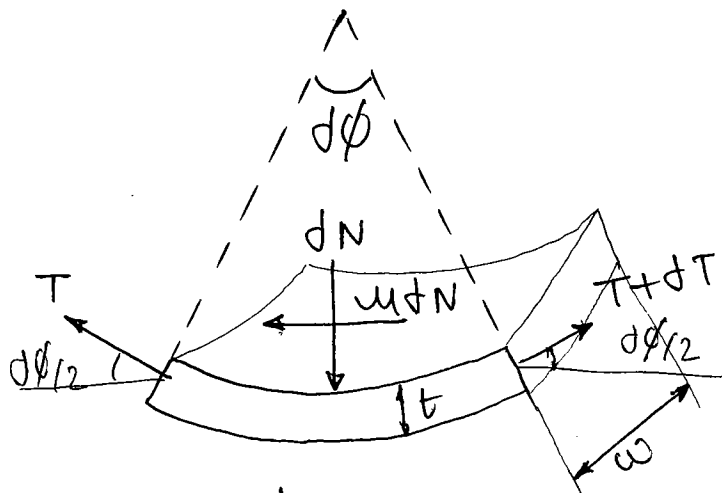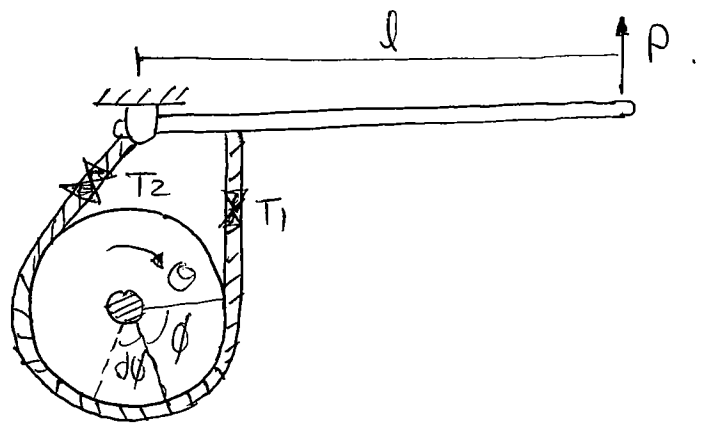
$$T \sin \frac{d\phi}{2} + (T + dT) \sin \frac{d\phi}{2} = dN.$$

$d\phi$ is very small

$$\sin \frac{d\phi}{2} = \frac{d\phi}{2}$$

$$T \frac{d\phi}{2} + \frac{d\phi}{2} = dN.$$

t = thickness

w = width

$$Td\phi = dN. - ①$$

Normal force = Pressure × area

$$dN = P \times (Rd\phi)\, \omega$$

$$dN = PRd\phi \times \omega$$

$$Td\phi = PRd\phi \times \omega$$

$$P = \dfrac{T}{\omega R}$$

P — pressure.

Max pressure

$$P_{max} = \dfrac{T_1}{WR}$$

Min Pressure

$$P_{min} = \dfrac{T_2}{WR}$$

Pg no 88

Qno 28 and 29.

$$R = 500\,mm$$
$$F = 300\,N.$$
$$\mu = 0.45.$$
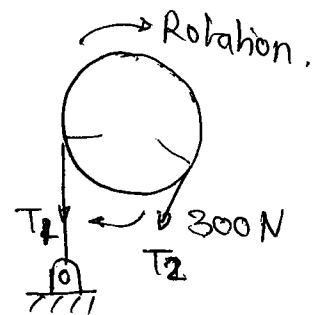
$$\dfrac{T_1}{T_2} = e^{\mu\theta}.$$

$$T_B = 800\,N$$

$$T_B = (T_1 - T_2)R_B$$

$$800 = \underbrace{(\cancel{300 - T_2})}_{T_1 - 300} \times 500 \times 10^{-3}$$

$$T_1 = \cancel{301.6}\ 1900.$$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{1900}{300} = e^{0.45\,\theta}$$

$$\ln \frac{1900}{300} = 0.45\,\theta$$

$$\text{rad } 4.101 = \theta$$

$$\boxed{\theta = 235.01^0}$$

$$P_{max} = \frac{T_1}{\omega R} = \frac{1900}{80\times10^{-3}\times500\times10^{-3}}$$

$$= 126.66\times10^3 \text{ N/m}^2$$

$$= 126.66 \text{ KN/m}^2$$

Q no. 1 and 2

Pg no 84

m = 1500 kg.
V = 108 km/hr.
S = 100 m
D = 80 cm

$T_b = ?$

$$T_b = (T_1 - T_2) R.$$

$$S = ut + \frac{1}{2}at^2.$$
$$v^2 - u^2 = 2as.$$

1500 kg

108 km/hr
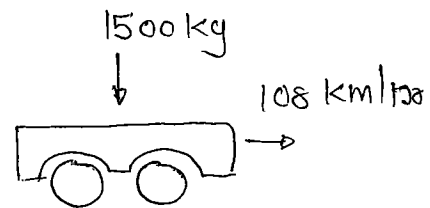
$$K \cdot E = \frac{1}{2}mv^2$$

$$= \frac{1}{2}\times1500\times\left(\frac{108\times1000}{60\times60}\right)^2$$

$$= 0.675 \text{ Kg } \frac{Km^2}{S^2}.$$

$$W \cdot D = F_t \times S.$$
$$0.675 = F_t \times S.$$
$$10^6 \times 0.675 = F_t \times 100$$
$$F_t = 6.75\times10^{+3} \text{ N}.$$

$$T_b = F_t \times d/2 = 6.75\times10^{+3}\times\frac{80\times10^{-2}}{2}$$

$$= 2.7\times10^{+3} = 2.7 \text{ KN-m}$$

② $$F_t = \mu R'.$$

$6.75 \quad \underbrace{2.7 \times 10^3}_{角} = \mu \times 1500 \times 9.81$

$$\frac{F_b}{4} = \mu \frac{R}{4}$$

$$\boxed{\mu = 0.458}.$$

Qno 3 and 4.

$$F \times 500 + -RN \times 200 \pm \mu RN \cancel{\overset{x}{150}} = 0$$

$$F \times 500 = RN \times 200$$

$$300 \cancel{5} \times 500 = RN \times 200$$

$$RN = 750N.$$

$$T = \mu RN \frac{D}{2} = 0.24 \times 750 \times \frac{300 \times 10^{-3}}{2}$$
$$= 27 \ N\text{-}m$$

200   300

$\overset{R}{N}$ 300N.

45°

F

150

100 rpm

D = 300 mm
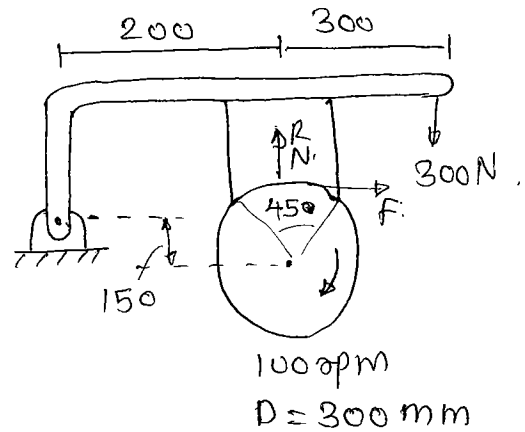
④   Heat generated

$$Hg = T \times \theta.$$
$$= T \times \left(\frac{\omega_1 + \omega_2}{2}\right) \times t.$$
$$= 27 \left(\frac{\frac{2\pi \times 100}{60}}{2} + 0\right) 5$$
$$= 706.85 \ Joules.$$

⑤.

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta}$$

$$= \frac{4 \times 0.3 \sin 45}{\left(\frac{\pi}{4} \times 2 \times \cancel{45} + \sin 90\right)}$$

do not put
45

put in rad.

$$\mu' = 0.330$$

Q 6 to 9

$D_r = 650mm$.

$D_b = 1000mm$

load = 2 tonnes.

$V = 9kmph$.

$S = 2.75m$

$\mu = 0.2$ (Four shoe)

$P = 0.3MPa$.
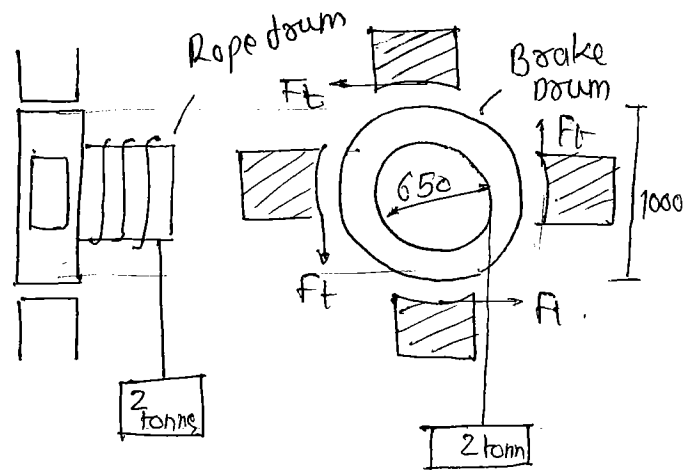
⑥ $u = \dfrac{9 \times 1000}{60 \times 60} = 2.5 m/s$.

$v^2 - u^2 = 2aS$.

$0 - (2.5)^2 = 2 \times a \times 2.75$

$a = -1.13 m/s^2$.

$F_{total} = W - (ma)$.

$= 2000 \times 9.81 - (2000 \times -1.136)$.

$= 21892.72 N$.

$= 21.89 kN$.

$F = \dfrac{\mu N}{4}$.

⑦ torque $= F_{total} \times \dfrac{D_r}{2} = F_t \times \dfrac{D_o}{2} \times 4$.

$= F_{total} \dfrac{650 \times 10^{-3}}{2} = F_t \times \dfrac{1000 \times 10^{-3}}{2} \times 4$

$= 21.89 \times \dfrac{650 \times 10^{-3}}{2} = F_t \times \dfrac{1000 \times 10^{-3}}{2} \times 4$

$\boxed{F_t = 3.55 kN}$

(8).
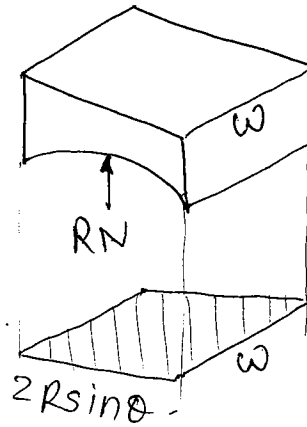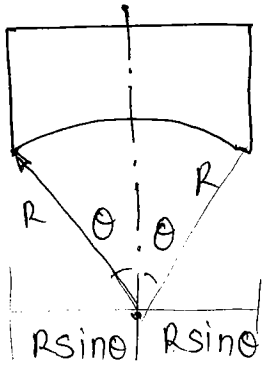
Pressure,

$$P = \frac{\text{Normal reaction}}{\text{Projected area}}$$

$$= \frac{RN}{\omega \times 2R\sin\theta} \qquad Ft = \mu RN$$

$$0.3 \times 10^6 = \frac{Ft/\mu}{\omega \times 2 \times 1000 \times 10^{-3} \times \sin\frac{45}{2}}$$

$$= \frac{17.175 \times 10^3}{\omega \times 2 \times \frac{1000 \times 10^{-3}}{2} \times \sin 45/2} \qquad \frac{N}{m^2} \quad \frac{N \times 10^6}{1000 \times 1000}$$

$$\omega = 0.1546 \, m$$

$$\boxed{\omega = 154.6 \, mm}$$

(9).

Heat generated

$$Hg = T_b \times \theta$$

$$= Ft \times \frac{D}{2} \times 4 \times \frac{\pi}{4}$$

$$= 3.55 \times \frac{1000 \times 10^{-3}}{2} \times 4 \times \frac{\pi}{2}$$

$$Hg/sec = \underset{\downarrow}{Ft} \times V \qquad \text{or } T\omega$$

$$\mu RN$$

time req. for stopping.

$V = u + at$

$0 = 2.5 - 1.136 t$

$t = 2.200 sec$

$Hg = \mu R N \times V \times t$

$T = F_{total} \times \dfrac{0.650}{2}$

$= 7.11 \ KN\text{-}m$

$Hg = T \times \theta$

$= 7.11 \times \left(\dfrac{\omega_1 + \omega_2}{2} \right) t$

$= 7.11 \times \left(\dfrac{7.69 + 0}{2} \right) \times 2.2$

$= 60.14 \ KJ$. —

X Torque $(T) = F_{total} \times R \times \omega \times t$ X

$= 21.892 \times \dfrac{1000 \times 10^{-3}}{2} \times 54.6 \times 10^{-3} \times 2.2$

$\theta = \left(\dfrac{\omega_1 + \omega_2}{2} \right) t$

$V = \dfrac{Dr}{2} \times \omega$ $\boxed{V = R\omega}$.

$\dfrac{m}{sec} 2.5 = \dfrac{650}{2} \times \omega$

$\omega_1 = 7.69 \ rad/sec$

$\omega_2 = 0$.

Q 10 and 11

$T = 450 N\text{-}m \ (\circlearrowright)$

$\Sigma M_{pivot} = 0$

$220 \times 200 + T_1 \times 50 = T_2 \times 100$

$T_1 50 - T_2 100 = -220 \times 200 - \textcircled{1}$

$T = (T_1 - T_2) R$.

$450 = (T_1 - T_2) \times \dfrac{150}{2}$.

$\dfrac{450 \times 2}{150} = T_1 - T_2$

$T_1 - T_2 = \dfrac{450 \times 2 \times 10^3}{150} = 6 \times 10^3 \ N\text{-}m \ \textcircled{2}$
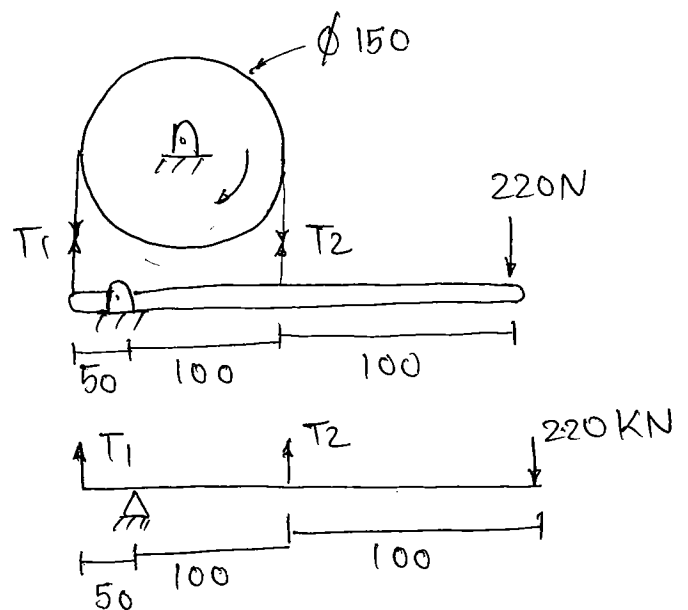
$$T_1 = 12800$$

$$T_2 = 6800$$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{12800}{6800} = e^{\mu \times \frac{\pi}{180} \times 180}$$

$$\boxed{\mu = 0.29}$$

(11) $\mu = 0.4$

$$220 \times 200 + T_2 \times 50 = T_1 \, 100$$

$$T_1 \, 100 - T_2 \, 50 = 220 \times 200 \quad \text{—①}$$
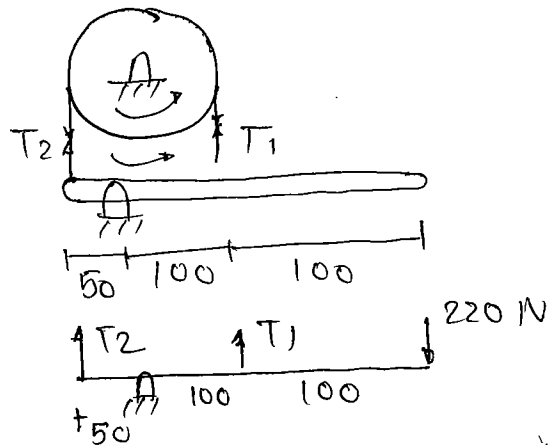
$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{T_1}{T_2} = e^{0.4 \times \pi}$$

$$T_1 = 3.51 \, T_2$$

$$T_2 = 146.179$$

$$T_1 = 513.089$$

$$T = (T_1 - T_2) \frac{150}{2}$$

$$= (513.089 - 146.179) \frac{150}{2}$$

$$= 27518.30 \; N\text{-}mm$$

$$T = 27.51 \; N\text{-}m$$

Q:- A force of 400N of is applied to the brake drum of 0.5m diameter in a band brake system as shown in fig when the wrapped angle is $180°$. If the coff. of friction is 0.25 the braking torque applied in N-m is.

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{T_1}{T_2} = e^{0.25 \times 180 \times \frac{\pi}{180}}$$

$$T_1 = 2.19 \, T_2$$

$$T_2 = 182.37$$

$$T_6 = (T_1 - T_2) \, D/2$$

$$= (400 - 182.37) \times \frac{0.5}{2}$$

$$T_b = \frac{870.49}{54.40} \, N\text{-}m$$
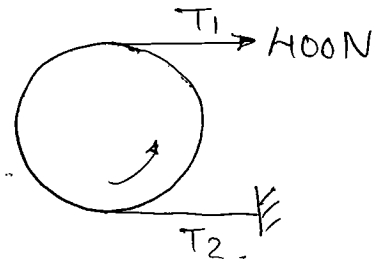
Direction of arrow in free space show T_1 direction

Differential Band Brakes.

$a > b.$

$a\theta > b\theta.$

$\boxed{a > b.}$

P effort.

Ball crank lever
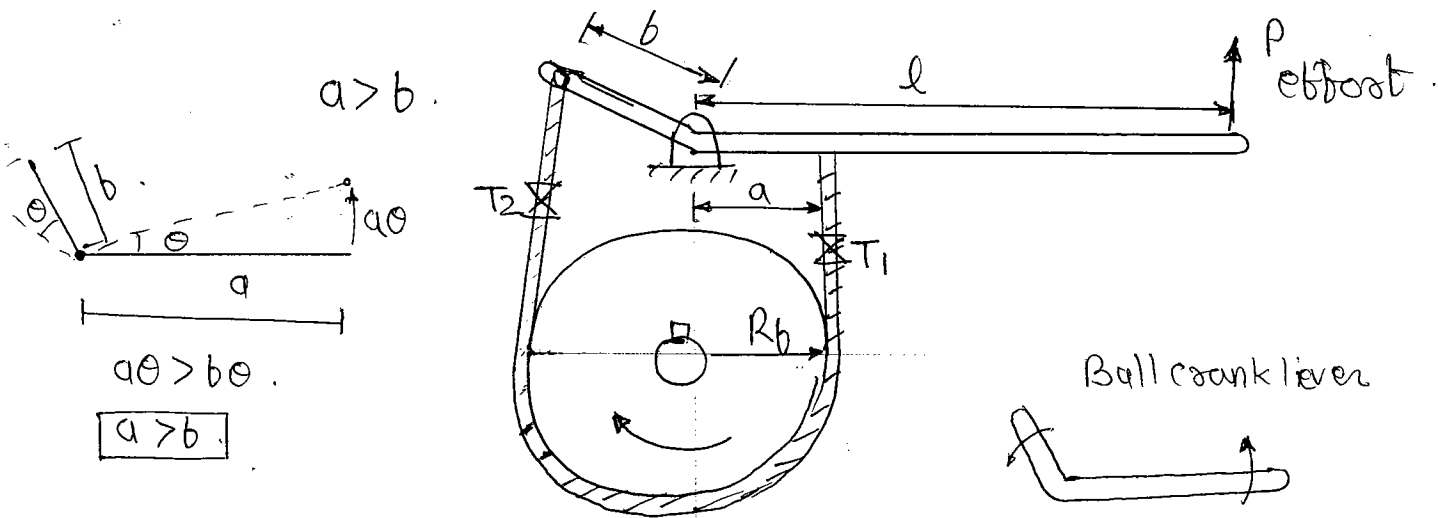
$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Torque, $T = (T_1 - T_2) R_b.$

$\sum M_{pivot} = 0.$

$$P \times l - T_1 \times a + T_2 \times b = 0.$$

$$T_1 a - T_2 b = P l.$$

For self locking $P \ngtr 0.$

$$T_1 a - T_2 b = 0.$$

$$\frac{T_1}{T_2} = \frac{b}{a} = e^{\mu\theta}$$

$b\theta > a\theta$

$b > a$
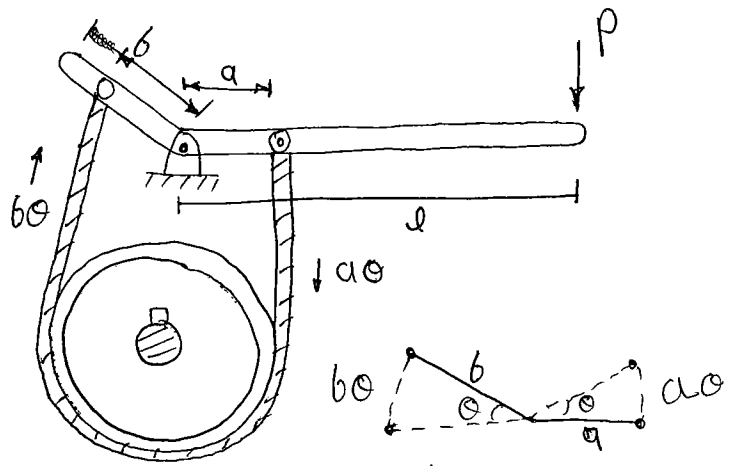
$b\theta$ is $>$ than $a\theta$ hence more

Internal Expanding Shoe Brake (Drum Brake).

$\Sigma M_{O_1} = 0$ (leading shoe)

$F \times l - N \times y + F_t \times x = 0$

Applied movement
AM

Friction movement
FM
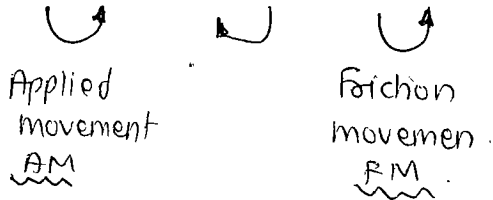
The direction of AM and FM is same hence it is self energizing brake.

Take moment about O_2.

$\Sigma M_{O_2} = 0$.

$F \times l - N \times y - F_t \times x$

AM

FM

AM and FM are in opposite direction (Trailing shoe).
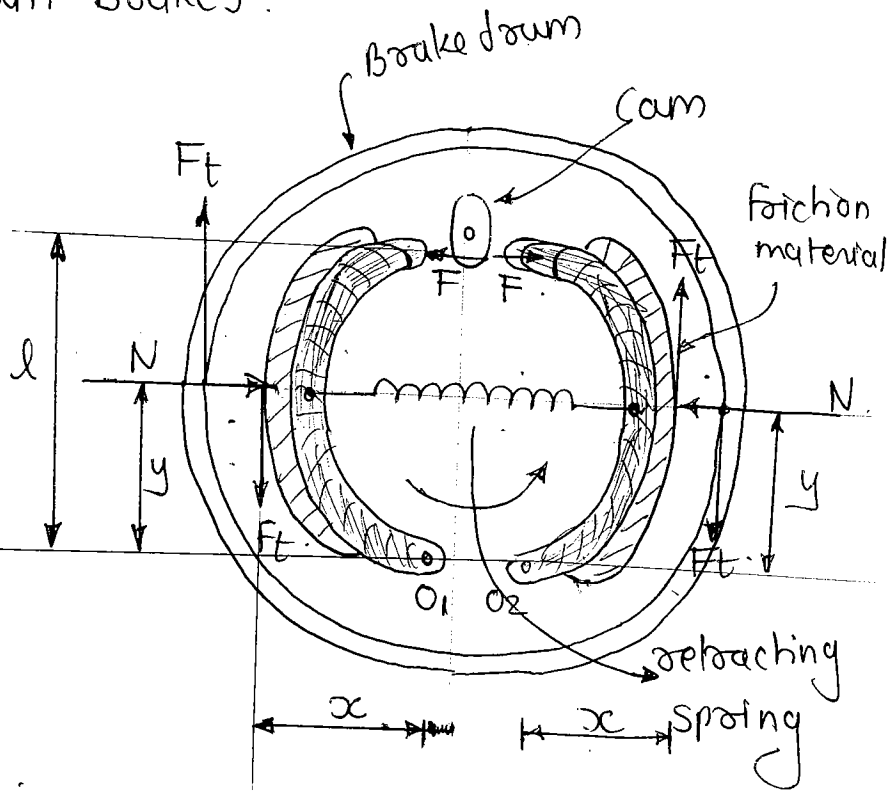
The direction of arrow between two shoe indicate leading or trailing shoe. arrow direction show the leading shoe direction between two shoe spare.

Both self-energising
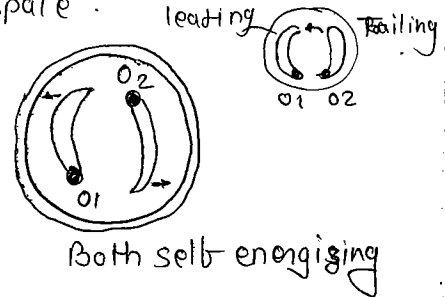
DESIGN OF CLUTCHES

A clutches is used to connect or disconnect the source of power from the remaining part of power transmittion at the will of operator.
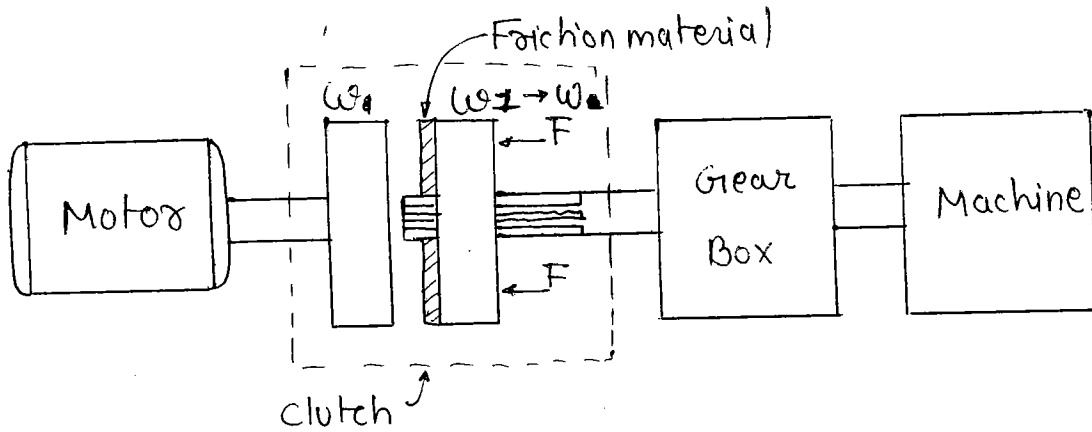
clutch

Amount of Heat generated (work done)

$$H_g = T \times \theta$$

$$\omega_2 = \omega_1 + \alpha t \quad \leftarrow \quad v = u + \alpha t$$

$$\omega = 0 + \alpha t$$

$$\alpha = \frac{\omega}{t}$$

$$F = m \cdot a$$

Torque

$$T = I \alpha$$

$$= I \times \frac{\omega}{t}$$

$$v^2 - u^2 = 2as$$

$$\omega^2 - 0^2 = 2 \times \frac{N}{t} \times \theta$$

$$I = m k^2 \quad - \quad k - \text{Radius of gyration}$$

$$= \frac{D}{2} \quad \text{For a rimmed Flywheel}$$

$$= \frac{D}{2\sqrt{2}} \quad \text{For a solid disc}$$

Types of clutches.

① Plate clutch
② Conical clutch
③ Centrifugal clutch.

Clutches
- Friction → Friction force is used and there is slip.
- Positive → No friction force is used and no slip.
  - ① Square Jaw clutch ② Spiral Jaw clutch.

Spline shaft.
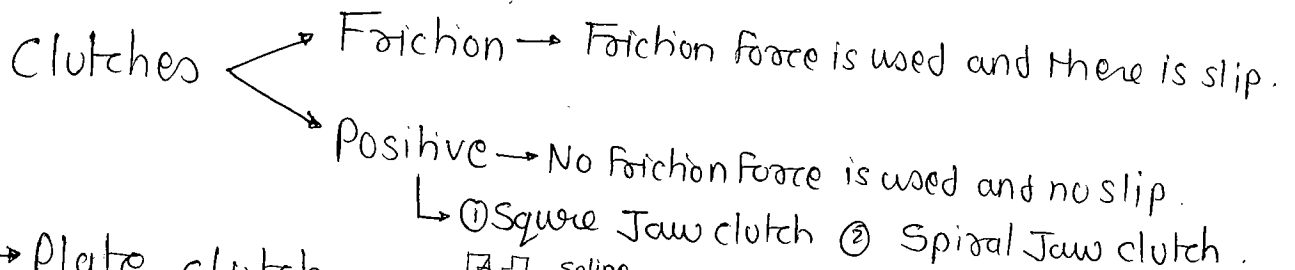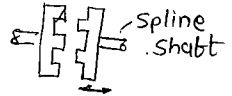
1] → Plate clutch

① Single plate
    Ex:- Used in tractors, Buses, lorries and cars.
② Multiplate clutch.
    Ex:- Machine tools, two wheelers.

2] → Conical clutch
    Ex:- Feed drives of m/c tool.

3] → Centrifugal clutch
    Ex:- Mopeds, washing m/c, coal crushers.

splineshaft
Square Jaw.

very slowspeed.

Spiral Jaw
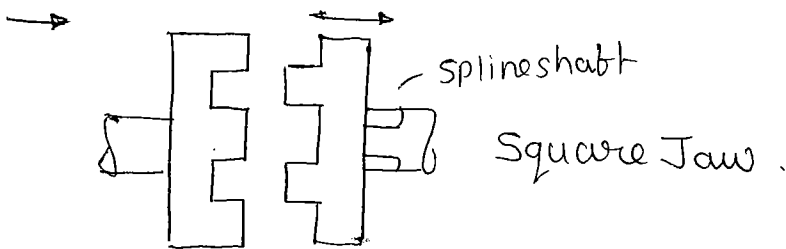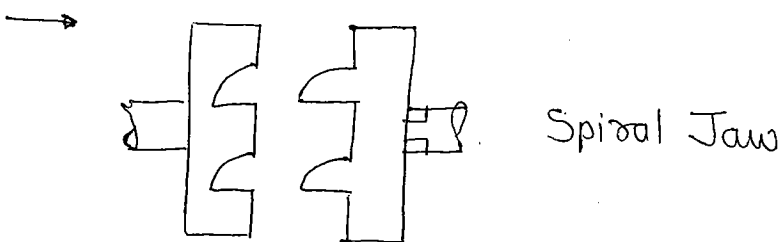
Bit high speed we can use this clutch.

2000 rpm (High speed side)   5000 rpm

motor — [clutch] — G·B — m/c
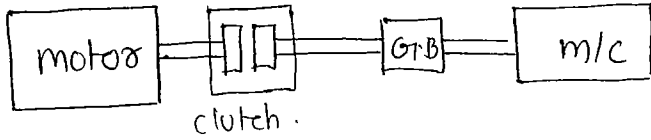
clutch.

$$P = \frac{2\pi NT}{60}$$

$P \propto NT.$

$N \uparrow T \downarrow.$

As we req. more torque if we place it abtor th G·B the size of the clultch will increse ($T \propto$ Radius of clultch) Hence we place clultch at high speed site ($N \uparrow T \downarrow$) which reduces the size of clultch.

Steel disc   spring   Ebbort.

S

S

Hob

clultch shabt

colleus

crank shabt

Pressure plate

S

S

Flywheel

P →
P →

W ←

$V_1 = r_1 \omega$
Ft
$V_2 = r_2 \omega$.

Ft

x

r_1

r_2

Ft

Ft

P →
P →
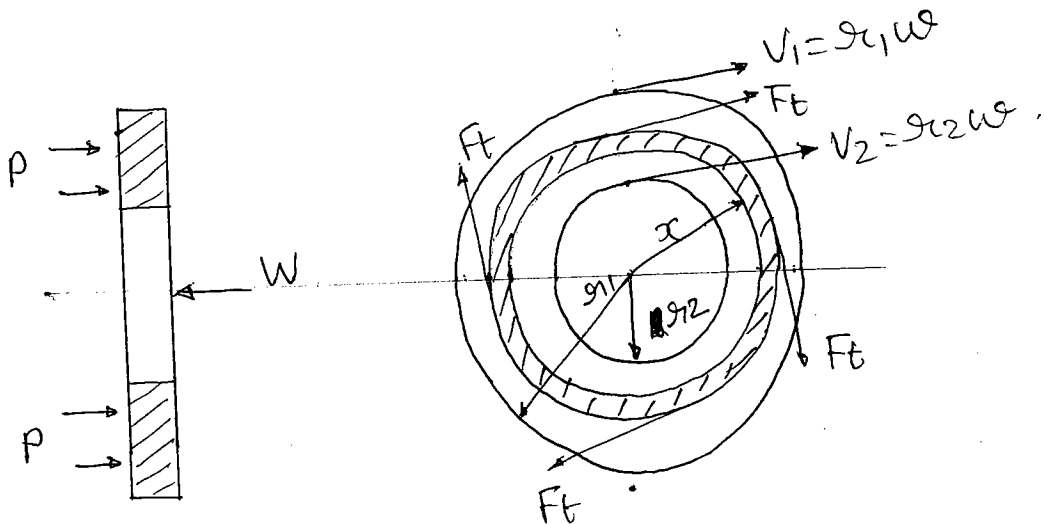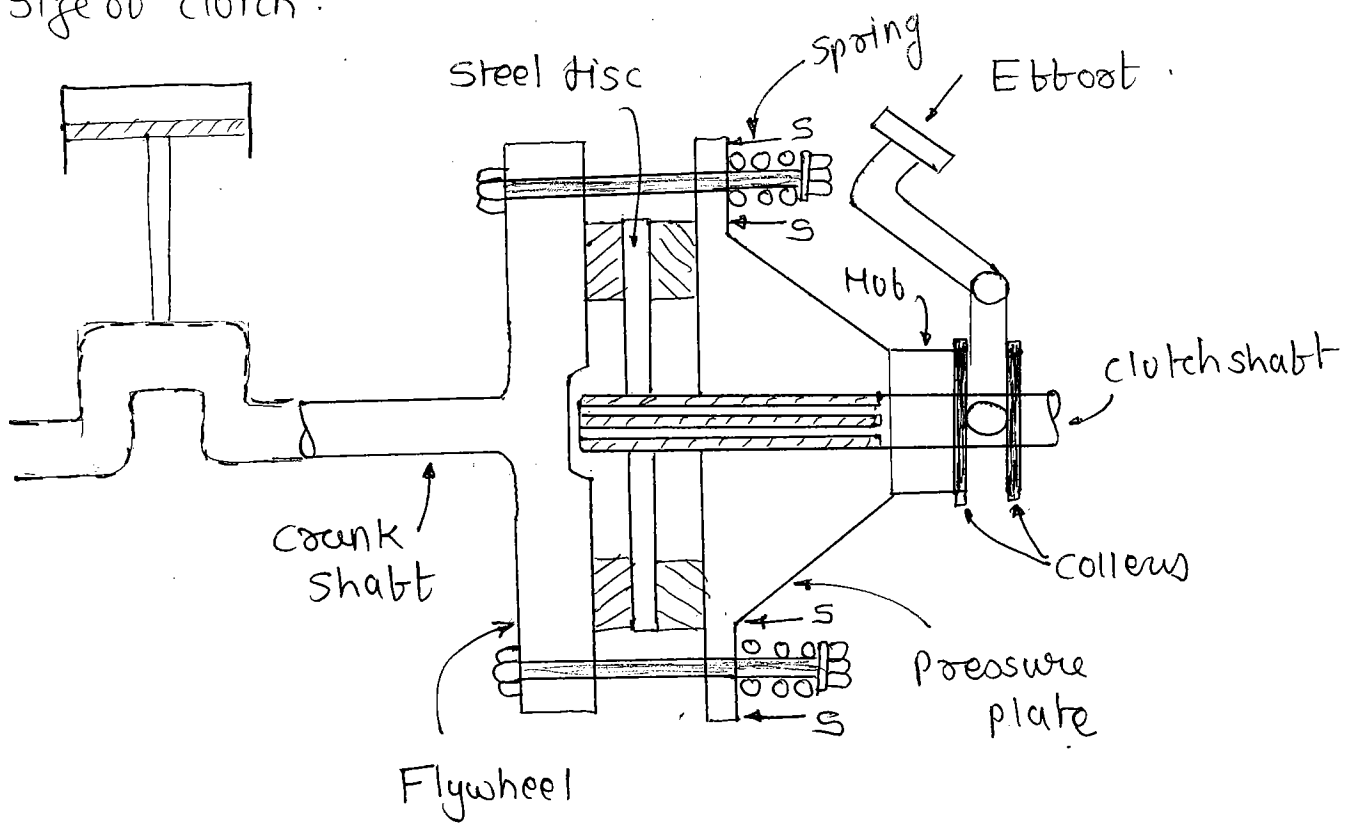
$\delta w = $ Pressure $\times$ Area

$\qquad = P \times 2\pi x\, dx$

Friction Force

$\qquad \delta F = \mu\, \delta w$

$\qquad\quad = \mu P \times 2\pi x\, dx$.

Torque

$\qquad \delta T = \delta F \times x$.

$\qquad\quad = \mu P \times 2\pi x^2\, dx$.

Pressure is not uniform or may be uniform according to clutch condition or wear portion.

1]   New clutch   —   Uniform pressure

$$T = \int_{r_1}^{} \delta T$$

$$= \int_{r_2}^{r_1} \mu P\, 2\pi x^2\, dx .$$

$$= \mu P\, 2\pi \left[x^2\, dx \right]_{r_2}^{r_1}$$

$$= 2\mu P\, \pi \left[\frac{r_1^3 - r_2^3}{3} \right]$$

$$= 2\pi \mu P \left[\frac{r_1^3 - r_2^3}{3} \right] .$$

If $\delta w = P\, 2\pi x\, dx$

$$W = \int_{r_1}^{r_2} P\, 2\pi x\, dx$$

$$W = P\pi \left[r_1^2 - r_2^2 \right]$$

$$W = P\pi \left[r_1^2 - r_2^2 \right]$$

$$P = \frac{W}{\pi \left[r_1^2 - r_2^2 \right]}$$

$$= 2\pi \mu \times \frac{W}{\pi \left[r_1^2 - r_2^2 \right]} \left[\frac{r_1^3 - r_2^3}{3} \right]$$

$$= \frac{2}{3} \mu w \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

② Used clutches — Uniform wear.

(Heat generated) $H_g = T \times \theta$.

$$H_{g/sec} = T \times \frac{\theta}{time}$$

$$" = T \times \omega$$

$$= \mu F \times r \times \omega$$

$$H_{g/sec/mm^2} = \mu F V.$$

$$= \mu \times pressure \times velocity.$$

$$H_{g/sec/mm^2} = \mu P V.$$

As $\quad$ wear $\propto H_{g/sec/mm^2}$

$$\propto \mu P V$$

$$\propto P V$$

$\therefore$ If $PV = $ constant wear is uniform.

$$PV = constant$$

$$V = r\omega$$

$$= x\omega$$

$$V \propto x.$$

$$Px = constant.$$

$$P_1 r_1 = P_2 r_2 = constant$$

$$P_{min} \, r_{max} = P_{max} \, r_{min}$$

$V_1 > V_2$
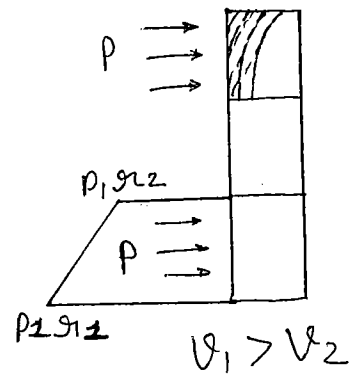
Initialy

$P_1 = P_2 \quad r_1 > r_2$

$P_1 r_1 > P_2 r_2$

After initial wear, wearing take place uniform

$P_1 r_1 = P_2 r_2 = C$

$$\delta T = \mu P 2\pi x^2 dx$$

$$T = \int_{r_2}^{r_1} \delta T$$

$$= \mu 2\pi \int_{r_2}^{r_1} P x^2 dx$$

$$= \mu 2\pi C \int_{r_2}^{r_1} x\, dx$$

$$T = \mu 2\pi C \left(\frac{r_1^2 - r_2^2}{2}\right)$$

$$\delta w = P \times 2\pi x\, dx$$

$$\delta w = \frac{C}{x} 2\pi x\, dx$$

$$W = 2\pi C \int_{r_2}^{r_1} dx$$

$$C = \frac{W}{2\pi (r_1 - r_2)}$$

$$T = \mu \times 2\pi \times \frac{W}{2\pi (r_1 - r_2)} \times \left[\frac{r_1^2 - r_2^2}{2}\right]$$

$$T = \mu W \left(\frac{r_1 + r_2}{2}\right)$$

$$T = \mu W\, r_m \qquad\qquad r_m = \frac{r_1 + r_2}{2}$$

$$T_{\text{uniform pres.}} = \frac{2}{3}\mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$$T_{\text{uniform wear}} = \mu W \left(\frac{r_1 + r_2}{2}\right)$$

For shorter period of clutch life it subjected to uniform pr. after that its wear is uniform hence consider uniform wear when condition is not given

Uniform Pressure

$$T_{up} = \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

Uniform wear

$$T_{uw} = \mu W \left(\frac{r_1 + r_2}{2} \right)$$

$$T_{up} > T_{uw}.$$

$$(r_F)_{up} > (r_F)_{uw}.$$

$$\frac{2}{3} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right] > \frac{r_1 + r_2}{2}$$

Multiplate clutch.

$$T_{wear} = \mu W \left(\frac{r_1 + r_2}{2} \right) n.$$

n = no. of pairs of surface in contact.

$$= n_1 + n_2 - 1$$
$$= 3 + 2 - 1$$
$$= 4.$$
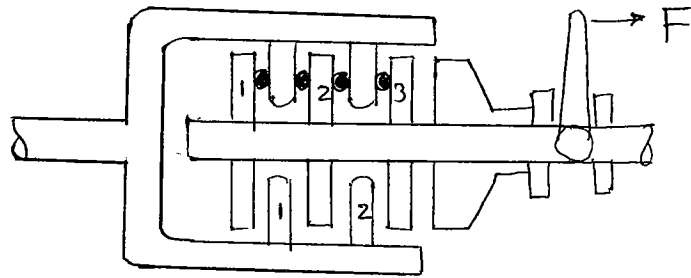
Pno 76.
Q no 6 and 7

$$r_F = 100 \, mm$$
$$\mu = 0.1$$
$$N = \frac{600}{\pi} \, rpm$$
$$W = 3 \, kN$$
$$T_{up} = ?$$
$$P = ?$$

$$T = \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

$$= \mu W R_F.$$
$$= 0.1 \times 3 \times 100$$
$$= 30 \, N\text{-}mm$$

$$P = \frac{2 \pi N T}{60} = \frac{2 \pi \times 600/\pi \times 30}{60}$$

$$P = 600 \, W.$$

$n_1 = 5$.

$n = n - 1 = 4$.

$T = 0.4 \, KN-m$

$= 0.4 \times 10^3 \, N-m$

$\mu = 0.5$

$r_1 = 150 \, mm$

$r_2 = \dfrac{200 \, mm}{2} = 100 \, mm$

load/spring.

$$T = \mu \omega \left(\frac{r_1 + r_2}{2} \right) n$$

$$0.4 \times 10^3 = 0.5 \times \omega \left(\frac{150 \times 10^{-3} + 100 \times 10^{-3}}{2} \right) 4$$
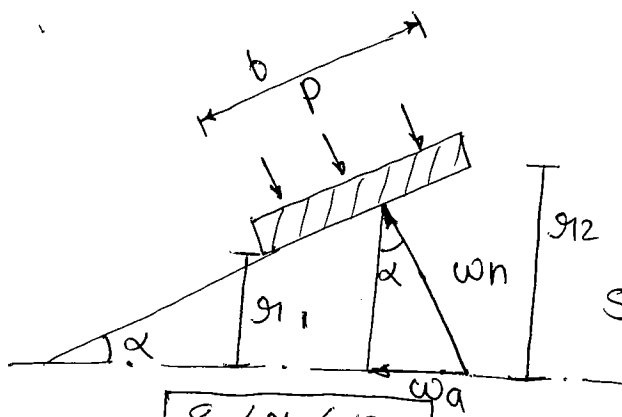
$$\omega = \frac{0.4 \times 10^3}{0.35}$$

$$\omega = \frac{\cancel{1142.85}}{1600} \, N.$$

$$\frac{load}{spring} = \frac{\omega}{no. \, of \, spring} = \frac{\omega}{4} = 400 \, N.$$

Cone / Conical Clutch.

$8 < \alpha < 12.5$ Smaller value of
α — leads to grabbing or self locking

Helical comp spring
(only one)
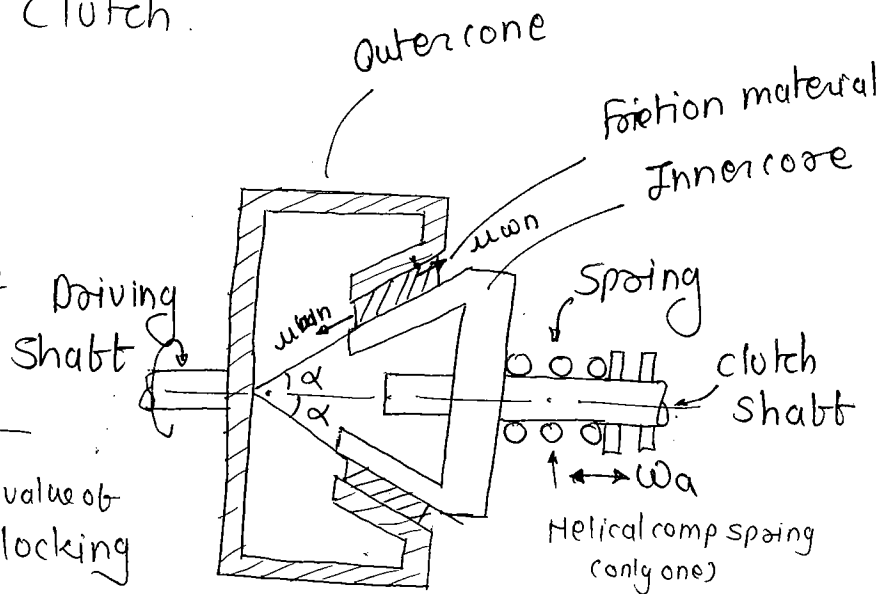
→ Uniform pressure

$$T = \frac{2}{3} \mu \omega_n \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

$$= \frac{2}{3} \mu \frac{\omega_n}{\sin \alpha} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

→ Uniform wear

$$T = \mu \omega_n \left(\frac{r_1 + r_2}{2} \right) =$$

$$= \mu \omega_n \left(\frac{r_1 + r_2}{2} \right)$$

$$\sin\alpha = \frac{r_1 - r_2}{b}$$

$$\sin\alpha = \frac{W_a}{W_n}$$

$$W_a = W_n \sin\alpha$$

W_a = Axial force applied during running.

$$T \propto \frac{1}{\sin\alpha}$$

$$\alpha \downarrow \quad \sin\alpha \downarrow \quad T \uparrow \cdots \cdots$$

Note –

① Driver and driven shaft must be co linear otherewise uneven wear take place.

②. Only one spring is used to balance the force even at reduced stiffness.

Axial force required to engauge the clutch.

$$W_{ae} = W_a + \mu W_n \cos\alpha$$

$$= W_n \sin\alpha + \mu W_n \cos\alpha$$
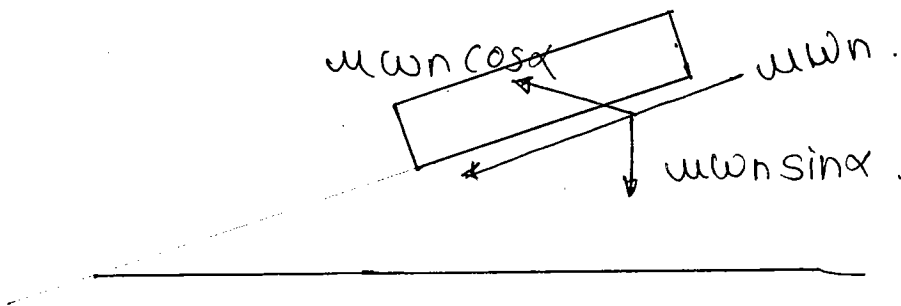
$$W_{ae} = W_n (\sin\alpha + \mu \cos\alpha)$$

Q 13 to 14.

$P = 20 \, kw$

$N = 1000 \, rpm$

$r_b = \frac{r_1 + r_2}{2} = 150 \, mm$

$2\alpha = 24°$

$\alpha = 12°$

$\mu = 0.2$

$p = 70 \times 10^3 \, N/m^2$

$T = \mu \, W_n \, r_b$

$= 0.2 \times W_n \times 150 \times 10^{-3}$ —①

$P = \frac{2\pi N T}{60}$

$T = \frac{P \times 60}{2\pi N}$ —②

① = ②

$0.2 \times W_n \times 150 \times 10^{-3} = \frac{20 \times 10^3 \times 60}{2 \times \pi \times 1000}$

$W_n = 6366.19 \, N.$

$W_{ae} = W_n (\sin\alpha + \mu\cos\alpha)$

$= 6366.19 (\sin 12 + 0.2 \cos 12)$

$W_{ae} = 2569.023$

$= 2.56 \, KN.$ Ans 6]

Pressure $= \frac{Normal \ force}{mean \ area \ of \ surface}$
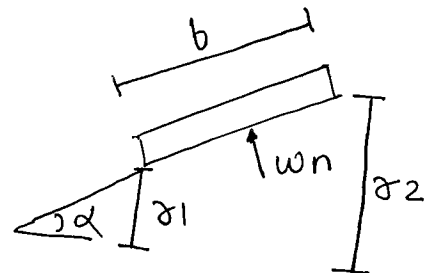
$= \frac{W_n}{2\pi r_b}$

$70 \times 10^3 = \frac{6366.19}{2\pi \times 150 \times 10^{-3} \times b}$

$b = 0.0964 \, m.$

$b = 96.49 \, mm$

Ans 6]

$r_b = r = \frac{r_1 + r_2}{2}.$

§ 38

$T_b = 140 \, Nm$ ~~$T_b = (T_1 - T_2) R$.~~

$\dfrac{T_1}{T_2} = e^{\mu\theta}$ ~~$140 = (T_2 k - T_2) R$~~

~~$140 = T_2 (k-1) R$.~~

$\dfrac{T_1}{T_2} = e^{0.2 \times \theta}$

~~$T_1 = k T_2$~~ — x —

$T = \mu \omega \left(\dfrac{r_1 + r_2}{2}\right) = \mu \times 2\pi C (r_1 - r_2) \times \left(\dfrac{r_1 + r_2}{2}\right)$

$C = P_{max} \, r_{min} = P_{min} \, r_{max}$.

~~Case 1~~ 27. $= P_{max} \, r_2 = P_{min} \times r_1$

$T = \mu \times 2\pi \times P_{max} r_2 \left(\dfrac{r_1^2 - r_2^2}{2}\right)$

$= \mu \pi P_{max} \, r_2 (r_1^2 - r_2^2)$

Case 1) $OD = 220mm, \; ID = 150mm, \; P_{max} = 0.25 \, MPa \; \mu = 0.3$

$T_b = 0.3 \times \pi \times 0.25 \times \dfrac{150}{2} \left(\left(\dfrac{220}{2}\right)^2 - \left(\dfrac{150}{2}\right)^2\right)$

$= 114.42 \times 10^3 \, N\text{-}mm$

$= 114.42 \, N\text{-}m < T$

not ~~sata~~ recomended

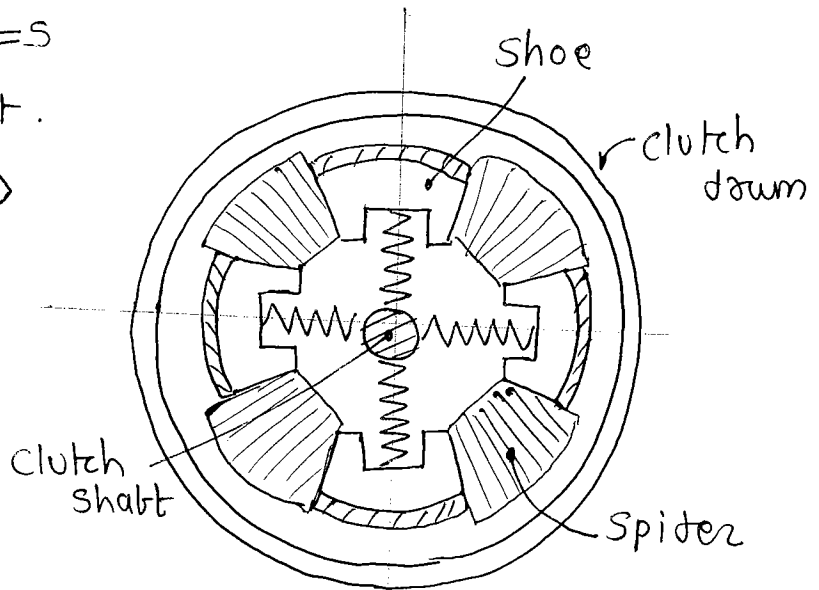Case 2] $OD = 250mm, \; OD = 150, \; P_{max} = 0.21 MPa \quad \mu = 0.3$

$T_b = 148 \, N\text{-}m > T$ recommented (very good)

Case 3] $OD = 275mm, \; OD = 150mm, \; P_{max} = 0.185 \; \mu = 0.3$

$T_b = 173 \, N\text{-}m > T$ recommented

Centrifugal Clutch.

$$F_{c2} = m \vartheta r \omega_2^2$$
$$F_{c1} = m \vartheta r \omega_1^2 = S$$
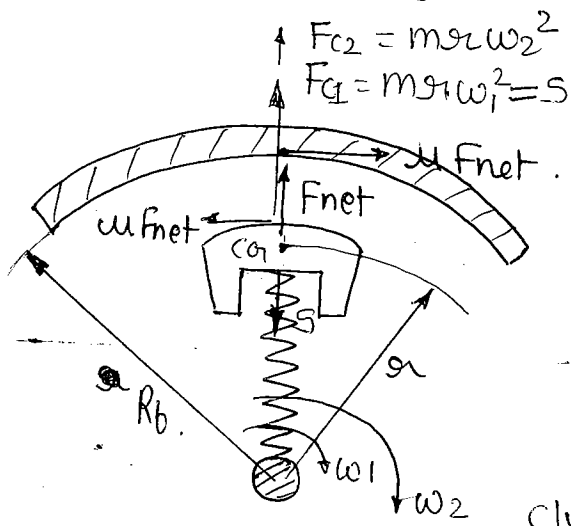
Provides time delay between starting of an engine and engagement of load

μ = coff of friction

m = mass of each shoe

ω_1 = engageing speed

ω_2 = running speed

ϑr = distance between center of shaft to C·G of mass

R_b = Inner radius of clutch drum.

$$F_{net} = F_{c2} - S$$
$$= F_{c2} - F_{c1}$$
$$= m \vartheta r (\omega_2^2 - \omega_1^2)$$

Torque

$$T = \text{Friction force} \times \text{radius}$$
$$= \mu F_{net} \times R_b$$
$$= \mu \times m \vartheta r (\omega_2^2 - \omega_1^2) \times R_b \times n$$

n = no· of shoes.

Pg no 78

Q. no 28

$T = 255 \, N\text{-}m$

$n = 4$
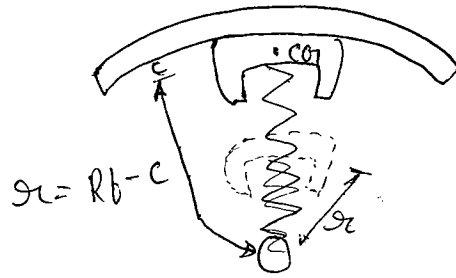
$\mu = 0.35$

$R_b = \dfrac{330}{2} = 165$

$r_1 = 140 \, mm$

$\omega_2 = 80 \, rad/sec$

$\omega_1 = 60 \, rad/sec$

$r = R_b - c$

$F_c = m r_1 \omega_1^2$

$F_{c_2} = m r \omega_2^2$

$r \lll \ggggg c$

$T = \mu \, m_{r_1} (\omega_2^2 - \omega_1^2) \times R_b \times n$

$255 = 0.35 \times m \times 140 \times 10^{-3} (80^2 - 60^2) \times 165 \times 10^{-3} \times 4$

$$\boxed{m = 2.48 \, kg}$$

GEARS

A gear is a toothed wheel used to transmit power at constant angular velocity ratio.

$$P = \frac{2\pi NT}{60}$$

$$P \propto NT.$$

$N \downarrow T \uparrow$

- Positive Drive (No slip)
- Short center distance.

Types of Gears.

1] Spur gear

① Tooth are cut parallel to axis

② Joins coplaner, parallel shaft.
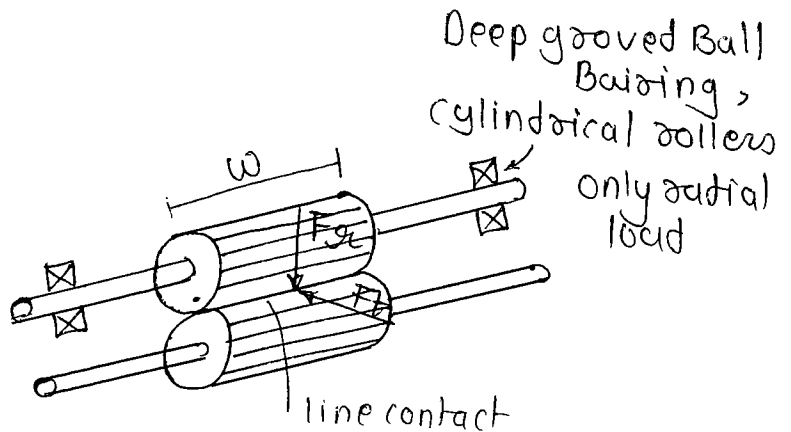
③ Line contact, sudden engagement and disengagement

④ Only radial loads.

⑤ Velocity ratio 3:1 to 6:1

Deep grooved Ball Bearing,
Cylindrical rollers
only radial load

line contact

2] Helical gear

- Tooth are cut at an angle to the axis.
- Joins coplaner parallel shaft
- Initially point contact but gradually become line contact
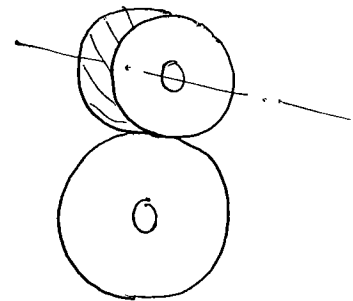- Velocity ratio 6:1 to 10:1
- loads both axial and radial

Tapered roller Bearing
Angle contact bearing

Point Contact

F_a
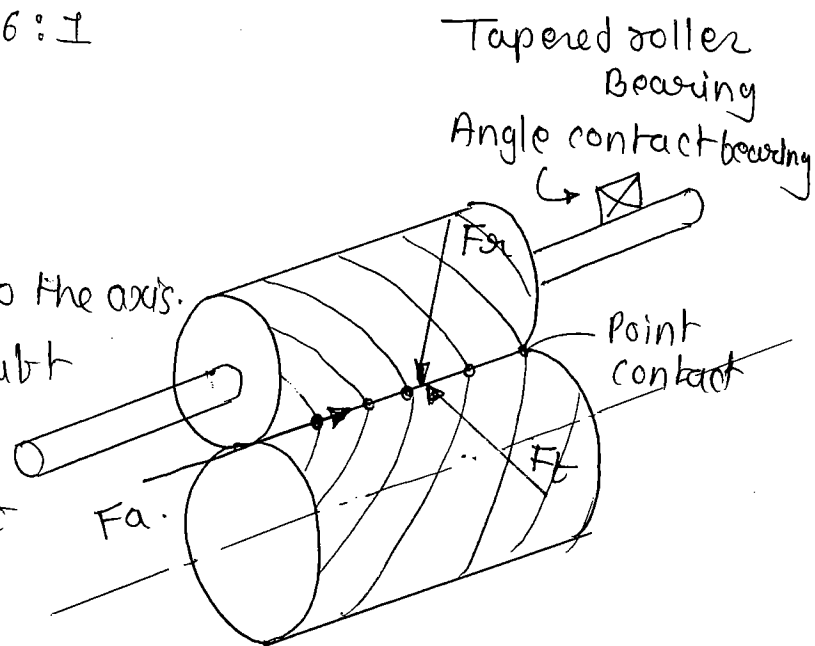
2] Herringbone gears.
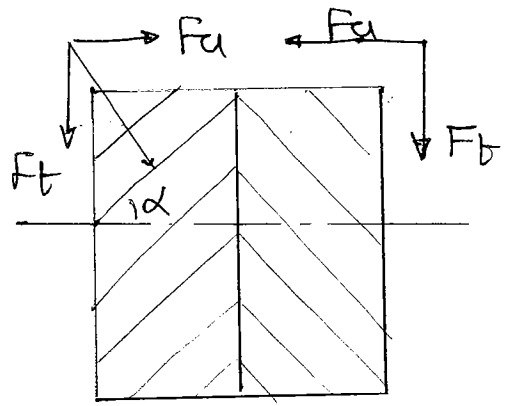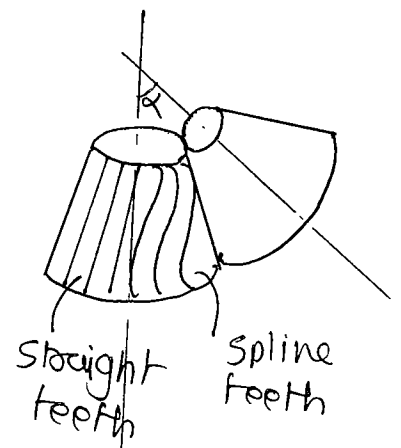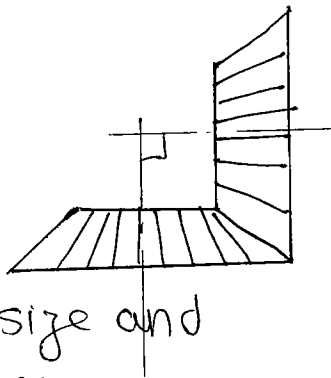

axial Force = 0.

3] Spiral gear / crossed helical gear.

- Joins non-coplenar and non
    interseching shafts (skew shaft)

- Always point contact.
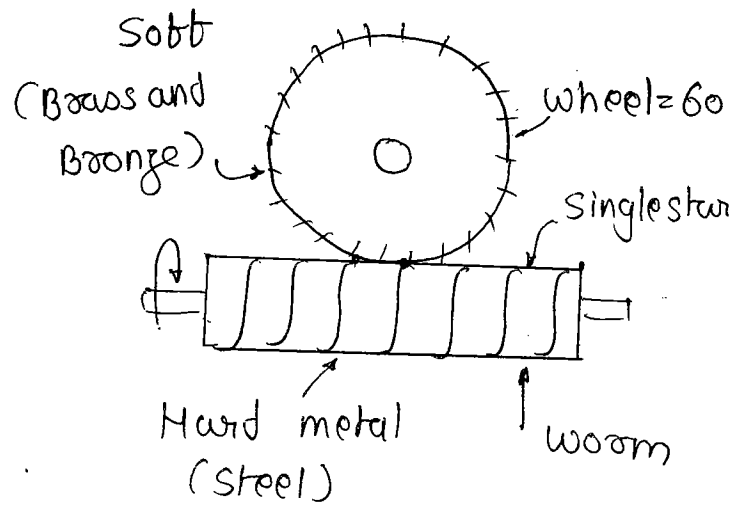
- Suitable for light loads.

4]

- Transmit power at angle
- Radial and axial loads
- Two bevel gear of equal size and
   at 90° are called mitre gear
- ∧ line contact.
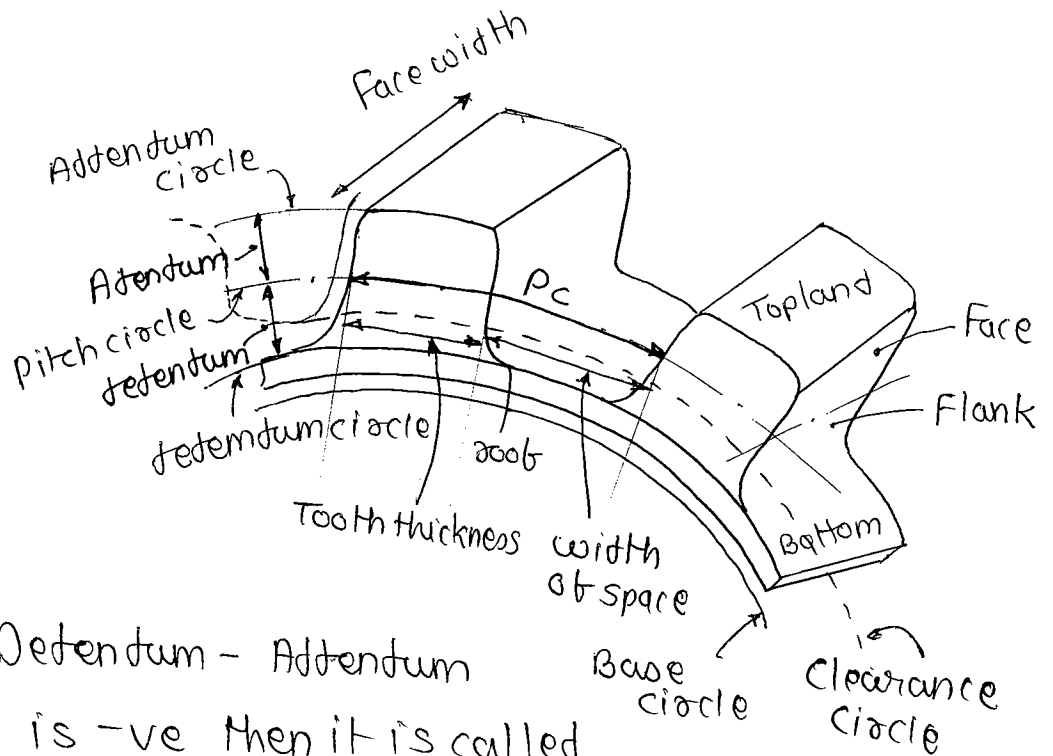

straight teeth

spline teeth

5] Woom gear

- Velocity ratio
  $$= \frac{1}{\text{No of teeth of wheel}}$$
- Skew shaft
- Velocity ratio upto 100 : 1 .
- Initially point contact than line contact.

Soft (Brass and Bronze), wheel=60, single star, Hard metal (Steel), woom

SPUR GEAR .

Gear Nomenclature .

Face width, Addendum circle, Addendum, pitch circle, Addendum, dedendum, dedendum circle, Tooth thickness, width of space, root, Pc, Topland, Face, Flank, Bottom, Base circle, Clearance Circle

Clearance = Dedendum − Addendum
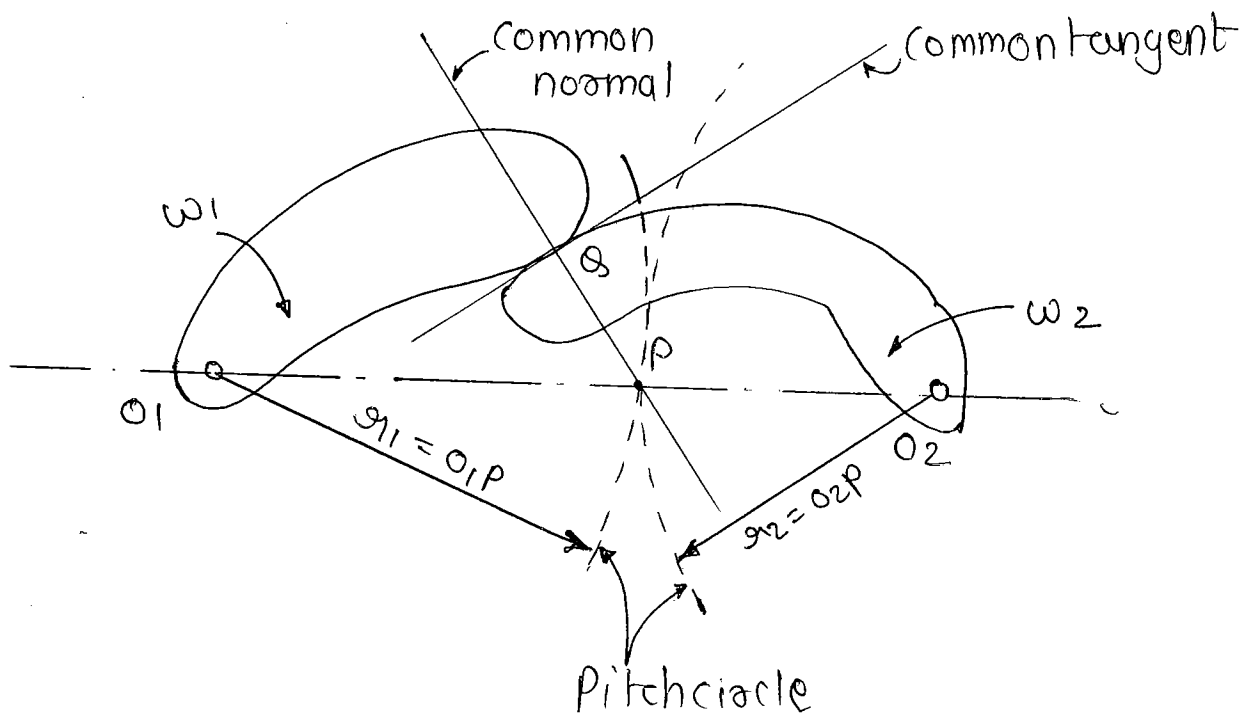If clearance is −ve then it is called interbariance .
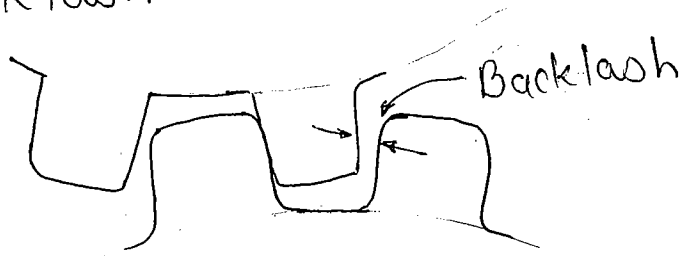
Backlash

Backlash
= Width of space − Tooth thickness

Law of gearing.

The common normal at the point of contact always passes through a fixed point on the line joining centers and devites the line joining centers in the inverse ratio of thier angular velocities.

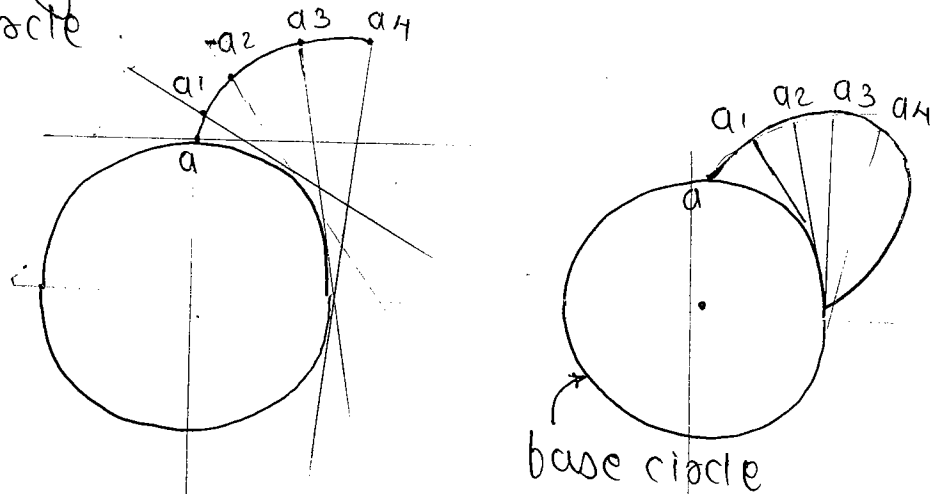$$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{O_2 P}{O_1 P}$$

Pitch line velocity, $V = r_1 \omega_1 = r_2 \omega_2$

Conjugate Problem → (Involute, Cycloid).

- Satisfy low of gearing.
- Maintain Constant angular velocity ratio.
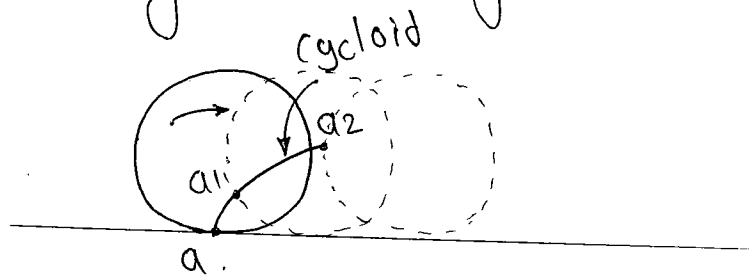
Involute.

The locus of a point on a straight line which rolls without slipping on a circle is called involute circle is called base circle.
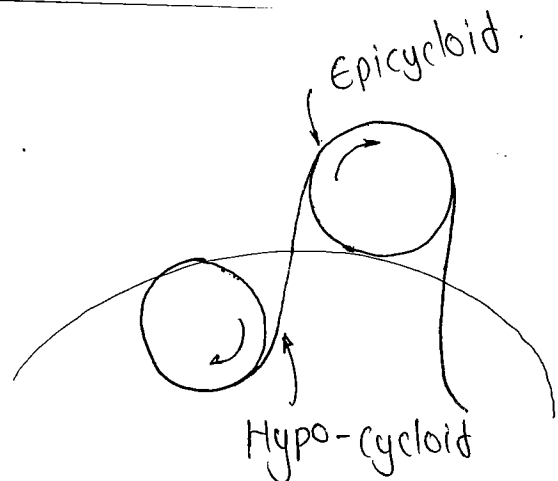

base circle

Cycloid

The locus of a point on a circle which rolls without slipping on a straight line is called cycloid.

cycloid

- More strength.
- zero interbarence.
- Less ware.
- Difficult to cut
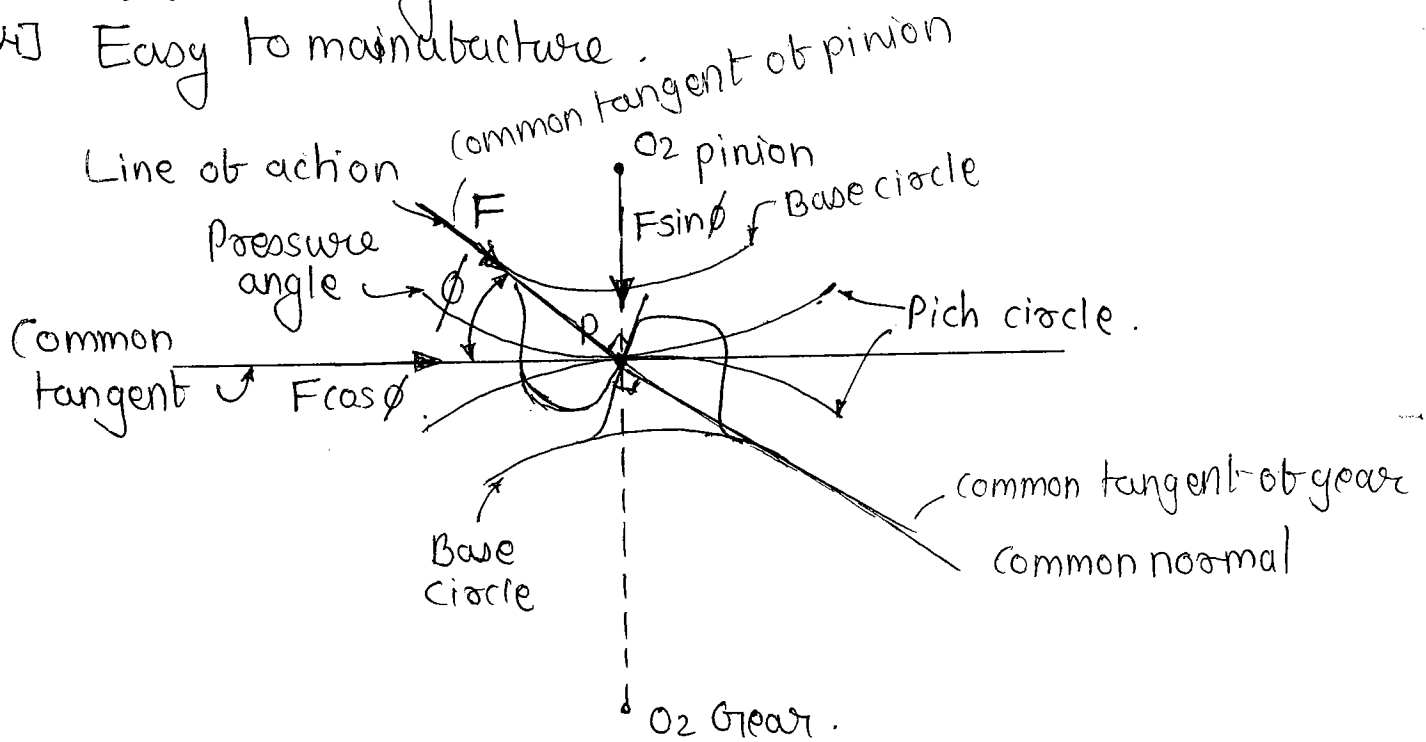

Epicycloid.

Hypo-cycloid

Advantages of involute

1] Conjugate in action

2] Constant pressure angle (ϕ)
(Pressure angle is zero at the pitch point and is maximum at tips for cycloid)

3] Small variation in the center distance will not effect velocity ratio.

4] Easy to manufacture.

Pressure angle (ϕ)

- Angle between common normal and common tangent
- Angle between common tangent to base circle and common tangent to pitch circle
- Angle b/w line of action and common tangent is pressure angle.
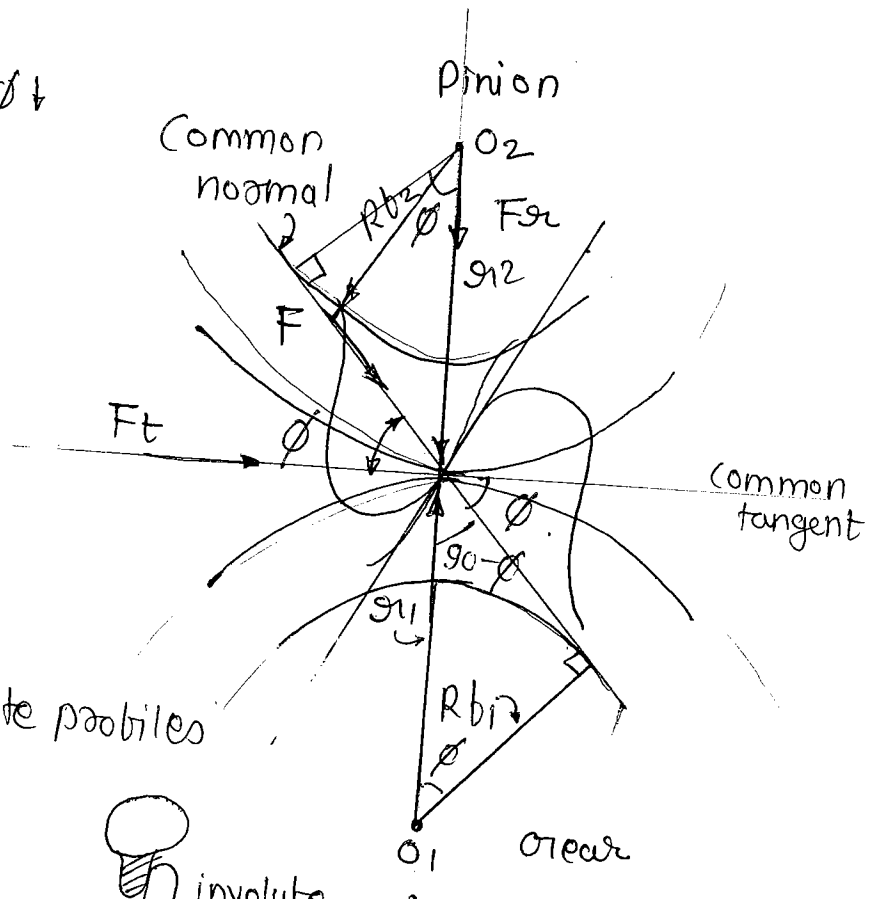
$$F_t = F \cos \phi$$
$$F_R = F \sin \phi$$
Torque, $T = F_t \times R$

$$T \uparrow \quad F_t \uparrow \quad \cos \phi \uparrow \quad \phi \downarrow$$

$\phi = 14\frac{1}{2}$ obsolute
$= 20°$ — widely used
$= 25°$

$$F_R = F\sin\phi$$
$$F_R \downarrow \quad \sin\phi \downarrow \quad \phi \downarrow$$

Pinion

Common normal

O_2

Rb_2 ϕ

F_R

$\mathcal{R}_2$

F

F_t

ϕ

common tangent

$90 - \phi$

$\mathcal{R}_1$

Rb_1

ϕ

O_1 gear

Interference

① Negative clearance

② Meshing of non conjugate profiles

Involute

non involute

velocity ratio in not constant

involute

constant velocity

$$F_t = F\cos\phi$$
$$\phi \downarrow \quad F_t \uparrow$$

$$\mathcal{R}b = \mathcal{R}\cos\phi$$
$$\mathcal{R}b \quad \phi \uparrow \quad \cos\phi \downarrow$$

we cant give or put $\phi = 0$

Then $\boxed{\mathcal{R}b = \mathcal{R}}$ then profile become

Involute

Pitch circle

non involute

$\mathcal{R}$
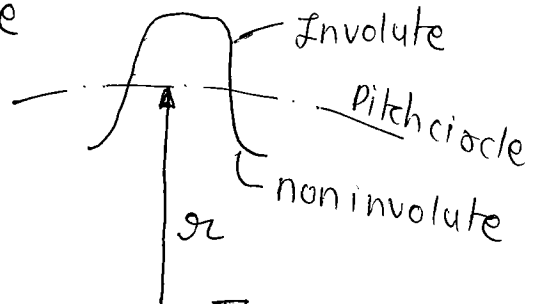
$$\phi = 20°$$
$$\mathcal{R}b = \mathcal{R}\cos 20°$$
$$\mathcal{R}b = 0.939\mathcal{R}$$
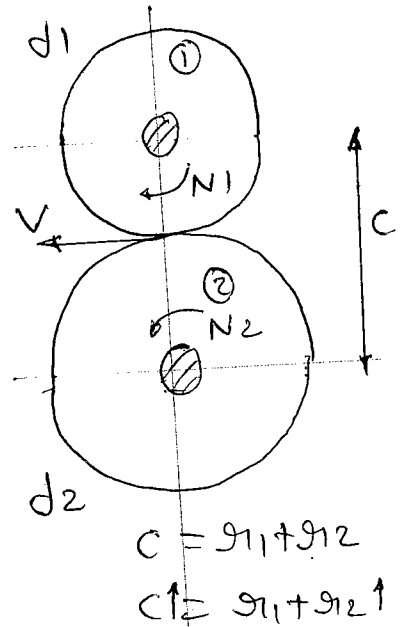
This cause Interference hence velocity ratio is not constant

If $\phi = c$

③ Small variation. in the center distance doesnot change v·P
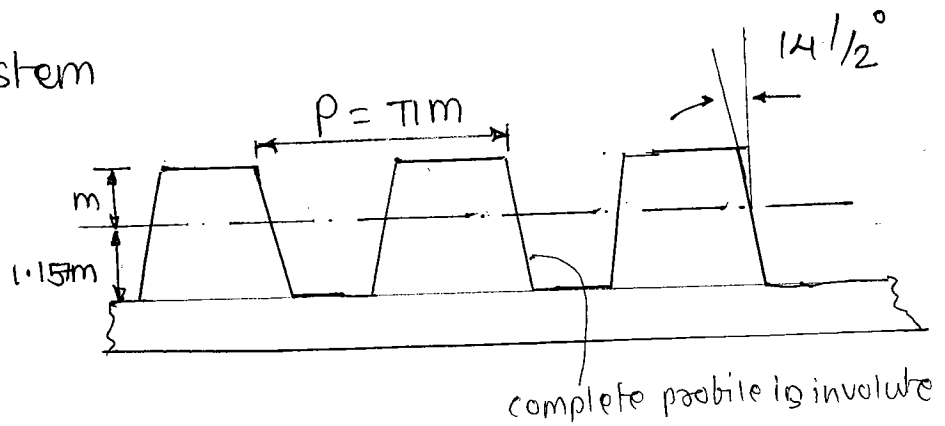
$$V = \frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60}$$

$$V \cdot R = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{r_1}{r_2}$$

Ex:- $C = 100mm$ - design

but actual assembly time

$C = 98 mm$

$C = r_1 + r_2$

$C' = r_1 + r_2 \uparrow$

Gear System

$P = \pi m$

$14 \frac{1}{2}°$

complete profile is involute

① $14\frac{1}{2}$ Full depth involute
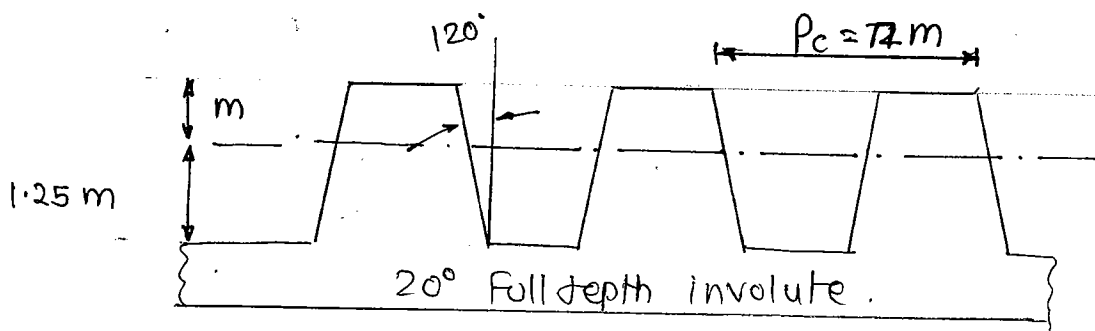
②→ Pitch of gear

① Diametral pitch, $P_d = \frac{No. of\ teeth}{pitch\ circle\ dia.} = \frac{T}{D}$ is difficult

② Circular pitch, $P = \frac{\pi D}{T}$ ⟵ difficult to express

     D- Pitch circle dia
     T- no. of teeth
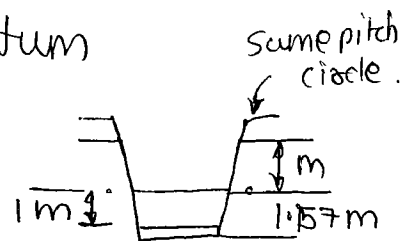
③ Module, $m = \frac{D}{T}$ mm

     $= \frac{1}{\wedge}$

Diagram: gear tooth profile with 120° angle, m, $1.25\,m$, $P_c = 7.2\,m$, labeled "20° Full depth involute."

① $14\frac{1}{2}$ Full depth involute

$$\text{Total depth} = \text{Addendum} + \text{Dedentum}$$
$$= m + 1.257m = 2.157m$$

$$\text{Working depth} = \text{addendum} + \text{addendum}$$
$$= 2m$$

Same pitch circle.

m

$1\,m$ $1.57\,m$

$$\text{clearance}, \ c = 1.157m - 1m = 0.157\,m.$$

② $14\frac{1}{2}$ Composite teeth.

Cycloid

m

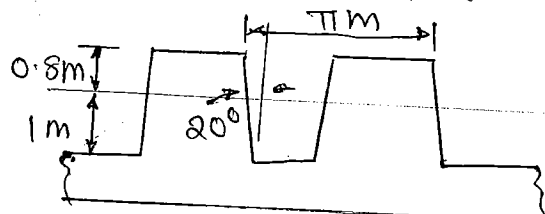$1.157m$

Involute

To avoid interference
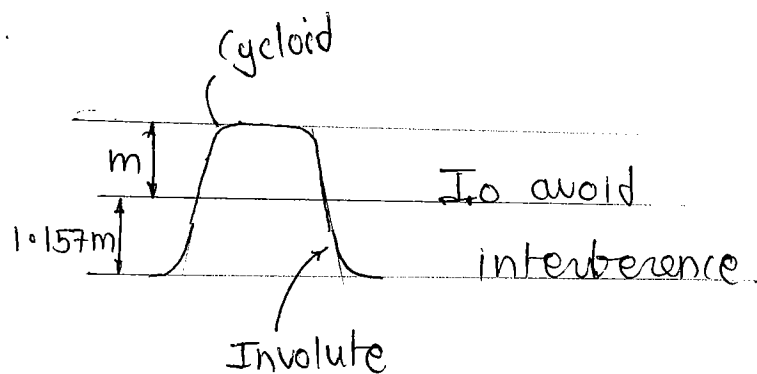
③ 20° Full depth involute

only clearance $= 0.25\,m$

④ 20° Stub involute

To reduce the bending stress because of high tooth hight in 20° Full depth we use 20° stub involute.
there is hight, limit to maintain the contact surface.

$$= 1 - 0.8$$
$$\text{Clearance} = 0.2\,m$$
$$\text{working depth} = 1.8\,m$$

Diagram: $0.8m$, $1\,m$, $20°$, πm

To avoid interference the minimum no of teeth on pinion

$$T_{min} = \frac{2}{\sin^2 \phi}$$

For $14\frac{1}{2}^{\circ} \longrightarrow T_{min} = 31.9$
$$\approx 32$$

$20^{\circ} \longrightarrow T_{min} = 17.09$
$$\approx 18$$

$25^{\circ}, \longrightarrow T_{min} = 12$.

Tooth thickness

$$t = \frac{P_c}{2} = \frac{\pi m}{2} = \frac{\pi}{2} \times \frac{D}{T}.$$

$$t = \frac{\pi D}{2T}$$

$t \uparrow \quad T \downarrow \quad \phi \uparrow \quad \cdots \cdots \quad T_{min} \approx \frac{2}{\sin \phi}$

6-9~ PF
9:30~1~ MD
English
HT.

** SAJIT CHINTAMKAR

Accepted that

NOT FALL IN LOVE WITH
ANY GIRL. IN LIFE

I HATE GIRLS*
I LOVE BOYS ONLY Sajit
3/6/ 2012
8. 10 AM

Dhanyawadam — Thanks.

① Ni perandi   wats your name ?

Adhi ② A'Wab pen   — My pen

③ Dine rate ...anta — How waths the cost

④ Bhagunnava
   Bhaunava   — How you are fine .

⑤ Minna ekkada untana ru — where are you living .

⑥ Mitho. Frendship cheyarha. — I want to be your friend.
                                        - waka .

⑦ Meru a college olo chatichadu vu thunna ru.

6-9 — Calculus
9.30-1 — M-S
2-3.36 — Production
6 — 8.30 thermal)

6-9 — PI
9-11 — TR
2-5.30 — TOM
6 — PI — PI

$$Z \theta = \frac{P \sin\theta (\cos\theta + \sin\theta)}{S l}$$

$$\frac{\partial Z}{\partial \theta} = 0$$

$$\frac{\partial}{\partial \theta} (\sin\theta (\cos\theta + \sin\theta))$$

Anohul.
9028372813